HYBRID IMMERSIVE MODELS FROM CUBICAL PERSPECTIVE DRAWINGS

MODELLI IBRIDI IMMERSIVI DA DISEGNI IN PROSPETTIVA CUBICA

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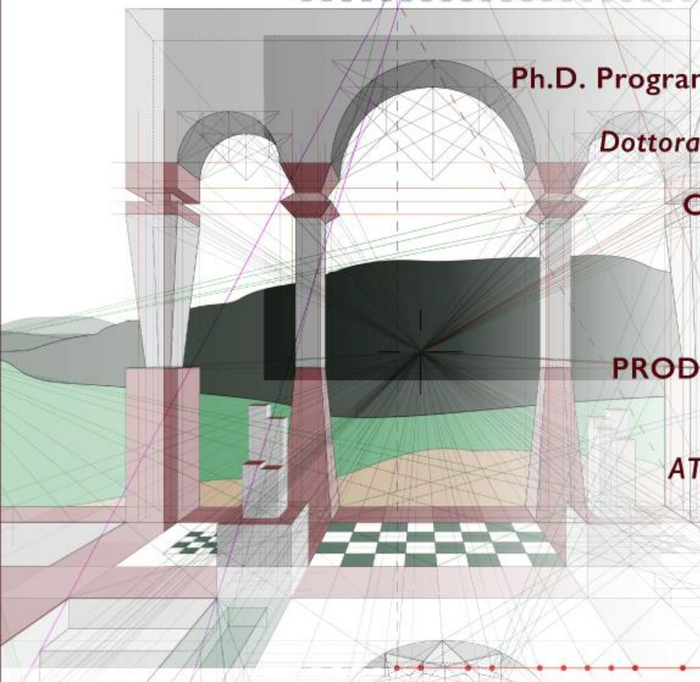
DOCTOR EUROPAEUS



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Hybrid Immersive Models from cubical perspective drawings

Modelli Ibridi Immersivi da disegni in prospettiva cubica

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SECOND PART

THIRD PART

SUMMARY (ITALIANO)

Il lavoro di tesi qui presentato sollecita un processo rappresentativo prima che illustrativo. Pone, infatti, in primo piano l'idea di rappresentazione come forma di conoscenza. Si serve perciò di concetti, estratti/astratti dalle analisi o dalle ipotesi, per manipolare letture sul costruito (cultura del rilievo) o, inversamente, verificare ritmi di composizioni ideali (cultura del progetto). A questo fine adotta una metodologia di cooperazione digitale (VR, AR). Utilizzando i principi di una buona pratica sperimentale integra tecnologie digitali e analogiche al fine di rivitalizzare i fondamenti scientifici della rappresentazione alla luce degli sviluppi informatici. Sulla scia delle premesse formalizzate dalla Scuola di Hochschule für Gestaltun, sviluppa la capacità squisitamente umana, di liberare l'idea dai limiti della materia, per ragionare sulle potenzialità che ne derivano. Affronta, perciò e innanzitutto, la riflessione sui modi attraverso i quali trascrivere nel piano (2D) il prodotto di una selezione critica filtrata dall'esistente (3D) o dallo spazio immaginato. Per far ciò si ricorre alla tradizione disciplinare: dallo schizzo autografico avanza verso la decodifica delle descrizioni allografiche, La procedura attraversa perciò la sequenza di rappresentazioni simbolicamente crescenti per costruire sulla carta il modo complesso e articolato della conformazione degli spazi e delle configurazioni dell'edificio sistema grafico-matematico di riferimento. Per verificare gli esiti si ricorre all'immersione degli spazi simulati.

Consapevoli che non basta il dominio delle tecniche digitali per cogliere le potenzialità delle rappresentazioni tradizionali e avanzate si ribadisce l'utilità di un grimaldello per gli esperti del settore: la geometria proiettiva.

Per soddisfare gli obiettivi si è utilizzato un metodo induttivo.

Dalla conoscenza acquisita mediante il contatto diretto con un determinato aspetto della realtà, si abbozza una sintesi dell'esperienza: bordi, linee e aree includono indicazioni di

profondità relativa all'osservatore, esattamente misurate (ed è questa una delle novità promosse) nelle restituzioni prospettive che a distanza si rielaborano.

In questo modus operandi, tra mano e mente, si interpone una procedura automatizzata, quella dettata dalle istruzioni del software deputato. La programmazione delle fasi limita senza dubbio la libertà dell'operatore ma nel contempo genera un nuovo assetto operativo, funzionale allo studioso attento. Le regole prescelte e imposte diventano un aiuto all'organizzazione dei dati per guidare le analisi sul già fatto o in alternativa orientare le sintesi sul da farsi.

Lo studio, pertanto, è stato organizzato in cinque unità: lo stato dell'arte, la prospettiva cubica, l'approccio olistico, le applicazioni e i risultati raggiunti/attesi. É stato definito un obiettivo specifico per ogni fase quindi strutturato lo sviluppo della ricerca in obiettivi secondari quali: raccogliere concetti fondamentali e metodi esistenti per la cooperazione digitale VR e AR; coprire il vuoto esistente nel campo disciplinare, ovvero, definire una ampia casistica di casi geometrici atti a fornire definizioni sistematiche per disegnare prospettive cubiche con strumenti semplici, sia da osservazione diretta che da schemi grafici presentati in pianta e sezioni associate; applicare il modello ibrido per analizzare le caratteristiche degli edifici esistenti e verificare ipotesi di progetto in modalità immersiva; sviluppare dei prototipi che aiutino a leggere l'impiego e l'utilità dei metodi proposti; e infine, verificare l'impatto del metodo attraverso campioni prototipali e a seguire ò loro capacità comunicativa dei dati raccolti e rielaborati.

Trascrivendo questa organizzazione delle fasi operative sul piano dell'articolazione del testo è parso opportuno articolare la tesi in tre parti contenenti due capitoli ciascuno di preminente carattere teorico seguita da una sezione pratica operativa.

In questa luce, ***il primo capitolo*** *(First Part CHAPTER I) riassume i principi e criteri alla base della prospettiva conica quattrocentesca, estraendo rapporti e relazioni fra gli elementi essenziali di ogni sistema di rappresentazione, ovvero l'osservatore, la forma dell'oggetto e la* *superficie di proiezione. Tali principi sono stati scelti con l'oggetto di essere ripresi in una fase successiva per lo sviluppo dei propri metodi per disegnare prospettive cubiche.*

Nel panorama storico, l'anamorfosi emancipa le proiezioni coniche del piano di proiezione della prospettiva lineare classica, ed estende le proiezioni coniche a situazioni con drastici cambiamenti di angoli, a casi che utilizzano più piani e a immagini proiettate su superfici irregolari. Nelle accezioni attuali, l'anamorfosi viene presa come il caso più generale delle proiezioni coniche, ovvero come il rapporto fra gli oggetti nello spazio e l'osservatore prima di considerare alcuna superficie.

A sua volta, la prospettiva cubica parte della prospettiva conica lineare e ne conserva alcune delle sue proprietà, ma ne altera altre come conseguenza dello scambio di superficie di proiezione. L'obbiettivo finale dietro tale sostituzione è quello di permettere una visualizzazione virtuale immersiva (VR), dove l'utente può percepire la raffigurazione di un'architettura come se fosse all'interno di essa, ovvero come parte attiva dello spazio percepito nonché con una allargata esperienza sensoriale.

A partire di questi parametri di riferimento, sono state indagate le specificità sulla superficie di proiezione, il centro di proiezione e l'area d'osservazione, l'occlusione radiale e procedure che definisco immagini e rappresentazioni. Inoltre, sono state prese in considerazione le variazioni di superfici proposte dalle anamorfosi, i panorami del diciottesimo e diciannovesimo secoli, e le prospettive sferiche, concentrando sempre lo sguardo sulla triade osservatore/forma dell'oggetto/superficie di proiezione. È stata particolarmente presa in considerazione la prospettiva sferica equirettangolare (I.6), marcatamente consolidata negli ultimi tempi e che oramai conta con un forte sviluppo in letteratura, un'accertata attendibilità e una larga casistica di applicazioni. Sono stati indagati soprattutto metodi per il disegno in tale prospettiva, in particolare per quello tradizionale e più in generale le alternative software per costruzioni raster (automatizzato e non).

Dall’analisi delle tecniche e dei metodi provenienti dalle proiezioni coniche è emersa una conoscenza di riferimento dunque traferita in forma analoga – con le riserve del caso – alla prospettiva cubica.

Nel **secondo capitolo** (First Part CHAPTER II) è stata proposta una definizione e posizionamento del modello ibrido immersivo (HIM) nel campo disciplinare del disegno. A tale fine sono state prese in considerazione delle definizioni esistenti e delle ricerche pregresse condotte da studiosi del settore come riferimenti essenziali (ad esempio, i modelli dinamici e interattivi).

Gli HIM legano il disegno tradizionale ai vantaggi delle tecniche immersive digitali. Il primo si configura come un’impressione grafica unica, uno strumento per riflettere mentre si crea, e che stimola lo sviluppo del ragionamento critico durante l'analisi o l'ideazione di un'architettura o di un prodotto. Da sua parte, le tecniche digitali, ad oggi, costituiscono la tecnologia innovativa e potente per raggiungere il massimo grado di immersione e interazione collaborativa.

Ulteriormente, sono stati descritti altri aspetti che fanno ai modelli ibridi, tali come le conoscenze di base che si necessitano per la sua elaborazione e la promozione e diffusione del modello come conoscenza aperta. Infine, si sintetizzano le potenzialità e caratteristiche dell’approccio olistico per la rifondazione intellettiva dell’operatore che osserva la stessa realtà con un nuovo sguardo e in modalità immersiva, dove “analogico” e “digitale” agiscono non più come tecniche antagoniste ma come forma di pensiero correlato.

Nel **terzo capitolo** (Second Part CHAPTER III) si raccolgono teorie e metodi di proiezioni coniche che utilizzano in alternativa al quadro della rappresentazione (prospettiva classica) una superficie articolata nello spazio derivata dalla proiezione di un poliedro. In questo capitolo e nelle sezioni I.4, I.5 e I.6 si indagano applicazioni che hanno spinto e preceduto l’immersione totale, proprietà perseguita storicamente dai panorami ma che ha raggiunto un apice di espressione soltanto grazie alla attuale realtà virtuale. Si analizza quanto presente in letteratura articolando lo studio in **tre gruppi**: le Perspective Box (scatole prospettiche), il Cubic Environmental

Mapping *(mappatura cubica di ambienti e ulteriori variazioni); e metodi intuitivi utilizzati per sviluppare la prospettiva cubica.*

Nel ***primo gruppo (III.2)*** *si prendono in esame antecedenti storici di tecniche manuali per la produzione di raffigurazioni parzialmente fruibili in modo immersivo. Nello specifico, è stato indagato l'esempio storico degli aggeggi del diciassettesimo secolo chiamati* Perspective Box *(danesi),* peeping karakuri *oppure* nozoki megane peep-box *(giapponesi). Tali dispositivi (poliedrici ma non necessariamente cubici) trovano l'osservatore in un punto determinato (il* peephole*) ma non necessariamente al centro geometrico dell'artefatto.*

Nel ***secondo gruppo (III.3)*** *sono state indagate le tecniche automatizzate per la fruizione del prodotto immersivo. Si focalizzano particolarmente applicazioni informatiche che utilizzano un cubo come superficie di proiezione e che trovano il fruitore in un punto necessariamente al centro di esso. Tale impostazione, proposta inizialmente da Ned Greene nel 1986 come forma alternativa di renderizzato informatico di ambienti virtuali, è apparsa come possibilità al panorama equirettangolare utilizzato come quasi l'unica opzione fino a quell'allora. La proposta di Greene dovette attendere qualche anno fino all'implementazione di hardware necessaria, ma rappresentò una rivoluzione nel campo dei CGI quando iniziò ad essere utilizzata, ed è così che ad oggi quasi tutti gli ambienti virtuali in cui navighiamo vengono convertiti in mappa cubico prima della sua visualizzazione, facendo uso della leggerezza di risorse che esso offre e la sua ottimizzata performance durante la navigazione.*

Fra le applicazioni di questo gruppo vengono raccolti lavori del mondo dei video giochi storicamente sviluppati alla fine del ventesimo secolo. In questo caso, il cubo si utilizza in rapporto ad una sfera per velocizzare e semplificare il così chiamato environmental mapping *o* reflection mapping. *Tale rapporto cubo/sfera ha avuto un forte sviluppo e diversi software fanno oggi la conversione fra uno e l'altro in forma semplice ed automatizzata.*

*Inoltre, sono stati inclusi in questo secondo gruppo alcuni artefatti CAVE (*Cave Automatic Virtual Environment*), ovvero apparecchi di disegno immersivo che non considerano le solite operazioni di proiezione sul piano, ma sono invece dei metodi automatizzati che utilizzano direttamente lo spazio immersivo digitale 3D come medium, creando un modello dinamico e interattivo a partire del seguimento wireless della traccia in aria dell'operatore.*

Infine, nel ***terzo gruppo (III.4)*** *sono state raccolte delle applicazioni strettamente collegate ai nostri sviluppi, ovvero esempi di metodi per disegnare sull'atlante cubico. In questo caso, il medium è sempre il piano ma si segue un atlante cubico che dopo verrà "ricomposto" nella forma 3D tramite l'utilizzo dei mezzi digitali. Si mantiene il rapporto cubo/osservatore precedente, ossia l'osservatore è in un punto necessariamente al centro dello spazio cubico.*

Il disegno cubico è stato spinto da operatori appartenenti al settore del disegno grafico che, richiamando la conosciuta azione di conversione fra cubo e sfera, approfittarono le bontà di entrambe le mappature per illustrare – anche di forma intuitiva – restituendo così un panorama a tutto campo. Ad oggi, la stessa conversione cubo/sfera è anche spesso utilizzata nella rappresentazione dell'architettura per il fotoritocco ed edizioni minori post-acquisizione di panorami fotografici e/o ottenuti da altri modelli di rilievo (ad esempio per cancellare il treppiede).

Da queste applicazioni temprane nacquero i primi tutoriali con principi essenziali scoperti a prova ed errore, ovvero metodi approssimativi, non normalizzati e scarsamente sviluppati di forma esaustiva. Questa conoscenza, tronca nelle sue definizioni in termini scientifici, segnalò la strada da fare con la ricerca, sia per la vigenza attuale del mezzo cubico ma anche perché la rappresentazione cubica risulta tipica del mezzo digitale, non avendo avuto motivi sufficienti per esistere prima dell'era informatica e per tanto ancora in via di sviluppo. La ricerca trovò per tanto il suo posto tra le scarse procedure scientifiche per il disegno cubico, e cercò di riempire il (quasi totale) vuoto di metodi sistematici con l'elaborazione di una casistica di soluzioni verificate con l'ausilio matematico.

Dallo studio dello stato dell'arte sono stati inoltre estratti concetti simili a quegli che si trovano alla base dei due metodi presentati nel capitolo successivo. Infatti, in questo gruppo e negli esempi della sezione di disegno con prospettive sferiche (First Part I.6.2) è stato indagato il rapporto fra sfera e cubo di forma analoga a quella che è stata di seguito utilizzata per le definizioni del secondo metodo (Second Part IV.5), nonché studi e fondamenti che richiamano le bontà informatiche per la rappresentazione immersiva.

Nel ***quarto capitolo*** *(Second Part CHAPTER IV) si inserisce la prospettiva cubica tra le modalità di rappresentazione grafiche fisse ma interattive. Infatti, la prospettiva cubica è un sistema relativamente recente che, pur basandosi sui criteri delle proiezioni centrali e i principi della prospettiva lineare, si sviluppa secondo una teoria autonoma che il presente studio cerca di ordinare e sviluppare alla luce delle conoscenze attuali. A tale fine si riprendono dei concetti dei capitoli precedenti per arrivare ad una descrizione scientifica più esaustiva, includendo le caratteristiche dell'atlante cubico ed una sistemazione (se non definizione) del sistema di riferimento.*

I metodi presentati in questo capitolo intendono ottimizzare il disegno cubico per la costruzione di modelli che raggiungono l'immersione totale. Una delle condizioni fondamentali in questo senso è che i limiti dell'atlante non siano visibili all'osservatore durante la navigazione, ovvero, che l'anamorfosi sia quella giusta in visione vincolata nonostante la sua apparente deformazione in visione libera. In tali condizioni, l'inganno visivo è il massimo e il visitatore coglie limpidamente l'illusione della terza dimensione durante la percezione dello spazio rappresentato. Di conseguenza è anche massimo il coinvolgimento dei sensi che perdono la nozione di differenziazione fra realtà virtuale e realtà materiale.

Uno dei problemi alla base della stesura di un'anamorfosi cubica è la frammentazione dell'immagine degli elementi proiettati (IV.2). Ovvero, quando due punti nello spazio A e B vengono proiettati di forma conica verso l'osservatore O posizionato al centro geometrico del cubo, ci sono due possibilità: la prima è che le loro immagini sul cubo A' e B' si trovino nella stessa faccia. In questo caso l'immagine del segmento AB è un singolo segmento rettilineo A'B' e la soluzione risale ai *regolari metodi per risolvere una prospettiva lineare classica. La seconda possibilità invece, è che l'immagine di uno dei due punti si trovi in una faccia diversa. In tale caso, l'immagine del segmento AB sul cubo risulta frammentata in due parti,* l_1 *da* A *a* C *e* l_2 *da* C *a* B*, dove C è un punto intermedio fra A e B la cui immagine coincide con lo spigolo fra le due facce un cui si trovano A' e B'.* l_1 *ed* l_2 *saranno due segmenti rettilinei ma con una fuga diversa dovuta all'orientamento di ogni faccia rispetto all'osservatore. Questo comporta che quando* l_1 *e* l_2 *vengono rappresentati sul cubo aperto, un minimo scostamento della direzione corretta di ogni segmento da come risultato una visualizzazione erronea durante la navigazione VR. Sono stati anche sviluppati dei dispositivi cartacei alla fine di comprendere meglio il comportamento dell'anamorfosi cubica e che, allo stesso tempo, possano aiutare a trasmettere i concetti di base di forma pratica (Second Part IV.2.2).*

Nel ***primo metodo*** *(IV.4), il problema delle frammentazioni è stato risolto considerando la prospettiva cubica come un gruppo di sei prospettive lineari indipendenti, ovvero trasferendo i punti di riferimento di ogni segmento misurati in pianta e sezioni associate. A tale fine, sono stati richiamati metodi tradizionali di composizione prospettica usando le diagonali a 45 gradi per calcolare la posizione in profondità di un punto dato. Questo metodo, seppur limitato a linee orizzontali e verticali (ovvero, parallele agli spigoli del cubo), è stato il prodotto di una prima esperienza pratica atta a risolvere il problema fondamentale della frammentazione. La procedura è un primo approccio nel passaggio dell'elaborazione della prospettiva con un unico piano (costruzione classica) al disegno nei quattro piani orizzontali (ovvero le facce panoramiche) e dunque alle sei facce al contempo. Gli esiti sono stati verificati e monitorati in modalità VR durante l'elaborazione del processo.*

Dopo l'elaborazione di questo primo approccio, si è dovuto affrontare il problema che la rappresentazione di una stessa linea può risultare frammentata fino a quattro volte, il che implica la misurazione di cinque punti (con i rispettivi errori) se si procede con i metodi tradizionali. Per tanto, l'obbiettivo del secondo metodo è stato quello di risolvere una rappresentazione omogenea, congiunta e compatta nell'atlante cubico a partire della misurazione di due punti soltanto.

In effetti, nel ***secondo metodo*** *(IV.5), la prospettiva cubica è stata considerata come un caso speciale di prospettiva sferica e con ciò si è riuscito ad andare oltre la soluzione dei singoli frammenti uno ad uno, arrivando a tracciare entrambi i segmenti partendo dal riferimento di due punti misurati sia da osservazione diretta sul posto (non contemplato nel primo metodo) che da pianta e sezione. In pratica, il metodo calcola il terzo punto (e successivi punti se esistenti) anziché misurarlo. In questo modo si evita l'induzione di errori sistematici, si mantiene una costruzione consistente, si garantisce una massima economia di riferimenti e quindi si arriva ad avere come risultato una visualizzazione VR coerente.*

Per raggiungere tale sintesi è stata considerata una superficie sferica ugualmente centrata sull'osservatore quindi paragonata con il cubo. In pratica, la correlazione fra cubo e sfera ha permesso l'unione con altre procedure per disegno immersivo già definite e verificate in letteratura, così come il trasferimento di principi matematici generalizzati per il caso sferico e con ciò la risoluzione effettiva dei problemi della rappresentazione frammentata. Infatti, una delle proprietà del disegno immersivo sferico focalizza il fatto che l'immagine di un segmento AB sulla sfera risulta nella geodetica fra A' e B'. Tale ragionamento ha offerto il vantaggio di poter utilizzare lo sviluppo della geodetica sul cubo ed elaborarla come una costruzione compatta ed omogenea in sé stessa. Oltre a ciò, se si considera il grande cerchio che contiene tale geodetica, risulta che ogni linea ha sempre esattamente due punti di fuga e che questi punti sono sempre all'interno del foglio di disegno, ovvero sono punti propri.

Per le prospettive curvilinee, tali come l'equirettangolare o l'azimutale equidistante, pur essendo più fluide che il caso cubico (nel senso che ci sono meno interruzioni nella mappatura) rimane ancora il problema del disegno curvilineo, il che fa l'elaborazione più complessa e ne toglie precisione. La prospettiva cubica risolve tali questioni, aggiungendo semplicità e precisione nel tracciamento delle geodetiche, grazie all'utilizzo di linee rette e la possibilità di adoperare con elementi semplici come il righello.

Pertanto, in questo metodo il problema della rappresentazione sull'atlante cubico si è centrato nello sviluppo e tracciamento delle diverse geodetiche. A tale fine, è stata sviluppata un'ampia casistica preliminare per comprendere i possibili metodi per disegnare nella mappa cubica. Quindi sono stati ristretti i casi in due gruppi: geodetiche di quattro e di sei lati. In sezione IV.5.3.2 si sviluppano tutti i casi particolari, dal più semplice (ovvero geodetiche di quattro e sei lati con A e B nella stessa faccia), a quello più complesso (ossia geodetiche di quattro e sei lati con A e B in facce diverse).

Inoltre, sono stati definiti dei metodi per tracciare punti all'interno dell'atlante facendo uso delle geodetiche e considerando i suoi due angoli caratteristici, ovvero λ (longitudine) e φ (latitudine). Sono state inoltre prodotte delle griglie di riferimento e specificate le sue costruzioni utilizzando i punti di fuga a 45°*, punti che risultano di facile localizzazione nella mappa cubica perché conservano una posizione costante e determinata per una stessa fuga.*

A questo punto della ricerca si era già stabilito un panorama abbastanza completo dei vantaggi e svantaggi dell'utilizzo del cubo rispetto ad altre prospettive immersive. Facendo focus negli svantaggi, sono stati sviluppati ulteriori approfondimenti atti a migliorarli ed ottimizzare le procedure sviluppate (IV.6 ed esercizi in sezione V.2.1). In particolare, una prima scorciatoia, sviluppata e verificata analiticamente, semplifica la forma di trovare il punto ausiliare C (ovvero quando A e B sono in facce adiacenti). La procedura evolve dalle prime operazioni di ribaltamento e proiezione ad un'unica costruzione con meno passaggi, più semplice ed auto contenuta fra A e B. Durante la verifica analitica è stato trovato che si trattava di una costruzione che richiama la stessa costruzione sviluppata in precedenza (IV.5.3.4), ma stavolta proiettandola di forma parallela su tre piani diversi: una volta su ogni faccia, e la terza sopra un piano posizionato a 45° *(IV.6.2). La costruzione verifica sia per geodetiche di quattro che di sei lati, semplificando il metodo ad un'unica costruzione.*

Ulteriori ottimizzazioni della procedura di disegno sono: la formalizzazione di alcuni punti caratteristici della mappa cubica come centri di rotazione per traslare corrispondenze fra gli spigoli più scollegati, ovvero dietro/sopra e dietro/sotto (IV.6.3), costruzioni per la ripetizione di *elementi regolari, e per il tracciamento di una geodetica di sei lati a partire da un punto misurato ed un punto di fuga. Queste ultime due mancano ancora della verifica analitica, ma hanno avuto successo ad una prima verifica grafico-vettoriale con costruzioni fatte su GeoGebra.*

Il ***quinto e sesto capitolo*** *(Third Part CHAPTER V e CHAPTER VI) si presentano emblematici casi studio in cui sono stati applicati i metodi e le scorciatoie matematiche precedentemente analizzate. Nel dettaglio: la fabbrica di Ceramiche Solimene a Vietri sul mare (cultura del rilievo) e la conformazione di ritmi ideali (cultura del progetto). Segue la ricostruzione remota e filologica della Chiesa di San Michele in Hildesheim, Germania e alcune sperimentazioni di utilizzo al design.*

Nello specifico, il ***primo*** *è orientato alla sperimentazione dei concetti di base per il disegno in prospettiva cubica, con un ulteriore verifica nella costruzione di uno spazio ideale (cultura del progetto). Per tanto, sono stati concepiti esercizi di complessità crescente atti a trasmettere la teoria precedentemente mesa a punto (V.2.1). L'operatore è quindi messo nelle condizioni di ideare una legge modulare per verificare la qualità dei rapporti e delle proporzioni studiate in piante e prospetti associati (metodo delle doppie proiezioni ortogonali). Partendo dagli schemi ideati, si sono costruite le prospettive cubiche, utilizzate poi come prototipi di spazi virtuali. Il contenuto viene verificato in modalità immersiva utilizzando ed adattando il work-flow dei software per il montaggio di fotografie immersive.*

A sua volta, il ***secondo*** *caso studio condotto presso la fabbrica di ceramiche Solimene a Vietri sul Mare, in Italia, va dall'ambiente reale, passa per la lettura delle leggi modulari sintetizzate in pianta e sezione e arriva alla verifica in modalità immersiva (cultura del rilievo). Il work flow di lavoro è partito dall'acquisizione di molti scatti individuali sul posto poi "cuciti" in una foto panoramica a tutto campo. Successivamente questo panorama è stato convertito al formato cubico e utilizzata come base per tracciare, evidenziando gli elementi più risalenti della geometria dello spazio, le sue relazioni e proporzioni. Quindi, sono stati interpretati criticamente i dati ottenuti e infine l'analisi è stata verificata in modalità immersiva.*

Utilizzando i principi di una buona pratica sperimentale, si confrontano gli esiti elaborati, proponendo nel contempo un dibattito sulle potenzialità dei campioni costruiti e virtualmente manipolabili.

Dalla verifica delle soluzioni matematiche applicate ai casi studio (cultura del progetto) è stato possibile passare alla conoscenza acquisita mediante il contatto diretto del costruito (cultura del rilievo) al fine di normalizzare le immagini equirettangolari, derivate dall'acquisizione di panorami sferici, in atlanti cubici su cui si selezionano elementi strutturali da mappare vettorialmente al fine conoscitivo comunicativo.

Per ultimo, nel capitolo Third Part VI.2 si presentano ***algoritmi*** *creati a partire delle conoscenze dei metodi del capitolo IV. Alcune delle definizioni matematiche sono state utilizzate per creare dei frammenti di software (ancora in sviluppo) atti ad automatizzare i processi di disegno cubico. Con partenza nel primo metodo si introduce un primo algoritmo completo per tracciare linee verticali e orizzontali e cioè, parallele agli spigoli del cubo. L'applicazione dell'algoritmo è stata verificata in modalità VR per una scena in cui l'osservatore ha un edificio avanti e uno dietro, entrambi con altezze le cui immagini sul cubo risultano frammentate in più di una faccia.*

Basati sul secondo metodo, sono stati creati ulteriori algoritmi con JavaScript e GeoGebra, fra cui uno script per ottenere di forma automatizzata la geodetica dati due punti in facce adiacenti (e cioè, per il caso più complesso). Questi due ed altri script si sono iniziati a compilare in una prima versione unificata che ne raccoglie a tutti quanti: la previsione del software finale introduce, oltre al disegno cubico, la possibilità di vedere un punto localizzato sulla mappa cubica, la sua posizione nello spazio e anche su un piano frontale (ovvero in una prospettiva classica) allo stesso tempo. È inoltre possibile modificare parametri del campo visivo dell'osservatore. La costruzione rappresenta una base per ulteriori sviluppi magari da essere finalmente sviluppata da specialisti IT in una fase successiva.

Prime conclusioni

A fronte di operazioni intuitive, ispirate a modi soggettivi di impostare e condurre a soluzioni problemi, il modello proposto consente di interagire con il sistema di aspetti della realtà presi in esame.

Nel ribadire i motivi per i quali si privilegia il disegno a mano e la costruzione virtuale dello spazio cubico, l'essenzialità degli schizzi mantengano la comunicazione ad un livello basso e costante; così che l'esemplificazione evita il sovraccarico percettivo, favorendo l'attenzione sulla strutturazione consapevole degli elementi configurati, poco importa se reali o ideali. La procedura è capace di compenetrare la figurazione di pochi e semplici schemi grafici, fornendo la chiave per manipolare un modo complesso e articolato qual è la costruzione in digitale e per traslato il cantiere reale. Malgrado la fissità della posizione del fruitore inchiodato al centro dello spazio cubico (e per traslato in quello sferico), lo sguardo è libero di spaziare a tutto campo per filtrare fattori che possono contribuire a definire la qualità di quanto appreso e classificato.

La procedura inoltre offre la possibilità di procedere ad una modellazione veloce e semplificata 3D (di consueto assai più elaborata poiché sostenuta da diversi obiettivi) funzionale alla rifondazione intellettiva derivata dalla mutazione delle relazioni tra elementi invarianti studiati nelle loro dimensioni rapporti e proporzioni assoggettate ai criteri di normalità e convenzioni di cui necessita il disegno di architettura per garantire l'esatta trasposizione delle idee in ipotesi di fattibilità costruttiva.

Procedendo in questa direzione si accenna, sia pure indirettamente, ai mutamenti capaci di essere forieri di risultati innovativi, derivati dalla necessità di tradurre codici grafici in codici digitali. L'astrazione numerica, garantita dal software, aiuta a generalizzare concetti: parametrizzando le variabili astrae criteri e principi così che, nella memoria del computer, una retta è una particolare linea manipolabile, non più solo ed esclusivamente luogo geometrico assume i connotati di linea generatrice di forme libere di qualunque natura (NURBS). Le *procedure entro le quali il pensiero si formalizza non sono indifferenti all'esito: lo ricordava nel lontano I secolo Plinio il Vecchio con il celebre racconto sul vasaio corinzio Boutades, lo confermano le tecnologie che, integrando funzioni di calcolo con funzioni di archiviazione e dunque comunicazione-consultazione, organizzano i dati, pianificando categorie entro cui definire classi di possibilità per guidare le analisi sul già fatto e verificare il da farsi. Se, dunque, la rappresentazione governa la modificazione del dato rilevato o progettato, il prototipo digitale costruisce un modello in divenire.*

Tra le occasioni colte restano alcune difficoltà imposte dal passaggio digitale/raster e viceversa: sebbene tali passaggi non stati ancora definitivamente sviluppati, è oramai possibile la definizione di una procedura completamente automatizzata richiamando la casistica definita in questo lavoro (ma in ogni caso, tale compito sarebbe il lavoro di supporto di un informatico). Il riconoscimento visivo del contenuto iconico può infatti indurre errori riducendo i vantaggi di una rappresentazione numerica che si risolve ricorrendo all'ausilio di medie calcolate. Sta di fatto che il modello rappresentativo tradizionale necessita ancora, per essere veramente integrato nel modello digitale (in modo che l'uno sostenga l'altro), di nuovi e ulteriori sviluppi. Diverse appaiono le categorie mentali da adottare anche quando occorre trascrivere nel linguaggio binario elaborati in applicazione delle regole classiche. La maggiore astrazione, infatti, consente di rimuovere vincoli e raggruppare compiti, isolare insiemi più consistenti di classificazioni. Operazioni queste che spingono verso il riscatto della rappresentazione manuale.

Parole chiave: *prospettiva cubica, immersione, modelli interattivi, rilievo architettonico, progetto architettonico, strategie di rappresentazione numeriche/analogiche, studio e divulgazione digitale*

ABSTRACT

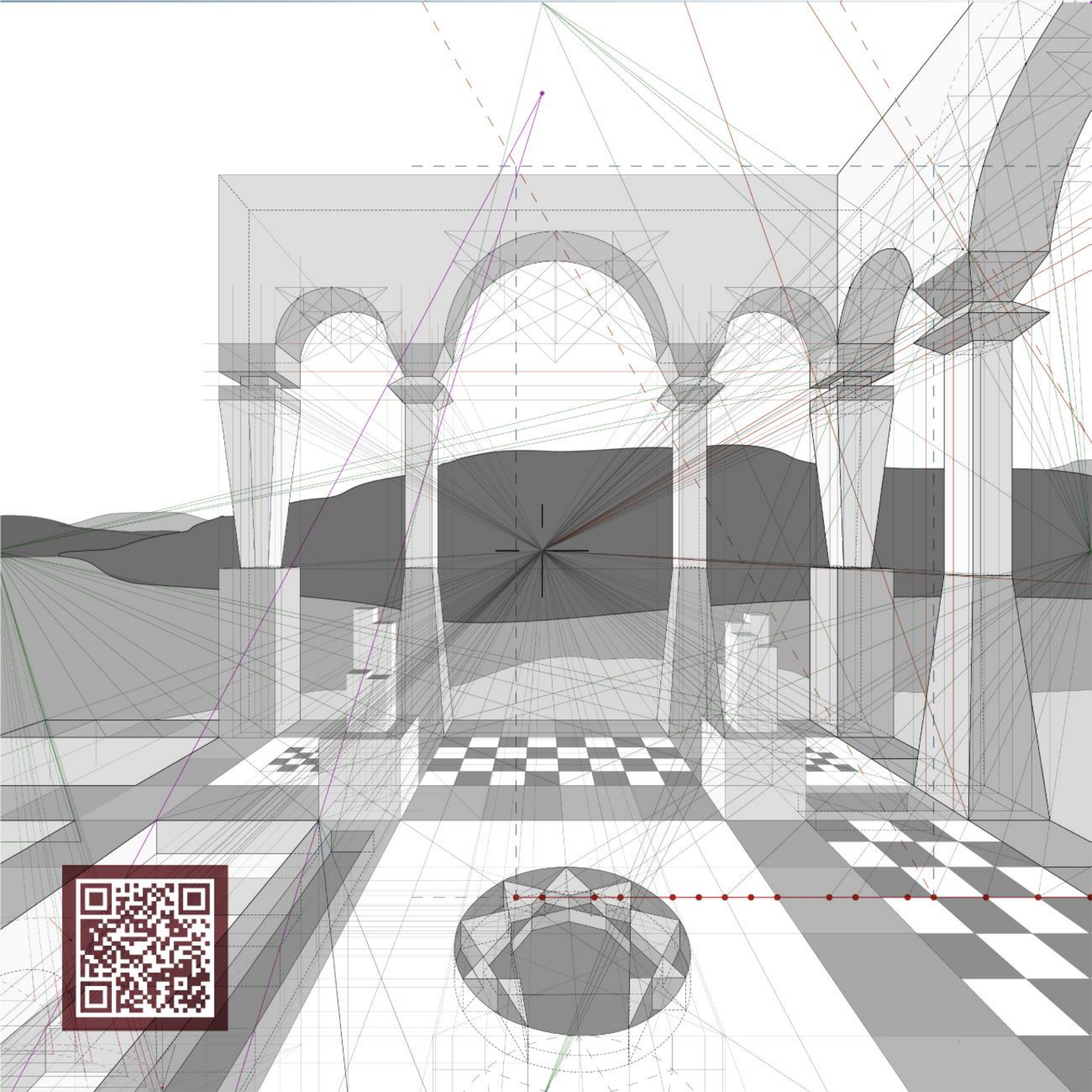
A digital cooperation methodology based in the scientific definition of cubical perspective is presented. The research gathered new methods for cubical perspectives drawing with their final VR fruition through digital techniques. The developments belong to the disciplinary field of drawing and are reinforced with definitions from the mathematical one. The utility of the approach was tested through cases study.

Among the outputs, the investigation defined the "hybrid immersive model" (HIM), a model that joins traditional drawing with digital immersive techniques. As well, the research developed new theories and methods for cubical perspective, starting to fill an existing gap in the field of drawing. To such an end, it organized two methods with systematic definitions for drawing cubical perspectives by using simple tools, both from direct observation and from associated floorplan and section. Specifically, the second method transferred several properties already in use by spherical perspectives (e.g., the use of exactly two vanishing points per line) within the compact and linear setup of the cubical map.

Four cases study were conducted, highlighting the advantages of a holistic vision in immersive modality, where both traditional and innovative techniques are a correlated way of thinking and not antagonists anymore. The applications developed prototypes using the new methods for cubical drawing, covering both survey and project cultures, and showing the impact of the procedure for the study, analysis and spread of architectural data.

Hence, the research used representation as a way of knowledge and study verified in the architectural composition plane. The operator highlighted and interpreted relations among extracted/abstracted elements using the hybrid model. Although, such a personal analysis resulted methodologically comparable in the plane of representation, revealing the procedure as a useful tool for the philological reading, the critical analysis of the spatial geometry' relationships and the definition of modular laws.

Keywords: *cubical perspective, immersion, interactive models, architecture survey, architecture design, numerical/analogical representation strategies, digital study and dissemination*



HYBRID IMMERSIVE MODELS FROM CUBICAL PERSPECTIVE DRAWINGS

MODELLI IBRIDI IMMERSIVI DA DISEGNI IN PROSPETTIVA CUBICA

INTRODUCTION

When this research started in November 2017, I expected to focus on the pros and cons of handmade immersive applications using the cube as projection surface. Yet, I would never expect that cubical perspective might represent a gap in the field of representation: I found several and well-developed approaches for using cubical environmental mapping for rendering purposes (Second Part III.3), a few examples for drawing by trial and error procedures in the cubical map (Second Part III.4), but I found no systematic methods for cubical drawing. Still, the gap was there during the definition of the state of art, so the investigation ended also defining the cubical perspective in a systematic way.

For a systematic method I mean an efficient procedure for illustrating an unfolded cube, that is, a system for representing using perspective's principles within that map. Once digitalised, such a drawing can be folded back in a 3D articulated surface surrounding the observer. Thanks to such a setup, to draw within a cube means to draw the whole environment around the observer, and this implies using all vanishing points at the same time. Thus, a proper method should include an efficient mathematical characterisation for the management of those vanishing points, groups of lines, rules for repeating elements, creation of grids, etcetera.

Still, I do not want to fall in the vanity of affirming the total inexistence of full procedures. I prefer to walk on tiptoe, leaving the door open to the possibility of a method described in some dusty book, hidden in the library of a remote Benedictine abbey in the Piemontese Alps. If such material exists, the content here presented should keep fresh its originality anyway, for mainly two reasons: the mathematical approach and the cases study's applications. Indeed, cubical perspective alone would have not been enough for a PhD research in the field of representation: there was also the big challenge of applying such definitions in an innovative way, so to have a clear impact within the scientific field of architectural drawing.

Both goals, I should confess, turned the investigation quite challenging, but the key to find the courage for facing it was teaming up: on the one hand, with **Adriana Rossi**, whose knowledge about the Italian disciplinary field of drawing made me understand the variables that I was playing with: she helped me to see the utility of the product (applying it critically for both survey and project), historical aspects of the investigation and what is expected from an academical research within the representation's field. On the other hand, with **António Bandeira Araújo**, who gave me the necessary mathematical knowledge to think in terms of the visual sphere surrounding the observer (Figure 1), and the background to join cubical and spherical perspectives. This international team enhanced the research's strengths with experience, specialised knowledge, a wide range of disciplines and a multicultural vision.

The methodology used for carrying on the investigation included the definition of the state of the art, the delineation of new theoretical insights and the applied research through case studies. I structured the content in six chapters plus a final section of conclusions: the current situation of conical projections (CHAPTER I), definition of the hybrid immersive model and its insertion within the disciplinary field of drawing (CHAPTER II), the state of art for immersive drawing with cubical projections (CHAPTER III), cubical perspective (CHAPTER IV), applications (CHAPTER V and CHAPTER VI).

Specifically, I gathered resources critically from a bibliographic research during the definition of the state of the art, investigating the current situation of conical projections and other existing immersive applications. For the former, I dove in the historical background of linear-conical projections, passing through the fifteenth-century perspective, the anamorphosis, cylindrical and spherical perspectives, virtual and augmented reality. The development of every unit gave me a collection of essential concepts and existing methods for VR and AR digital cooperation through drawing thus used for composing the own proposal.

On the other hand, I dug other methods for creating virtual immersive environments with drawings. In particular, I focused on those cases using a sphere and a cube as a projection surface.

A paradigmatic case that I followed among the spherical projections was the equirectangular perspective. Such a case has been fully developed recently in literature, leading to a consolidated practice for handmade drawing with a verified reliability. Following this case, I structured a system to be analogically referenced to the cubical case.

As in the treatises of Piero Della Francesca for linear perspective [1] or Barre & Flocon's book for spherical one [2], I aim (not without some ambition), to define rules and methods for plotting a cubical drawing either with simple tools, i.e., ruler and compass or by free handmade drawing, e.g., sketching on-the-spot (Figure 2). Such essential and traditional constructions put the draughtsman in a reflective position where the result is a meditated human-made procedure, output of a (new) holistic vision of the same reality (First Part II.4).

This way, I sought to enhance and promote the benefits of the traditional pencil within the full immersive representation, the same pencil that never runs out of battery and gave us a direct tool to exteriorise thoughts: *"sketching has been an essential practice in all creative and artistic processes through externalization, reflection and self-communication"* [3, p. 101]. Besides the chosen technique, *traditional drawing* itself demonstrated largely to be a way of thinking through experimentation (more than just a tool), organising ideas and shaping the project[1] [4, p. 159], [5, p. 1491] [6, p. 1].

In following a "minimalism[2] with traditional tools", I seek to define elegant and self-coherent constructions for cubical drawing. This makes possible to start the drawing with the smallest quantity of key-points thus building the image following an analytic reasoning. Such a principle

[1] As often came out during my conversations with Adriana Rossi

[2] The term minimalism has been hardly battered lately and it has currently a banalized meaning as the "minimal quantity" of actions for designing or materialising a project (e.g., the less possible furniture in a space). In other words, as the result of the minimal effort and reflections thoughts or almost without them. Instead, I recall it here in Ludwig Mies van der Rohe's logic, that is, as the last depurated action, the optimised solution after a long path of research, reflection and development

helps to ensure a consistent VR visualisation, but it also facilitates the reading or the composition of modular laws. In the former case, getting information from the reality (culture of survey). In the latter, the project might be composed by following previously defined geometric relations and proportions among elements (culture of project).

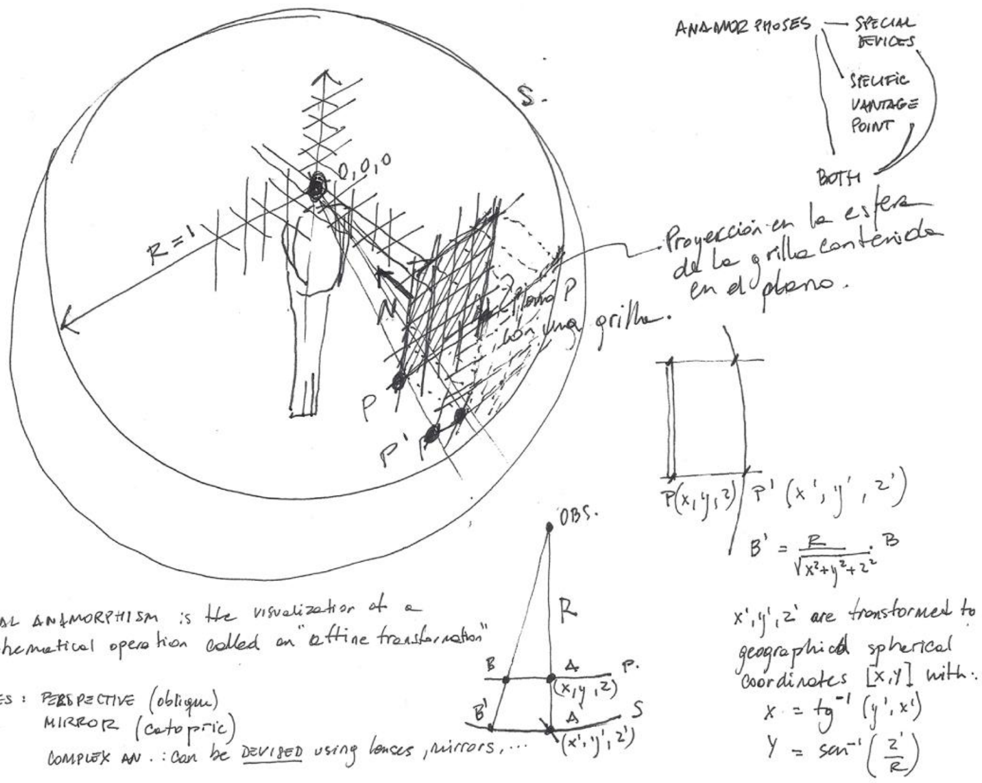


Figure 1 . Understanding the visual sphere and spherical perspectives. Drawing by the author3, 2018

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3 From now on, I write the author's name just if it is not me

Among the new theoretical insights, I positioned and defined the hybrid immersive model using previous definitions of the disciplinary field (First Part II.3). As well, I faced the analytical definition of the cubical perspective with two newfound methods: the first one (Second Part IV.4) emerged from a 2018 experimental study and re-applying Renaissance's methods. The procedure considers cubical perspective as a set of six independent linear compositions and uses floor plan and section for its construction. This method turned to be useful to understand the dynamics of the cube map, to get shortcuts and to the elaboration of a first algorithm.

The second method (Second Part IV.5), came from the alternative of centring a cube within a sphere. In such a set-up, cube and sphere may share the same graphic information; mathematically they share a topology, a homeomorphism. The purpose of this interpretation was the analogous translation (to the cube) of the general scheme already defined for other spherical perspectives and the characterisation at an anamorphic level with every line having two vanishing points (First Part I.6.2.4) [7]. Thinking the cubical drawing in space opens an attractive utility: the possibility of managing a line with whichever of its two vanishing points. This gives a new dynamic for drawing and a key for an innovative result, and therefore, for thinking the space. This property has been explored previously by spherical perspectives, but cubical perspective is the first linear case, that is, a conjunction of all the available resources previously defined for classical methods (e.g., multiplication of regular elements) with full immersive compositions.

Hence, in this method cubical perspective is considered a special case of spherical perspective, or to better say, characterised it as the cubical spherical perspective. This method can be applied for drawing and gathering points angles' information both from direct observation or from floorplan and sections.

The two methods are synthetised in short exercises that allow an efficient transmission of the theory to neophytes through a progressive passage from current classical perspective drawings to immersive ones. I also developed shortcuts for the second method, seeking for an optimisation of the procedure.

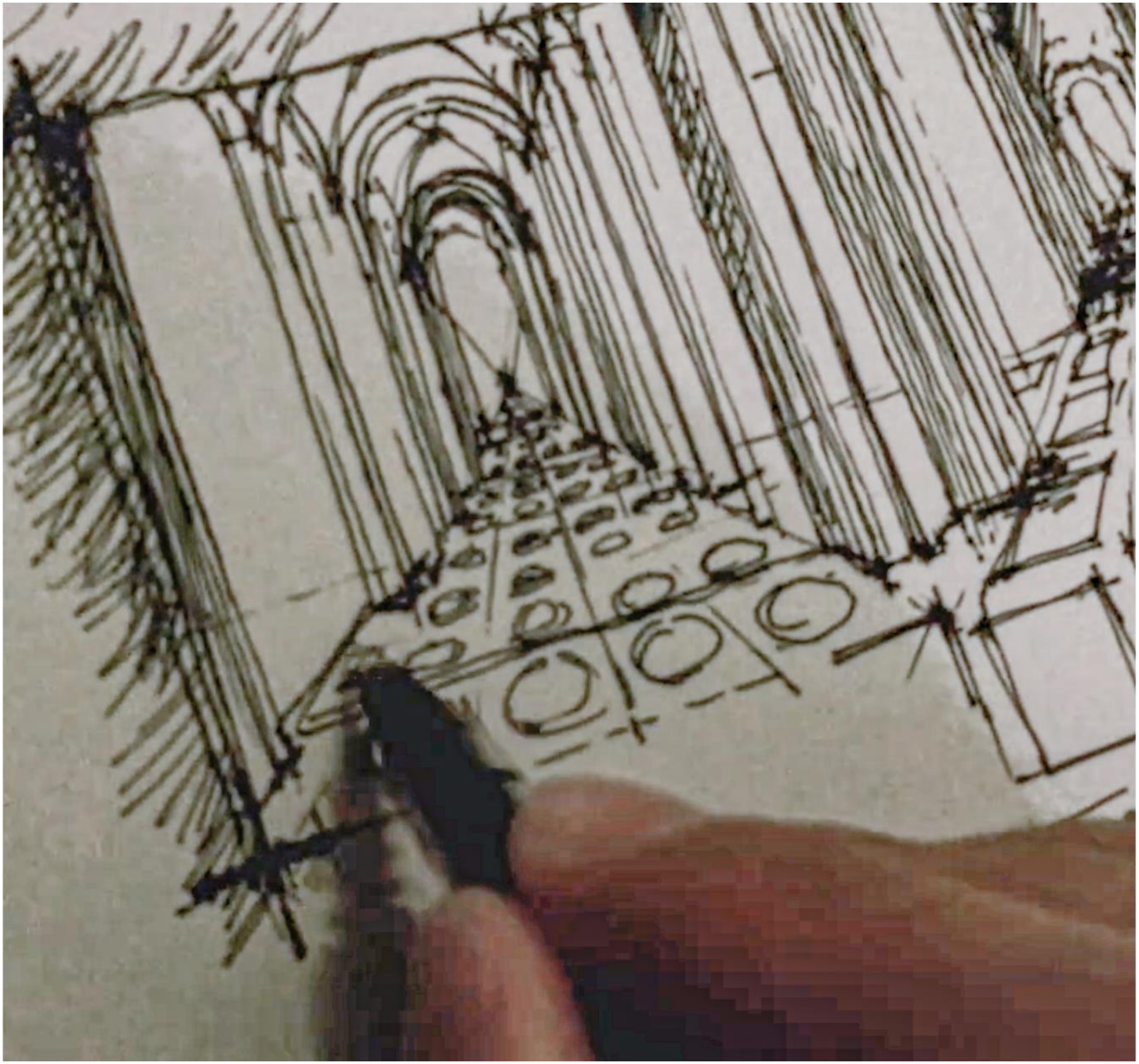

Figure 2 . Drawing a cubical perspective without instruments

As well, I develop shortcuts (Second Part IV.6) and short exercises (Third Part V.2.1). The former, seek for an optimisation of the procedure while the latter a simplification of the full theory for retransmitting the knowledge.

A further chapter introduces some algorithms (Third Part VI.2) created with Geogebra and JavaScript for the further programming of a cubical perspective software (the full software is not among the goals of this investigation).

Finally, I verify the previous definitions with applied research through applicative cases, bringing to the light the utility and reliability of the product. I described a total of four cases study: the first two developed as an interactive and circular path, the third applying for the remote reconstruction of a heritage building, and the fourth for the design of products.

In detail, the first one goes from the theory of cubical perspective up to its VR implementation (Third Part V.2). The operator learns methods for drawing, defines a geometric relation and finishes composing an ideal architecture. A second path applies hybrid immersive models for surveying the Solimene's Factory at Vietri sul Mare, Italy (Third Part V.3). In this case, cubical perspective shows its utility for reading and exploring the reality in immersive modality. The operator extracts conclusions of the architecture's composition modular laws and reports them back to floorplan and section. A third application uses available information of St. Michael's church in Hildesheim, Germany (Third Part V.4). It obtains information from a philological study and it reconstructs with that data a remote architecture heritage. Finally, a fourth example (Third Part VI.1) shows an application in product design, where the immersive model itself is the communicational and informative product.

The cases study outlined the operators' operational criteria, offering the possibility to manipulate the point of view, to examine the cognitive potential and the geometrical relations among architecture elements. The final model gives these possibilities, supporting the simultaneous and interactive perception as well.

With this research, I constantly pointed to the idea of promoting an open knowledge more than just a software tool, or, to the *knowledge that constructs algorithms in our minds*[4]. In this sense, whoever wants it, it can reuse the same algorithms and shortcuts presented here for creating a new cubical perspective software from the mathematical definitions.

As well, I gathered the necessary digital steps to mount the immersive model and watch it in VR modality (Second Part IV.7) since cubical perspective would hardly exist without CGI, and it is just with digital technology that one can reach its complete fruition. Therefore, the procedure will not be complete without a basic approach for mounting the VR environment. Yet, I do it in a generic way more than promoting a specific software, such that an operator may use any similar software and obtain the same results.

The research aims to keep a balanced conjunction between traditional and digital techniques. It is from such equilibrium that Hybrid Immersive Models are coined, that is, a model amid traditional and digital techniques that considers both as a complementary way of thinking, experimenting and doing, and not as antagonists as they were up to not much time ago. Moreover, since the research points to reasoning while drawing, I have tried to not link the knowledge to any technology in particular, leaving to the reader the option of using either traditional, digital or mixed techniques. The whole investigation intends to define a polyvalent knowledge for drawing a cubical perspective using a vector-based program, an architecture board or simple sketching on-site with nothing but pencil and paper.

Hereinafter, the results, which I wish the reader may discover stimulating as I found exciting its experimentation and writing.

Lisbon, February 2021

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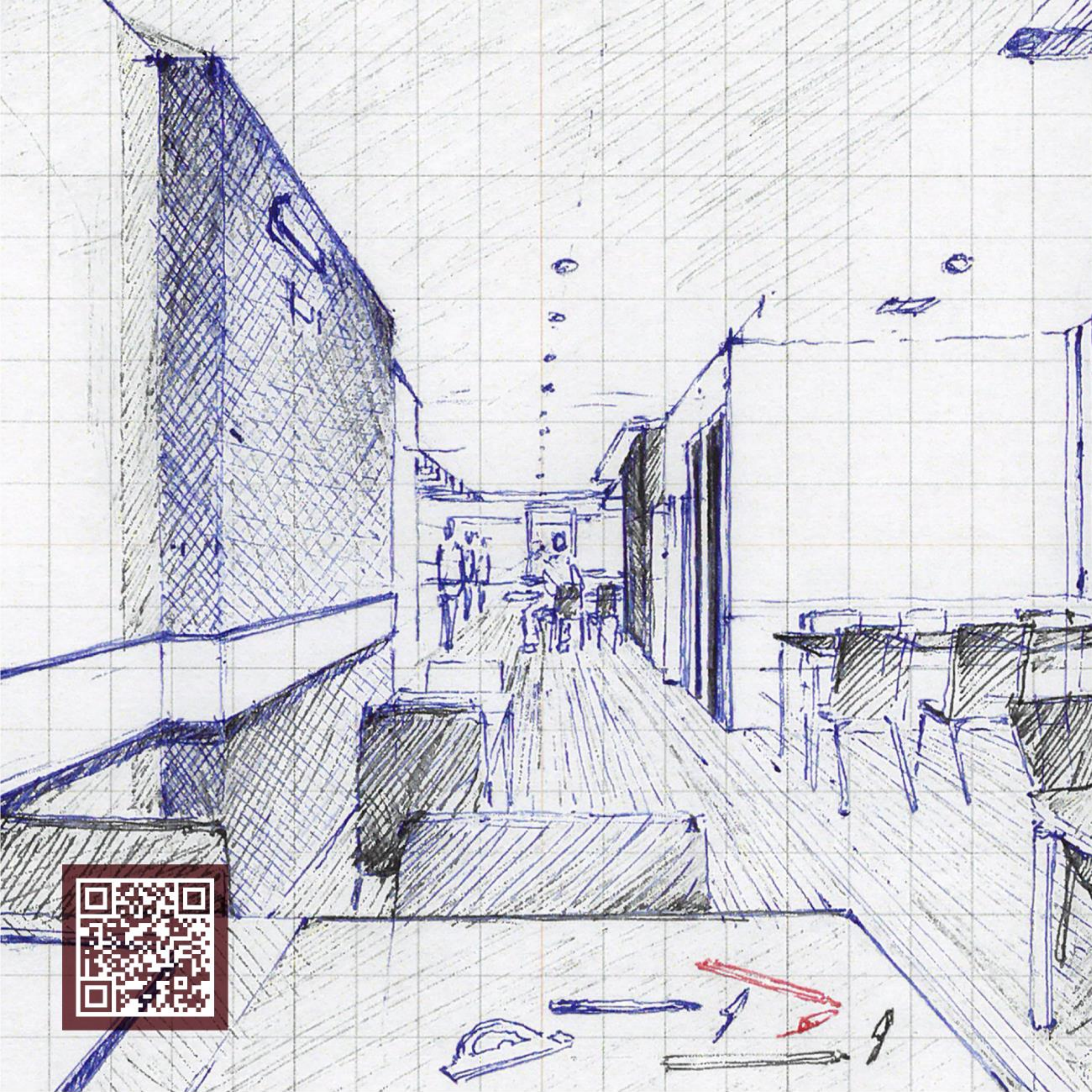
[4] Concept that António Araújo expressed during the congress UID-2019 held at Perugia, Italy

FIRST PART



CHAPTER I

CURRENT SITUATION OF CENTRAL PROJECTIONS: FROM LINEAR PERSPECTIVE TO SPHERICAL PANORAMAS

It is not easy at all for me to talk about perspective among researchers that dedicated their whole career to it. Any used word or recalled terminology seems to be omitting essential details or falling in a wrong path of interpretations. Until today specialised architects, engineers, philosophers, art historians and mathematicians deal with the very origin of perspective and its many different characteristics [8]–[13].

Nevertheless, I must face the task, since the first big part of the methodology seeks to get ***many linear perspectives*** *(in the screen) from one* ***"special kind" of linear perspective****. So, to make it fair, in the chapters to come I will try to browse the most neutral features of perspective, without the intention neither to change the specialised researcher's mind nor to give any lecture. Between all the different – and sometimes radically opposite – positions, I extract perspective's principles, criteria, as well as their cultural and scientific components. From there, I stop in the elements after reused for the cubical case. Such features indeed, help us in the second section to define the cubical drawing as a proper and independent perspective system.*

I.1 FIFTEENTH-CENTURY LINEAR PERSPECTIVE

Perspective, *to see through* from the Latin original *perspicere*, finds its origins in classical optic's studies and Euclid's geometry applications: *"Perspective is the application of Euclid's visual cone to a glass plane intersecting it*" [14, p. 6]. It aims to depict *a scene* in a 2D *plane* (Alberti's window), in such a way that when we see the composition from the observation point, it evokes the illusion of seen 3D geometries in space: "*we are meant to believe we are looking through this window into a space*" [13, p. 27].

During Renaissance, the representation system of linear perspective was born, emancipating the *Perspectiva artificialis* from the *Perspectiva naturalis.* The former provides geometric constructions to create a flat image simulating depth, while the latter deals with physical and physiological aspects of the *act of seeing* [15, p. 37], [16]. At the beginning of the 15th century Filippo Brunelleschi (1377 – 1446) defined practical principles and some years after Leon Battista Alberti (1404 – 1472) formulated the mathematical properties of such principles [12, p. 158], [14, p. 6]. The mathematical view of Alberti[5], gave to Brunelleschi's practical principles: "*a method by which pictorial space was continuous, optically accurate, mathematically coherent and related to the scale and position of the spectator*" [12, p. 158].

With the development of linear perspective, architects and designers changed their way to think the project, passing from distributing real objects in a space, to a *flat space equivalent* (Figure 3) that can exists *before* its geometry, and therefore, it can also be defined before its construction [18, p. 2]. The way perspective affected and changed our mentality was impressive and nowadays is considered "*a major event in the history of mankind*" [8, p. 2].

[5] As a parallel consequence, by the moment I started to study the cubical representations, I preannounced that working with mathematicians was a *must* for an accurate definition [17, p. 36]

In particular reference to drawing disciplinary field, linear perspective represents a first historical core of projective geometry and therefore also an important step in the history of science [15, p. 19]. Since then, the conical linear perspective using a plane as projection surface[6] has a big place among the theoretical, practical, and philosophical developments in the field of representation. It is the mimetic resource *par excellence* and almost the unique *proper* way of representing a scene in three dimensions "*at least in Occident, this method (linear perspective) became the unique criterium for constructing the figurative space since Renaissance*" [19, p. 15][7].

In particular, Hubert Damisch interpretates the many advantages of Renaissance perspective. He dives carefully in Manetti's descriptions about Brunelleschi's *tavolette* and enhances some features, such as: the importance of such experimentations moving the knowledge towards the *manipulation* of the point of view; the significant aspect that the point of view can be *deduced* from the painting, i.e., the emancipation from the mere *image* towards the elaboration of a representation system; how perspective constitutes an instrument for reasoning, a way of composing rightfully by disposing dimensions and measurements; the first step that perspective made towards the interpretation of the architectural elements' compositive order, with such an understanding clearly visible in graphical schemes; and the speculative value of the method, i.e., its utility to model and shape ideas [9, pp. 89–98], [20].

After Alberti, Piero della Francesca wrote his *De prospectiva pingendi* explaining perspective's scientific foundations and, changing in an implicit way, "*the claims for proof from Euclidean geometry to a combination of geometry, optics and surveying*" [21, p. 411]. As this book was mainly destinated to (Renaissance) artists, Piero della Francesca took care of its readability by describing two methods: the first, aimed to represent simpler elements, was connected with

[6] Hereinafter I may refer to such a setting as "classical perspective"

[7] Original quote: "*A partire dal Rinascimento questo metodo diventa, per secoli, l'unico criterio per costruire lo spazio figurativo, almeno in Occidente*"

the theoretical relations of perspective with Euclidian geometry. The second, was a method more mechanic, that used – what we know today as – floorplan and section, and it pointed to complex geometries representation [1, Bk. III], [21], [22, p. 25]. In both cases, the two methods do the same and reach the same result, but the second method should avoid lines stacking with intricated volumetric representations: "*it is easier to demonstrate and to understand [and] in terms of complex objects, it is less cumbersome in terms of lines*" [21, p. 409].

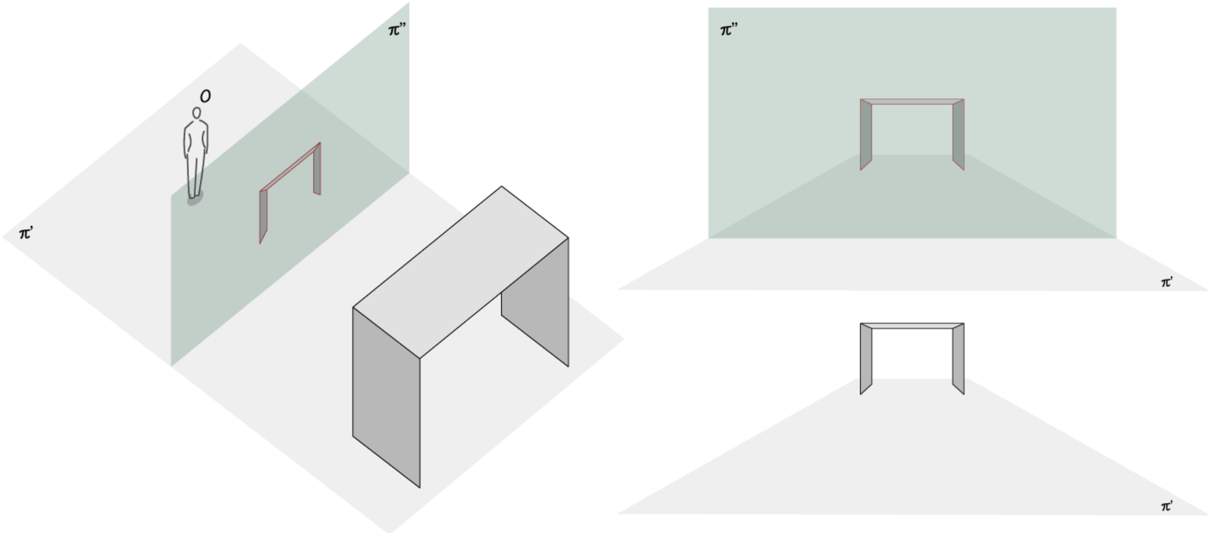


Figure 3 . The image in π'' is a flat equivalent of the real geometry: they look the same when seen from O (right)

Nowadays it is clear that there were many more than two methods during Renaissance (according to Veltman, Guidobaldo del Monte identified not less than 22 methods), but the two chief methods reply to the "legitimate construction" and "distance point construction" [15, pp. 153–156], [21, p. 407], [23, pp. 599–601]. The treatises and methods published during Renaissance aimed to give to artists a tool that would represent spatial objects with the right proportions among them and with the right size according to their distance to the observer, that is, a method to depict reality as *this appear to the human eye* [8, p. 2], [22, p. 1]. Being an eye-centred system, it is quite clear why linear perspective got so closely connected with the motto *"as a person sees"* [24, p. 409].

I.1.1 The five elements of Piero della Francesca

The legitimate construction (Piero's second method with floorplan and section), took the first place thanks to "*Leonardo's practical demonstrations […], a tendency confirmed by Serlio and codified by Danti in his commentary on Vignola's Two Rules of Pratical Perspective*" [21, p. 412]. Such a method was the progenitor of the future "*architect's method*" or "*projecting planes*" method and it contained, most probably in an implicit way for Piero della Francesca, key elements to understand the relation between the observer and the image [22, p. 4]. Such elements are[8]: the **eye**, the **shape of the object** to be represented, the **distance** among them, the **rays** joining the extremes of the geometry eye with the eye, and the **projection support** (Figure 4) [1, p. 81].

Three of these five elements condensed every representation system since Piero, namely, the **projection centre** (eye), the **object** and the **projection support**. Nowadays, the different possible relationships among them determine **central**, **parallel** or **double orthogonal projections**" methods. Applications of such methods are the **perspective**, **axonometric views** and **associated floor plan and sections** respectively.

[8] "*La prima è il vedere, cioè l'ochio, seconda è la forma de la cosa veduta, la terça è la distantia da l'ochio a la cosa veduta, la quarta è le linee che se partano da l'estremità de la cosa e vanno a l'ochio, la quinta è il termine che è intra l'ochio e la cosa veduta dove se intende ponere le cose*" [1, p. 81]

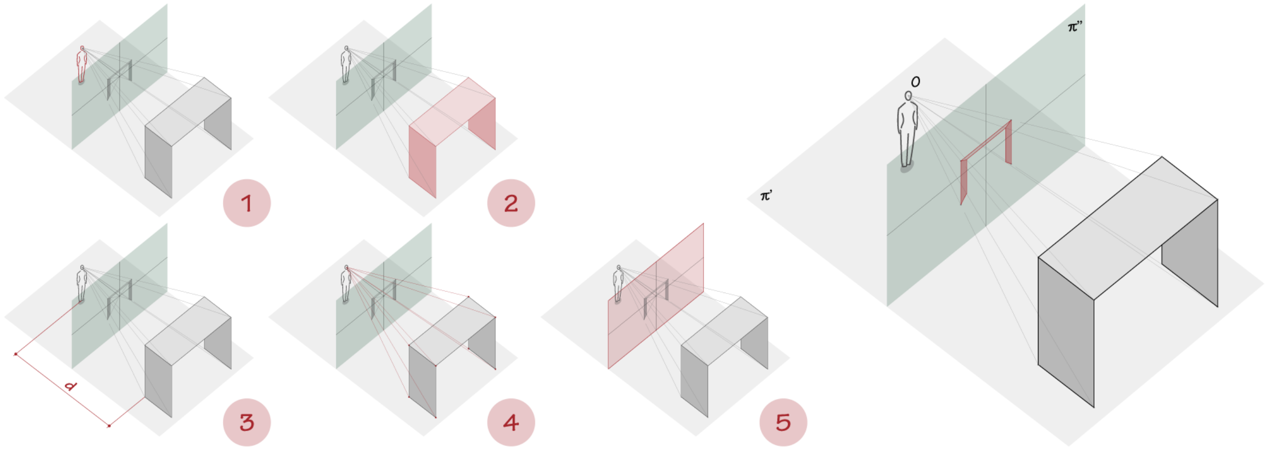


Figure 4 . The five elements of Piero della Francesca

I.1.1.1 About the surface

In a current definition, the perspective of a given object depends on its position regarding the observer, and it is related to the projecting surface among them. Furthermore, this perspective image derivates from projection and intersection operations (see I.2), it uses straight lines and gets shaped in a **flat surface** [22, pp. 2–4]. A large literature covers different methods for such procedures. For a guide so to follow the last century's developments, I forward to [25], [26].

Using a plane as **projecting surface** restricts linear perspective for wide fields of view. Indeed, Leonardo da Vinci's noticed a paradox: moving an object parallel to the plane of drawing (Figure 5), its image in the plane increases the size until in some point becomes bigger than the size of the object itself [8, p. 106]. Therefore, classical perspective limited itself to the scene contained in a cone of 90° or less (with 60° "in focus"), assuring the "no existence of marginal aberrations", those parts of the drawing usually associated with *deformations*.

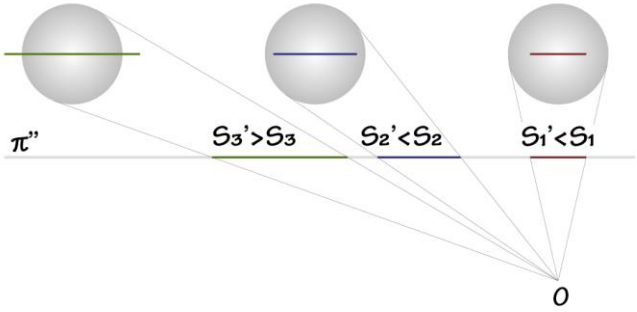


Figure 5 . Leonardo's paradox: the further the objects are, the bigger is their image in the plane

This limitation of the classical perspective to a small field of view, is an evident boundary in architectural, archaeological, artistic and heritage dissemination applications when we desire an overall view, e.g. during the reconstruction of an archaeological site, or to preview a non-materialised architecture idea [27]. In these cases, 3D modelling and 360-degree photography have a real advantage since they can offer at once both immersive and interactive views [28], [29]. Nevertheless, it is also possible to represent wider angles by drawing, as an alternative way of thinking the project and saving a large amount of time and resources for 3D modelling. In the chapters to come, I use anamorphosis as a key to extend the drawing field of view and still keep using conical projections.

I.1.1.2 Projection centre and observation area

A confusing debate took place from considering the element *centre of projection* as a "*fixed-point O*". Indeed, the legitimacy of perspective was questioned when the image is not perceived with *one and immovable eye* [13, p. 29]. Nevertheless, it has been demonstrated the triviality of such limitation and even the existence of an **observation area**. Indeed, Decio Gioseffi replied Panofsky's thesis, pointing to the immobility of the eye as something *necessary* for the *composition* of the image, and as something *relative* for the *perception* "*perspective can be seen with both eyes and a person can freely move his eyes and his head within limits without* *losing the illusionary effect of the perspective"* [30, p. 30]. Within the observation area, the observer can slightly move and the depth effect still prevails, even with the real environment behind so to compare it with the perspective image [30, p. 52].

Any punctilious reader can verify by itself this "margin" inspecting the fresco of Andrea Pozzo at *Casa Professa del Gesú*, made between 1681 and 1686. Orseolo Fasolo and Riccardo Migliari analyse this example: the visitor finds five perspectives on the vaulted ceiling and in the walls, all of them composed from a unique shared centre of projection. Standing at such a point (marked in the floor), the user can perceive the depth of the scene with binocular vision and free of rotating and moving. The illusion effect will not disappear even if the observer moves near the point within the mentioned margins [30, p. 50].

Aligning with such studies, this research considers *centre of projection* and *observation area* as two different things. The former, it is essentially a point for *setting* the right geometrical construction of the perspective. The latter, regards the *perception* of such a construction and it has a margin of movement: considering the main distance to the drawing *d*, the observer can translate 1% of *d* laterally and up to 5% of *d* forward/back and notice no difference [31, p. 35]. Considering this margin and the colossal dimensions of the rotundas, the *observation area* was even materialised as a walkable stage in the old panoramas (see I.5). Moreover, the same Damisch calls the example "*The Flagellation of Christ*" by Piero della Francesca to point out how such a movement during the perception is actually *necessary* for the deep understanding of the good perspective artwork [20, pp. 13–14].

I avoid stepping deeper in this philosophical swamp since this is not the place for such a task and it is not certainly my work, I quoted such assertion with the sole intention of analysing it later for the cubical case. Those who intend to go deeper into this matter, I forward to scholars who have talked (far better than me) about this question and about stereoscopy in perspective [10], [30]–[33].

I.2 RADIAL OCCLUSION

The basic principle behind linear perspective is the equivalence between a spatial point A and its image in the projecting plane A''. To find it, ray R is projected from A to a fixed observer O. Between A and O there is the drawing surface, plane π''. A'' lies in the intersection of R with π'' (Figure 6 left).

An observer standing in O should notice no differences between A and A'' **since A'' occludes A**. Thanks to such a radial occlusion, it is possible to create a scene that gives a mimetic effect of the spatial geometry behind iterating the process for every ray inside an eye-centred cone. Hence, image and real geometry are *equivalent* for the given observer/plane/geometry conditions (Figure 6 right).

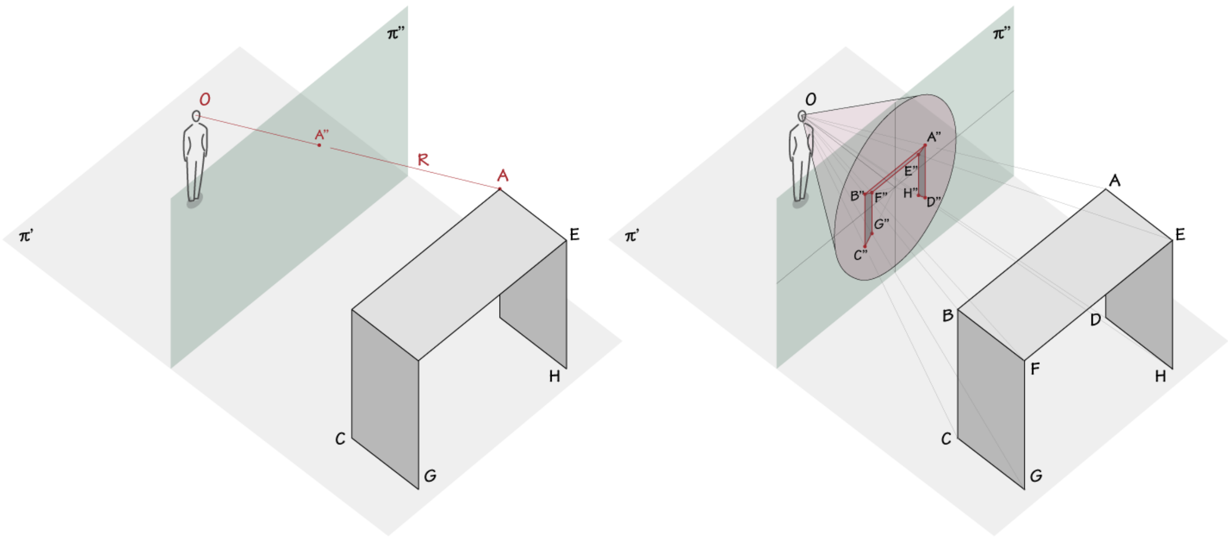


Figure 6 . Radial occlusion (left) and construction of the image projecting rays within the cone (right)

Note that for a given point A, *infinite* points along ray OA look the same from O. In the example of Figure 7, for every point from A to H there are image points A'' – H'' in the intersection of rays OA - OH with π''. I define the perspective of the object in π'' joining such points and using the central vanishing point (Figure 7 left). Yet, considering an irregular surface or not surface at all, it is possible to take a third projection A''' to H'''. With such points, I can compose a second equivalent image of the object. For example, the red and the blue wireframe drawings, and the object itself look like equal geometries from O (Figure 7 right). Thus, the represented object has infinite equivalents or *anamorphs* that *look the same* when seen from O [34, p. 148].

As a consequence, among the many conical projections, classical perspective is one particular case of the radial occlusion principle. While the former uses a plane as a projection surface (so to say, to make the representation concrete and possible), the latter has no such limitation (but it stays as an abstract concept). A bit later than linear perspective, a parallel discipline started to deal with such variations of conical projections: the **anamorphosis**.

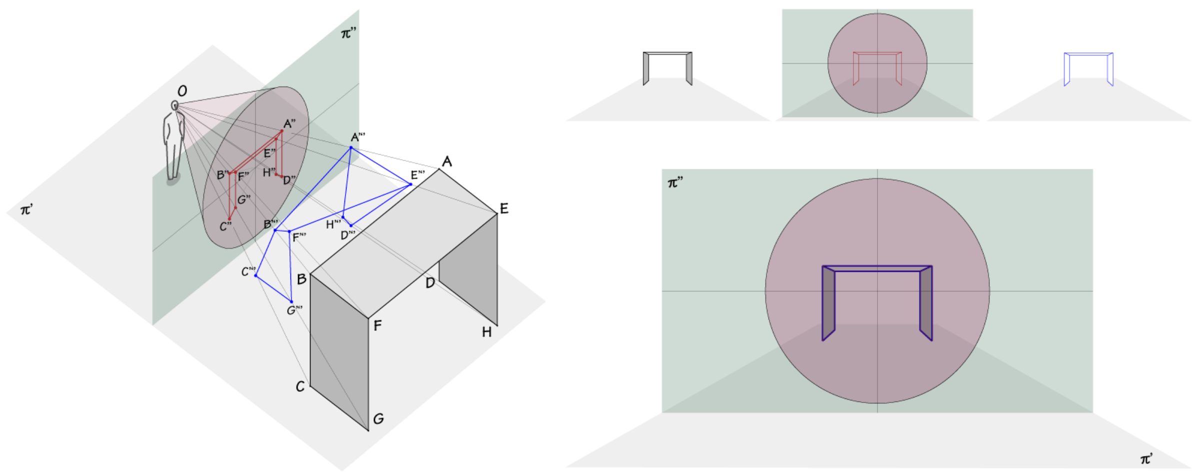


Figure 7 . Anamorphs or equivalent images of an object

I.3 Procedures

Cubical perspective is, somehow, an extended and special case of linear perspective. Nevertheless, one must be aware that not all linear perspective systems are proper scientific procedures even if they may look like one. I illustrate the concept with Jan Van Eyck, Maurits Cornelis Escher and Pompeii's frescos as examples.

Figure 8 . The Virgin of Chancellor Rolin (1434-35 ca) by Jan Van Eyck. Image under public domain [35]

Jan Van Eyck (1390-1441) created unusual depth sensations in his paintings using light and shadow effects, but his perspective composition was just approximate as no less than six different hypotheses demonstrate [36, p. 53]. Indeed, for those who know how a classical perspective works, it is easy to understand where the errors are: multiple vanishing points for elements that should share the same one, the repetition of regular elements not matching the right proportion (missing the diagonal) (Figure 8).

Yet even non-specialised observers may perceive something “eye uncomfortable” after a deep review of his paintings. Jan Van Eyck was not entirely aware about linear perspective principles, and his compositions were guided “by eye” as James Elkin concludes: “*Jan van Eyck presumably had no interest or awareness of these analytic finesses, and we need not assume he had any “system” in mind, the differences […] are great enough to argue against that. Yet he accomplished by eye, and with consistency between paintings, a compromise between medieval and Renaissance sensitivities”* [36, p. 62].

By his part, **Maurits Cornelis Escher** (1898 – 1972) had much more than the knowledge of one representation system: he had a path led by the mathematical logic “*In a consistent manner, Escher follows the road that the mathematicians show him, going beyond the familiar world of Euclidean geometry and codified drawing techniques*” [37, p. 308]. He teased the paradoxes of perception, playing with the ambiguity of projective systems [37, p. 307]. Such a way of composing, in words of the mathematician Albert Flocon (who is among Escher’s influences), put him among artists that are also *penseurs nets* (net thinkers), such as Piero della Francesca, Leonardo da Vinci, Albert Dürer, Jamnitzer, Bosse-Desargues, Jean-François Nicéron and many others [38].

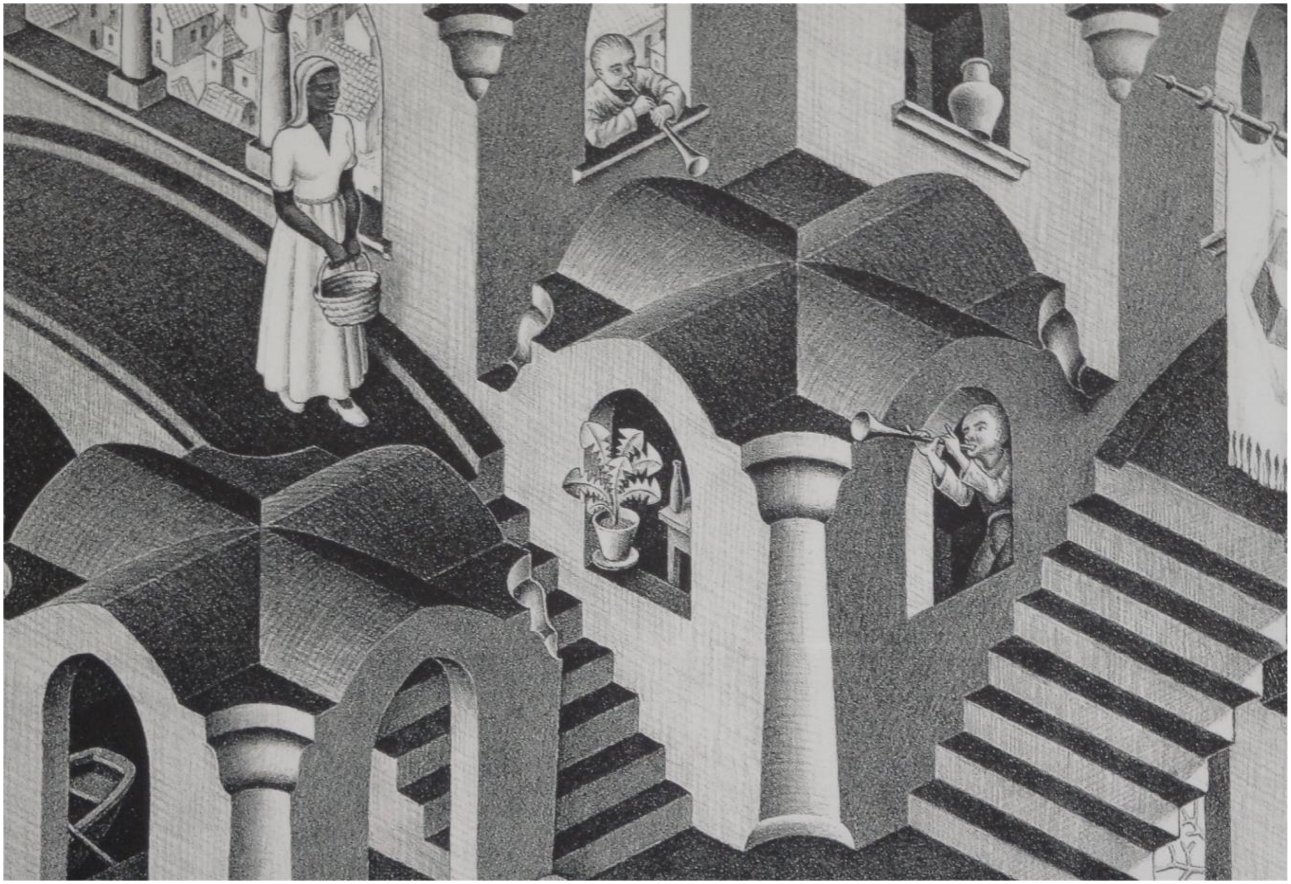


Figure 9 . Concave and convex (detail) by M.C. Escher (1955). Picture taken by Esparaz Marco Aurélio and distributed under Creative Commons licence [42]

Escher's interest and instinct for going deeper into representation rules was manifest in artworks such as *Hand with Reflecting Sphere*. This lithography was first printed from the original drawing made on the stone in 1935 [39], which means 30 years before Escher met his "French kindred spirit and life-long penfriend" Albert Flocon [40], [41, p. 10]. Two years after that meeting, Albert Flocon characterised the azimuthal equidistant perspective up to 180° and published *La Perspective curviligne de l''espace visuel à l''image construite* with illustrations of André Barre [2].

Figure 10 . 1st century wall fresco in Villa Poppea, Torre Annunziata, Italy (n.d., unknown author). Orthophoto (2014) by Alfonso Paolo, Alaniz Julieta, Cetti Maria Victoria, Falcone Salvatore, Saviello Saverio and Vicidomini Maria

Such a pity for Escher that he met Flocon only 30 years after he composed *Hand with Reflecting Sphere* since the contact with Flocon and his knowledge about spherical perspectives must have been certainly revealing for his work. In any case, Escher developed a big innate capacity to understand and construct accurately complex representations, not just spherical but also ambiguous linear compositions (Figure 9) [43, p. 60]. *Hand with Reflecting Sphere* can be wrongly considered as a spherical perspective and confused with Flocon's system, but actually it is not, it is a master spherical reflection artwork [7, pp. 167–168].

The third example is from almost two thousand years before Escher: the frescos in the walls of Pompeii, Herculaneum and the *Villas* around (Figure 10). Although they look as central perspectives and there was already a canon of rules that allowed painters to depict spatial relationships, these rules were not based on any scientifically proven insight [44, p. 27].

Summarising, there are cases where an illustration may follow a logical and maybe even accurate geometrical definition, but this does not imply for them to be a full perspective system, even if they may really look like one. In these cases (see also *Drawing and guessing* at I.6.2.2 and I.6.3.1), the operator may or may not have a real management of the figurative compositive elements. The latter should not be the case of the designer since the representation must bring back to the paper the accurate correspondence between the mental model and the real object, i.e., the final image should be a scheme re-presenting what the designer has in mind. A designer that draws fancifully and finds shapes *after*, may induce negative consequences during the materialisation of the object or space.

Designers and architects use representation systems to pop up possible geometries from the whimsical immaterial world of ideas to the three-dimensional physical space. Such a task gives them one less degree of freedom than artists (whom do not need to leave the canvas) or mathematicians (whom think in terms of logic and not of statics) [37, p. 306]. Nevertheless, to dive in mathematics and think in terms of logic (as Escher did), gives another level of consciousness to the draughtsman. Indeed, managing "meta-structures" of conical projections help to make easier the composition of the image. An example of this is the general scheme for *thinking* spherical perspectives as a composition made by a conical projection onto the sphere's surface and the flattening of that sphere [7, p. 148]. Once defined, the same scheme was used for the generalisation of the azimuthal equidistant [7] and the equirectangular [34] perspectives and also the cubical case (see Second Part IV.5).

Knowing general rules, it is possible to play, break and distort them, not just with straight linear systems, but also with curvilinear ones; and not just with Euclidean dimensions but also in n-dimension spaces. It is clear that also artists and enthusiasts of drawing may reach “by eye” some level of accuracy in the perspective structure, but it is more difficult in this case to rightfully master spatial geometry and lighting.

I.3.1 Image and representation

Overcoming the modern age, the so-called practical method of the architects gains scientific status with Girard Desargues (1591-1661), Guarino Guarini (1624-1683), Amédée François Frézier (1682-1773) and the same Gaspard Monge (1746-1818). Indeed, one of the chapters of descriptive geometry dedicated to central projections merge studies of perspective [45, p. 272]. In such a method for linear conic projections, a point *A* is represented by the intersection of two straight lines. These two lines are the projecting line where P lies, and a second line individuated in the projecting plane. Working with these two known lines, it is possible to complete the passages from *P* to *P”* and from *P”* to *P*, or in other words, what makes *scientific* a representation is the biunivocal correspondence between point *P* and its image *P”* (Figure 11). For a detailed exam of the essential passages that converted the practical method of the architects in an autonomous chapter of the so-called *Sciences of Representation*, I forward the interested reader to [16], [46]–[48].

However, a perspective *tout court* (e.g., a photography) does not verify for itself the biunivocal relation: it is possible to get *P”* from point *P,* but not the other way around. Indeed, to get *P* from *P”* it is necessary to add extra information since ray *OP* contains infinite representations of *P* equally seen form *O* (i.e., due to radial occlusion). From the single perspective image (e.g., a photographic survey), it is necessary to complete the biunivocal condition by adding metric and angular information of the image.

Avoiding the mere pictorial image of medieval age, Renaissance’s architects played with such a relation getting the representation from surveying the material reality and from the representation the real size and spatial position of an object. In addition, the criticism highlighted how perspective, in synchrony with the culture at the time, moved the attention towards a new way of reading and interpreting history: the classical root of the occidental traditions is rediscovered, becoming a new model of civilisation able to make alive a world in which the human being takes the central place. The choice of a central point of view highlights the role of the observer, conferring it communicative and expressive strength.

In any case, perspective needed to deal with the observer's physical and psychical limits: the visual sensation and the visual perception. In the former, the eye records how much light impresses the retina. The latter instead, is an intellectual act in which the mind participates actively in the rationalisation process of giving significances to such images. The spatial acquaintance of an architecture uses the temporary factor between these two moments, the mental images indeed precede any kind of graphical re-presentation. Presuppositions in architecture, engineering and design base on the modality we organise the psychophysical experience.

I mentioned before that Brunelleschi credits the technical invention of linear perspective. Such a recognition came thanks to his two famous tablets (*tavolette*), sadly lost but thoroughly described by his biographer and pupil, Antonio Manetti. The principles of the method demonstrated by Brunelleschi (and deductible from Manetti's book) are: first, parallel straight lines in the reality look as straight lines converging to one point in the representation; and second, fixing the observation point and one point of distance, it is possible to calculate the decreasing of the real dimensions in proportion to the observer's distance [49].

Following these criteria, Brunelleschi overlaps the material reality of the Baptistery with the architectonical project. The perfect match between material and immaterial has a doble effect: on the one hand it describes the current state of the building (*lo stato di fatto*) without any personal reduction of the visual data. On the other hand, the device forces the observer what the designer thought and in a depiction of the project overlapped to the reality. These actions orientate the look of the observer in a specific direction, focalise a determined scene and highlight the geometry of the project, all of them operations signed by the architectonic culture of the moment and the intentions of the operator [50, pp. 24–27].

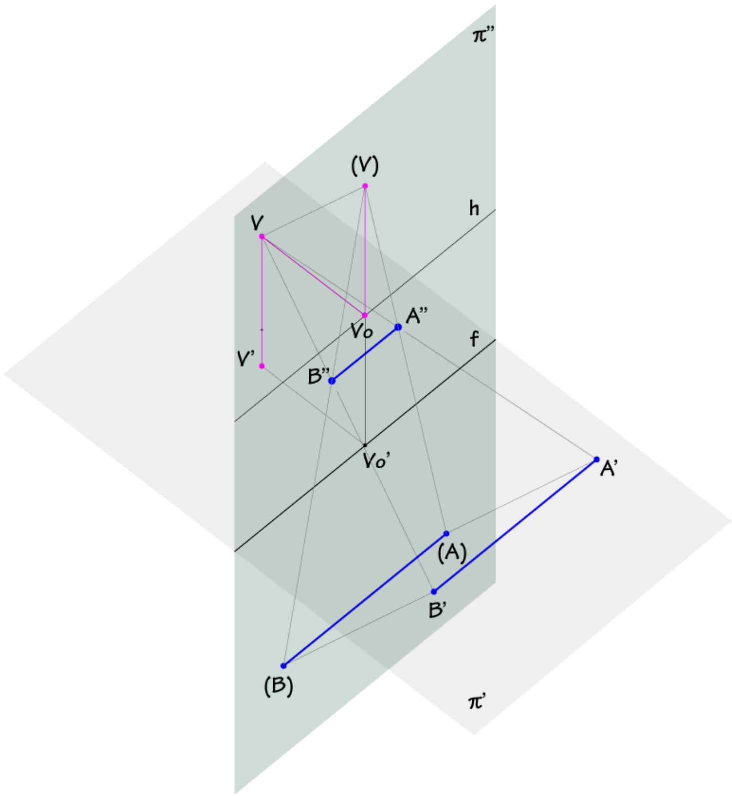


Figure 11 . Homology of overturning with centre V and axis f (fundamental straight-line intersection of two planes). The line of horizon is one of the two straight lines limits of the homology

I.4 ANAMORPHOSIS

Just as a linear perspective, an anamorphosis is a conical projection aiming to compose a scene equivalent to a real geometry [51, p. 2], [52, p. 15]. Probably thanks to the Greek root of the term meaning "transformation", anamorphosis is commonly associated as the "ugly" and "bizarre" sibling of classical perspective, a game or a distortion of it [18, p. 3], [31, p. 34], [53, p. 137]. Moreover, from dictionary entries and definitions from the disciplinary sector of drawing, anamorphosis has been reduced to a *deformed* drawing which *makes sense just when is viewed from the correct point of view* [9], [18, p. 3], [31, p. 34], [54]–[56].

I.4.1 Current definition

The adjective *deformed* brought users and scholars to the misconception that a classical linear perspective is not, while even this looks *deformed* if seen outside the area of observation ([32], [57]. For instance, in "The Ambassadors" of Hans Holbein The Young, if the observer looks the painting perpendicular to the projecting plane, then the "perspective" looks fine and the "anamorphosis" (the skull) as stretched (Figure 12 left). If the observer moves, the roles inverted: the ambassadors look distorted and the skull looks fine (Figure 12 centre and right).

Considering notions introduced with the principle of radial occlusion in I.2, a current definition presents anamorphosis as the **most general case of conical projection** happening among **objects** and the **centre of projection**, i.e., is the "*equivalence relation between three-dimensional objects, not necessarily flat*" [57, p. 1]. In this view, vanishing sets are defined at the anamorphosis level, that is, *before* the definition of **any perspective system** and therefore **independently of any projecting surface**.

Figure 12 . The Ambassadors (1533) by Hans Holbein The Young

I.4.2 About the surface

A better key to understand anamorphosis resides around the used **surface of projection**: on the one hand, a classical perspective considers exclusively a plane surface, a visual field of view limited by a cone of 90° or less and, generally, an observer perpendicular to the projecting plane. In turn, an anamorphosis may use any regular or irregular surface, a set of them or even none (Figure 13) [9, p. 217], [32, p. 137], [57, p. 1], [58].

It is clear from this latter appreciation that the single plane is included among the possibilities. Therefore, fifteenth-century linear perspective is the flat representation of what is happening at the anamorphic level, thus "*every perspective constitutes an anamorphosis*" [32, p. 137]. Many other surfaces have been used, such as the cylinder, the sphere, the composed geometry of the ceiling in a church, and, as in the case of this research, the cube.

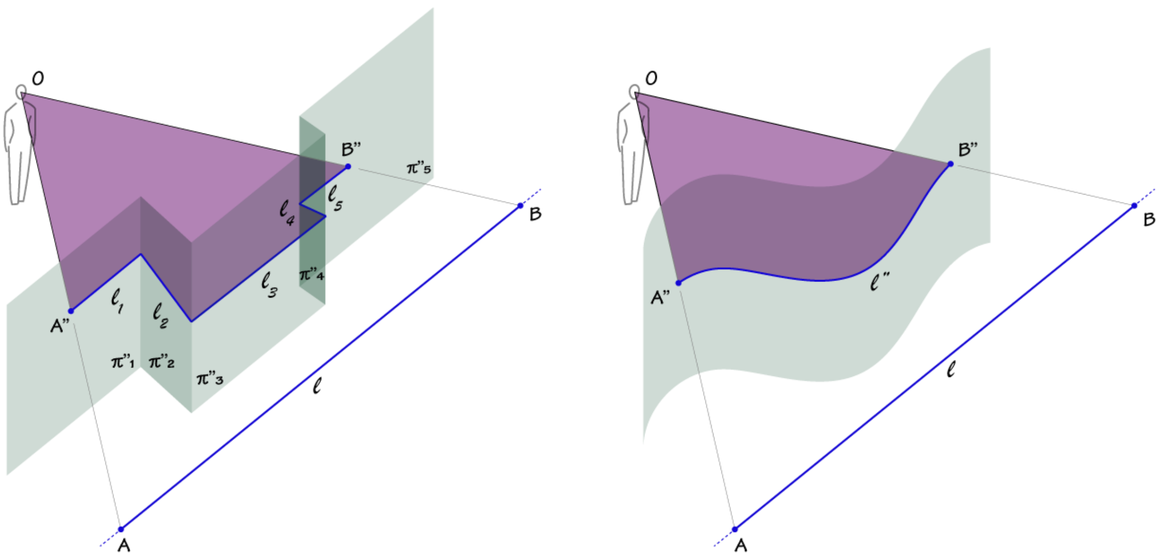


Figure 13 . An anamorphosis may use any regular or irregular surface, a set of them or even none

Historically, the so-called solid perspective and “*trompe l’œil*” explored the option of using several surfaces, integrating drawing with architecture. In such direction, the movement “quadraturism” for example, exploded these conical projections distributing one same painting in a combination of walls, vaults and ceilings. Such explorations started to cover fields of view larger than the one of linear perspective.

These first “traditional” works (e.g., the plane anamorphosis of Figure 12), are yet restricted to linear perspective limitations. In this case, the final surface is not considered as part of the drawing process: the painter prepares an original drawing by following linear perspective’s rules, thus creates an anamorph in the final destination using a grid (see I.4.3). This same mechanism of the grid was enough to translate drawings to *any* final surface.

A second group of anamorphoses emancipates from the plane and uses a different surface, e.g., a cylinder. Such illustrations changed the process of drawing itself: here the draughtsman also does a *flat* illustration, but this time following the plane development of the final surface *a priori*. In this (as in the following group), the topological characteristics of every projecting surface have a decisive role for drawing [58, pp. 73–74]. In this group, the field of view may be slightly bigger than the one of classical perspective.

With the arrival of the computer, there is a third group that considers the plane development of the final surface as well. However, the folding artefact is digital, adding the possibility of working with surfaces that surround completely the observer, e.g., a sphere. Such action reaches something never achieved before: the field of view covered around the observer is total, which creates a full immersive virtual reality.

I.4.3 Grids

The quadraturism was born in Italy during Baroque and its name derives from the term "*quadrettatura*" since they used a **grid of reference** to project, translate and trace the illustration on top of any surface [53, p. 149], [59, p. 120], [60], [61]. The documented, studied and extensive work of the Quadraturists is the result of an intense investigation from the *practical* field. Nevertheless, this practice was conducted with a rigorous and curated *thought* that follow geometry, optics, architecture and arts [62, p. 45]. For a detailed historical compendium about these subjects, I forward interested readers to the treatises [63]–[66] and the historical analysis conducted by Pedro Cabezos [32, Ch. 6].

The grid mechanism showed its utility for either maximising or reducing the dimensions of a drawing. In the former case for example, during the 17th century several anamorphoses were created using the grid and the light of candles in the generous dimensions of churches and convents [59], [62], [67]. Later on, during the 18th and 19th century, the grid was also used to paint panoramas in the *rotundas* (see I.5). In both cases, there was the disadvantage of requiring a huge infrastructure or a dedicated cumbersome device for the translation and visualization of the paintings. Also, every combination of surfaces and painting needed a special arrangement, resulting every case a unique and highly specialised work. All these factors combined reduced the availability of such paintings and raised their production cost [11], [68], [69].

On the other hand, the grid was used for creating miniatures: during 17th century existed polyhedral devices called “Perspective Box” (see Second Part III.2). In this case, anamorphic illustrations enhanced the first-person perception and, at the same time, drastically reduced the size of the artefact. Even today some current examples revive the utility of the perspective box for architecture interiors” perception [70].

A paradigmatic case is given by the work of Andrea Pozzo, who wrote about using three reference grids (Figure 14) in his treatise explaining the fresco “*Gloria di Sant’Ignazio*” made at Rome between 1691 and 1694. The first grid goes on top of the project drawing which was previously prepared following linear perspective treatises. The second was a physical grid, extended before the drawing surface. The third reticule was useful to paint the final surface, and Pozzo got it using a light source positioned at the observer’s point and tracing the cast shadows of the second grid. He proceeded by mean of a wire in the case where the distance between the point of observation and the drawing surface was too big. In this latter case, the wire may anyway have associated a light source near the point of contact with the grid [15, p. 53], [65, pp. 220–221].

Another example of Pozzo is the Refectory of the *Trinità dei Monti* Convent, painted in 1694 also in Rome. In this case, Pozzo used the grid and projected from the main point to a hypothetically convex surface. Yet, in some parts he modified the references on-the-fly, following his artistic-intuitive side more than the mathematician one. A hypothesis suggests that with this painting Pozzo tried to build what has been called a “perceptual dynamism”, or the involvement of the user both as observer and observed actor. In doing so, the user interacts and perceives the artwork within the full horizontal field of view around, action that anticipated the panorama that Barker would “invent” a century later [59, p. 121], [67, p. 131].

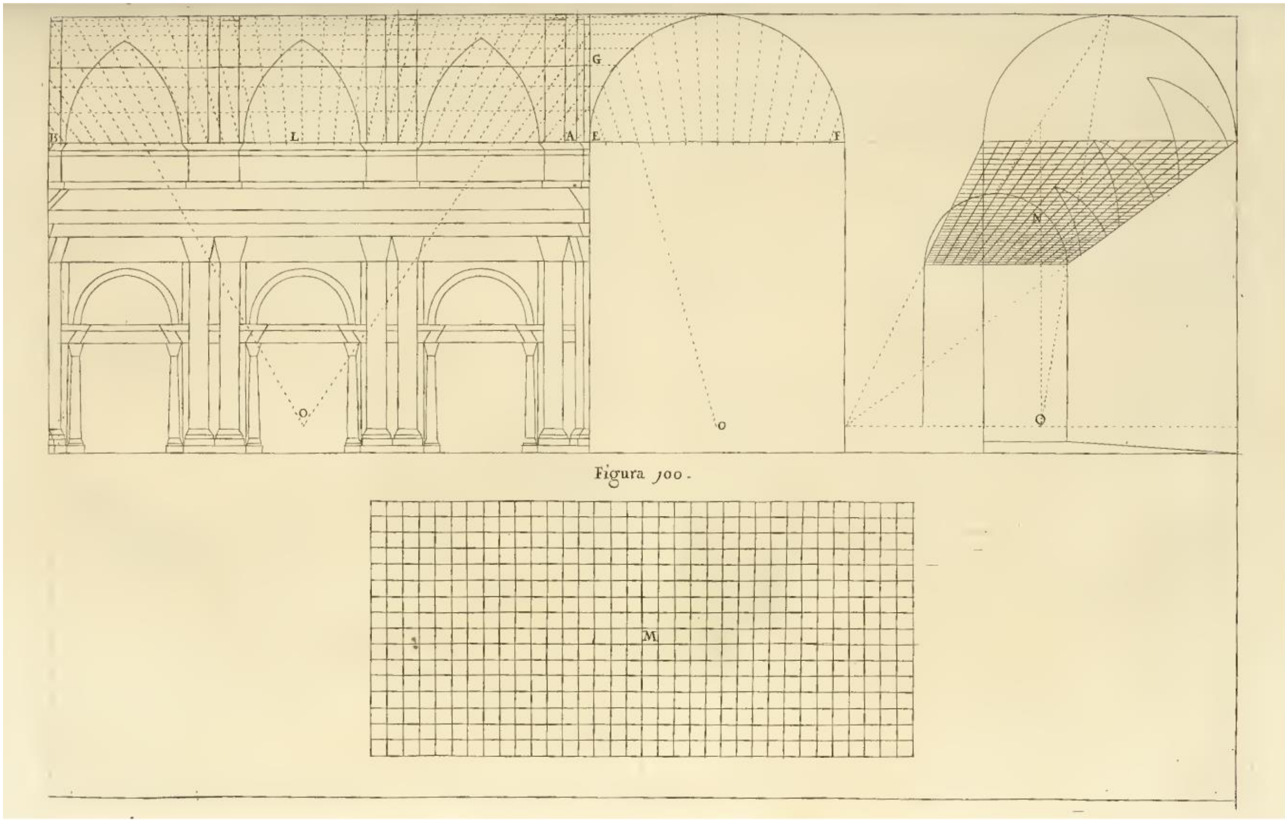


Figure 14 . The use of reference grids by Andrea Pozzo. Image with public licence [65, p. 220]

I.4.4 Projection centre and observation area

In I.1.1 I separated projection centre from perception area. Such definitions apply as well to rightfully produce and appreciate an anamorphic illustration. Two examples of this are the "*Camera degli Sposi"* of Andrea Mantegna made at Mantua between 1465 and 1474, and the above-mentioned fresco "*Gloria di Sant'Ignazio*" of Pozzo. The latter case has an additional feature: the illusion of the perspective keeps intact inside the margins around point *O*, and furthermore, it is practically impossible to differentiate (especially at naked eye) where does the painting start, which makes confusing to separate the real geometry from the faked one (Figure 15). Moving away from *O*, it is conceivable to differentiate painting and construction, but it is hardly possible to find the transition point between them. Examples like these "*involve the observer in a genuine experience of virtual reality* ante litteram*, and without using technological devices*" [30, p. 50].

Nevertheless, due to the many possible surfaces that an anamorphosis may use, the centre of projection of a scene is not always as intuitive and easy to get. Indeed, for the perception *sometimes* it is also necessary the use of artefacts (such as mirrors), to fold the centre back in space: "*the pictorial content can only be seen when the picture is viewed awry or through some appropriate optical device like a cylindrical mirror*" [14, p. 7], [71]. Such games misled to think the difference between classical perspective and anamorphosis as the altered position of the projection centre: "*In anamorphic art the appropriate viewpoint differs from normal or perpendicular to the picture planer*" [14, p. 7], [71].

The manipulation of the projection centre and the area of observation was a common practice promoted by the Quadraturists. They used many projection centres to provoke an augmented appreciation of the artwork and, on the other hand, to smooth the *deformations* on the sides of the painting [62, pp. 44–45]. The former games of putting back or forward the projection planes, aimed to exaggerate depth's effect either accelerating or delaying the perception. A paradigmatic example of this is the drawing attributed to Bramante at Saint Mary near Saint Satyrus in Milan [32, p. 138]. Regarding the smoothing effect, Pozzo negated such a practice in some of his paintings and concentrated the vision in a single observation point marked in the floor. With this, and sometimes even giving an added value to the marginal deformations, the Jesuit seek a perception of the final image even more effective, spectacular and amazing to the observer with its discovery [59, p. 120], [62, pp. 46–47, 52].

As an early conclusion, it is more proper to consider the presumed deformations behind anamorphoses as "apparent", more than a synonym of something erroneous or poorly performed. Indeed, they may look *more* twisted than a linear perspective if observed in **free vision** (*visione libera*). However, they cause the same effect to the observer than any other linear perspective when observed in **linked vision** (*visione vincolata*) from the correct area, no matter how warp and stretch they may look in free vision [72, pp. 50–60].

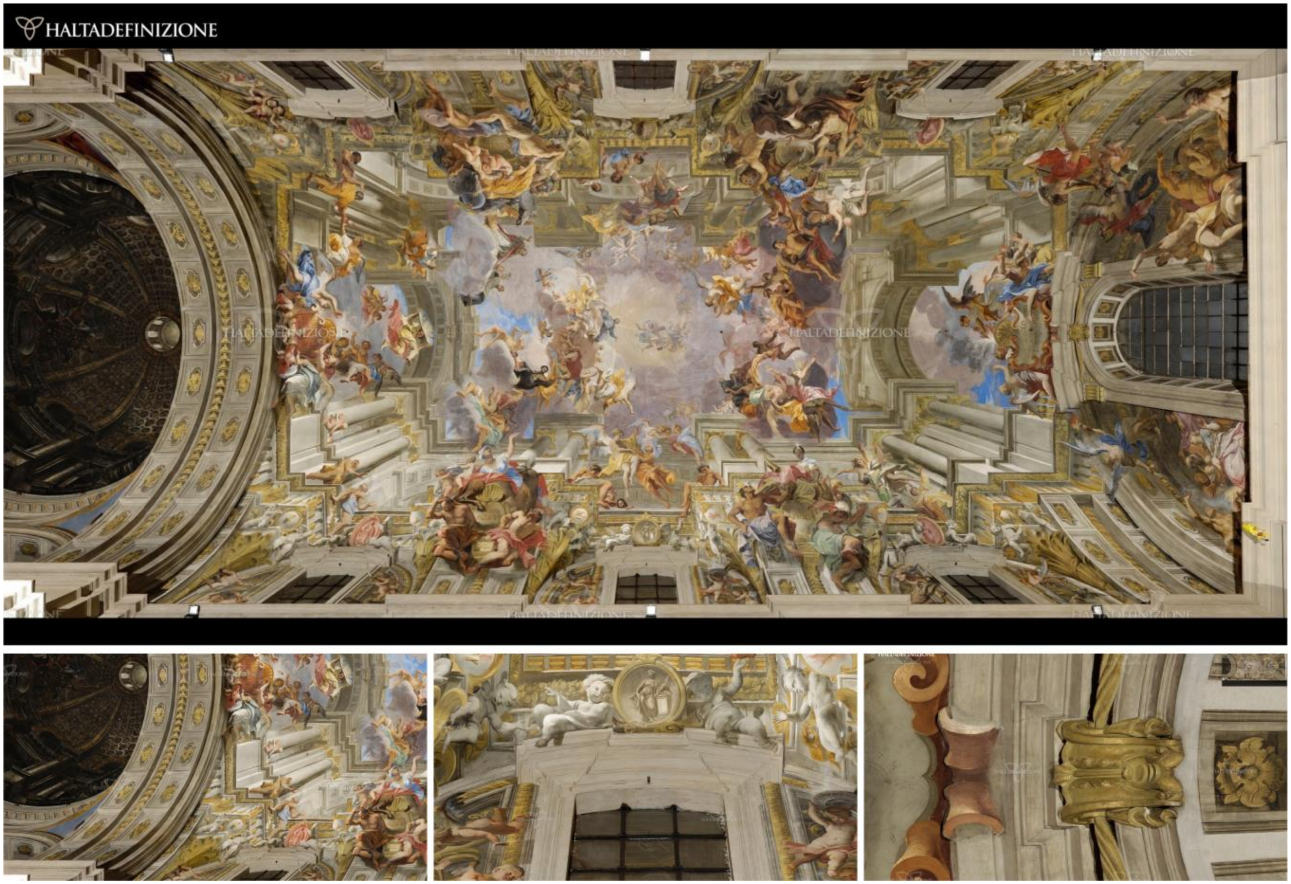


Figure 15 . Gloria di Sant'Ignazio (1691-94) by Andrea Pozzo. Screenshots (2021) from Haltadefinizione website (free use for didactic non-profit purposes) [73].

I.5 PANORAMA

The full immersive experience of a virtual reality changed our way of perceiving the space, leaving consequences in every field of daily life. One of the clearest influences in this direction was the invention of the panorama and the associated *Cyclorama*, started with the Edinburgh's Calton Hill of Robert Barker in 1787 [69, p. 134]. Barker patented the painting used for the occasion as *Panorama* (from the Greek *"an all-embracing view"*), which was made on top of a cylindrical surface (Figure 16) [74, p. 73], [75].

An interesting aspect from the field of architecture is the vision of Robert Barker for his invention. In the description of the patent, he explained many details and characteristics for the best building aimed to host the painting and increase the immersive experience and perspective perception [75, pp. 165–167]. Barker's invention included all the innovations appearing further away, including all kind of contraptions such as *faux terrain*[9] and lighting effects: "*the panorama developed into a presentation apparatus that shut out the outside world completely and made the image absolute*" [52, p. 59].

I.5.1 About the surface

Historical panoramas use a cylinder while current full panoramas use a sphere. The former, allows a free rotation around the area of observation giving larger field of view than classical perspective, but the viewport results cropped at the top and the bottom parts. The latter, collects all the visual data around the observer, allowing a free navigation around the observation point (see I.6). One remarkable innovation is the emancipation of the artwork from the physical limitations of the frame and the mutual relation of art and architecture to one same

[9] According to Grau firstly used by the French Charles Langlois in 1830 in his battle panoramas [52, p. 59]

purpose [76]. Rotundas became a fashion during the 1800s and the *Household Words* of Charles Dickens quipped on it, creating the imaginary character *Mr Booley* to boost the idea of visiting the world ("*going up the Mississippi and Missouri rivers*") within the confines of the *Cyclorama*[10]. This Victorian satire introduces - after two centuries and with the due differences – some of the most interesting characteristic of visiting and interacting with panoramas.

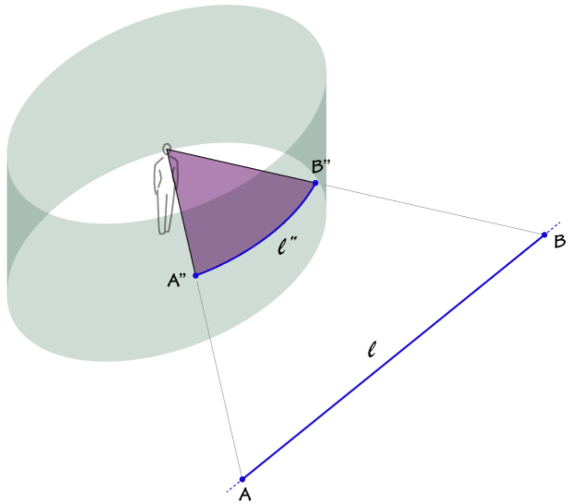


Figure 16 . Panoramas projected on top of a cylindrical surface.
In the old Rotundas, the observation area is materialised as a walkable stage

[10] Charles Dickens founded Household Words, a miscellany in the weekly *"Some Account of a Extraordinary Traveler"*, published as leader on April 20, 1850

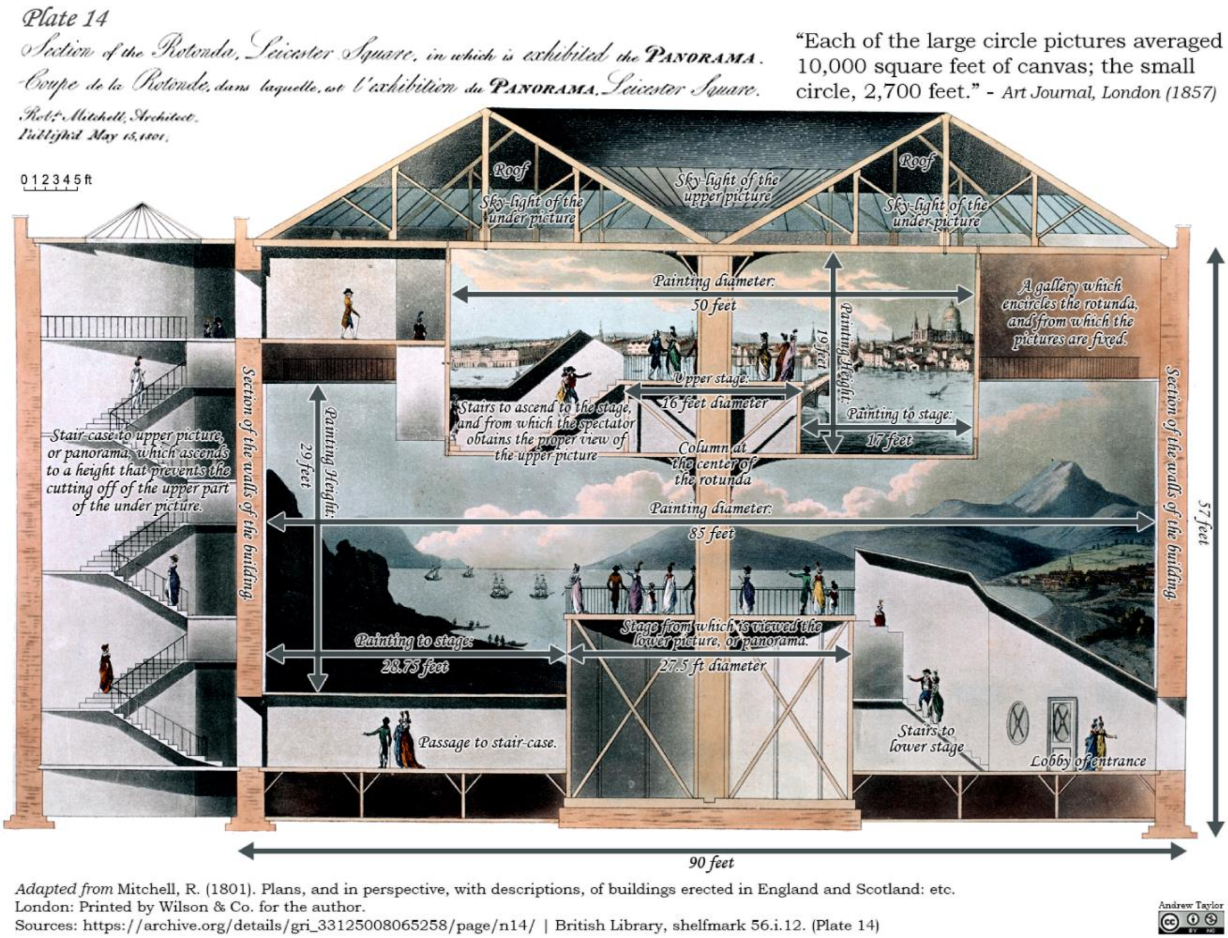


Figure 17 . Section of the Rotunda (1801) designed by Robert Mitchell under Barker's indications [77, p. 58]. Image edited by Andrew Taylor (2019) and shared under Creative Commons License [78]

I.5.2 Projection centre and observation area

If compared with a classical perspective, a panorama also considers the observer positioned in one spatial point, but in contrast with the former it allows from partial to full rotation. Such a movement frees the painting from the framework and allows the user focusing an infinite quantity of accidental perspectives around O.

In 1801 Barker employed architect Robert Mitchell to build a circular theatre at London's Leicester Square: the Rotunda (Figure 17), in which many more architectural details of the patent were finally materialised, enhancing the immersive illusion [68], [77, pp. 8, 59]. The visitor explored the panorama from a central area after having walked through a dark corridor and climbed a spiral staircase, a whole mechanism aimed to disorient the observer and boost the first impact [79, p. 81].

I.5.3 Drawing a panorama

About the panoramas' graphic elaboration Oliver Grau comments that "*using empirical methods, he (Barker) developed a system of curves on the concave surface of a picture so that the landscape, when viewed from a central platform at a certain elevation, appeared to be true and undistorted*" [52, p. 56]. With such a system, Barker who had an accurate knowledge about perspective, created an apparatus that may do the work easily if not automatically joining (today "stitching") single drawings. This means that he used a similar mechanism to those used by Quadraturists and Pozzo, i.e., a classical linear perspective drawing to the base and then produced the final illustration.

With the time, panorama's drawing progressively pointed to the industrial production: the measures of the building were standardized and some artefacts following the camera obscura simplified the acquisition of landscapes and the painting itself. Indeed, devices such as the *Panoramagraph* (1803), the *Camera Lucida* (1806), *Diagraph*, *Daguerrotypy*, and *Photography* "*could be used without prior knowledge of perspective and with only basic drawing skills*" [52, p. 115].

The “technologization” arrived to a workflow: photography – drawing – photography – projection – tracing – painting [52, p. 119]. That is: a first photography in the original place intended to represent (e.g., Waterloo); second, a linear drawing on top of the picture; third, a photography of the drawing; fourth, a projection with light sources; fifth, tracing the projected drawing on top of the walls; and finally painting. In the specific, the base drawing was made “*according to the rules of descriptive geometry*” whereas its tracing on the walls of the building followed a director guiding from the observation stage and many painters working in the walls [52], [80, p. 75].

An extra complication were the colossal dimension of the illustrations that, according to panoramist Hans Bohrdt, made the artist helpless near the canvas since “*he cannot even assess a straight line and when he has drawn one, it looks wrong*” [52, p. 119]. Effectively, the panoramas of those years were giant monstrosities: *“At the end of the nineteenth century, the norm for panoramas was around 2000 m2 and the paint used amounted to several tons”* [52, p. 119]. These installations characterised a moment in history in which panoramas aimed to the pure exhibition, a bourgeois luxury for just a few people enjoying it.

I.6 SPHERICAL PERSPECTIVES

Since Renaissance applications such as *The Arnolfini Portrait* (1434) of Jan Van Eyck (Figure 18) or Parmigianino's *Self-portrait in a Convex Mirror* (1524) tried to represent the curved reflection in spherical mirrors [36], [81], [82]. More recently Maurits Cornelis Escher explored spherical *reflections* with thorough precision (see II.3.3). Indeed, during the last century spherical perspectives expanded their use even more with new theoretical studies [83]–[86].

Even more recently, spherical perspectives are being used in photography to acquire full panoramas and generate virtual reality environments with them [86, p. 4]. With the high diffusion of this medium, it is possible to acquire such environments either using compact 360-degree cameras or a regular camera with almost no effort. Generally, there is a further stage of elaboration with software, e.g., for stitching single shots.

Figure 18 . Portrait of Giovanni Arnolfini and his Wife (1434) by Jan Van Eyck. Photography (2017) by Saiko. Image distributed under Creative Common Licence [87]

With this recent spreading, also emerged new theories and practical methods for **drawing** spherical perspectives. These methods focus into simple drawing, i.e., using ruler, protractor, compass and descriptive geometry. They find application in art, architecture and design (Figure 19) [7], [19], [34], [88]–[93].

I.6.1 About the surface

The early experiences after Renaissance opened the research to understand the relation of classical linear perspective with curved surfaces (Figure 18). In the last century, Barre and Flocon used already semispherical surface and published “*La Perspective curviligne: de l”espace visuel à l”image construite*” in 1967, developing the azimuthal-equidistant perspective up to a field of view of 180° (today better known as *fisheye* perspective) [2]. After some attempts [88], [94], such a perspective was successfully extended up to cover the whole field of view around the observer, that is, using entirely the visual sphere [7], [19].

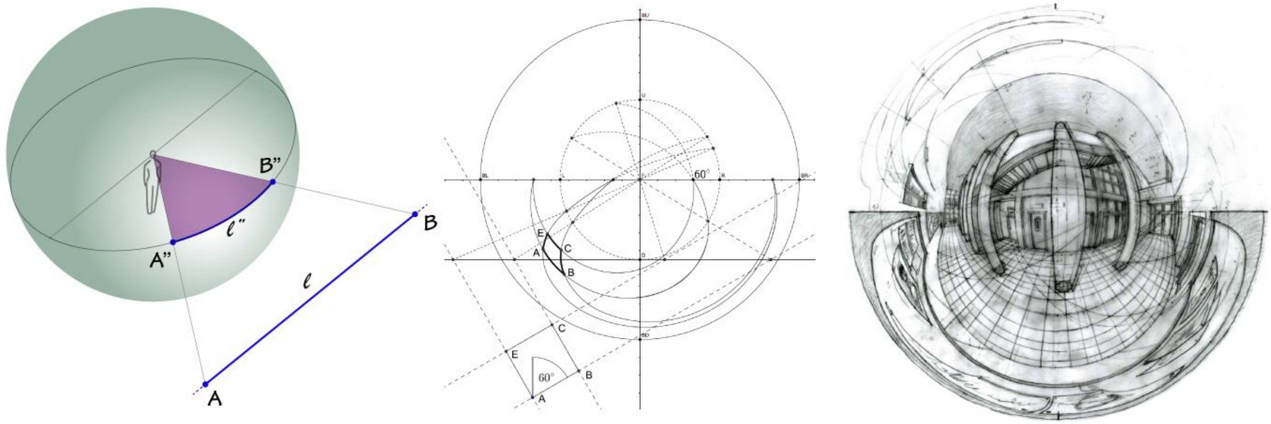


Figure 19 . Construction of an azimuthal-equidistant perspective (2018) by António Bandeira Araújo

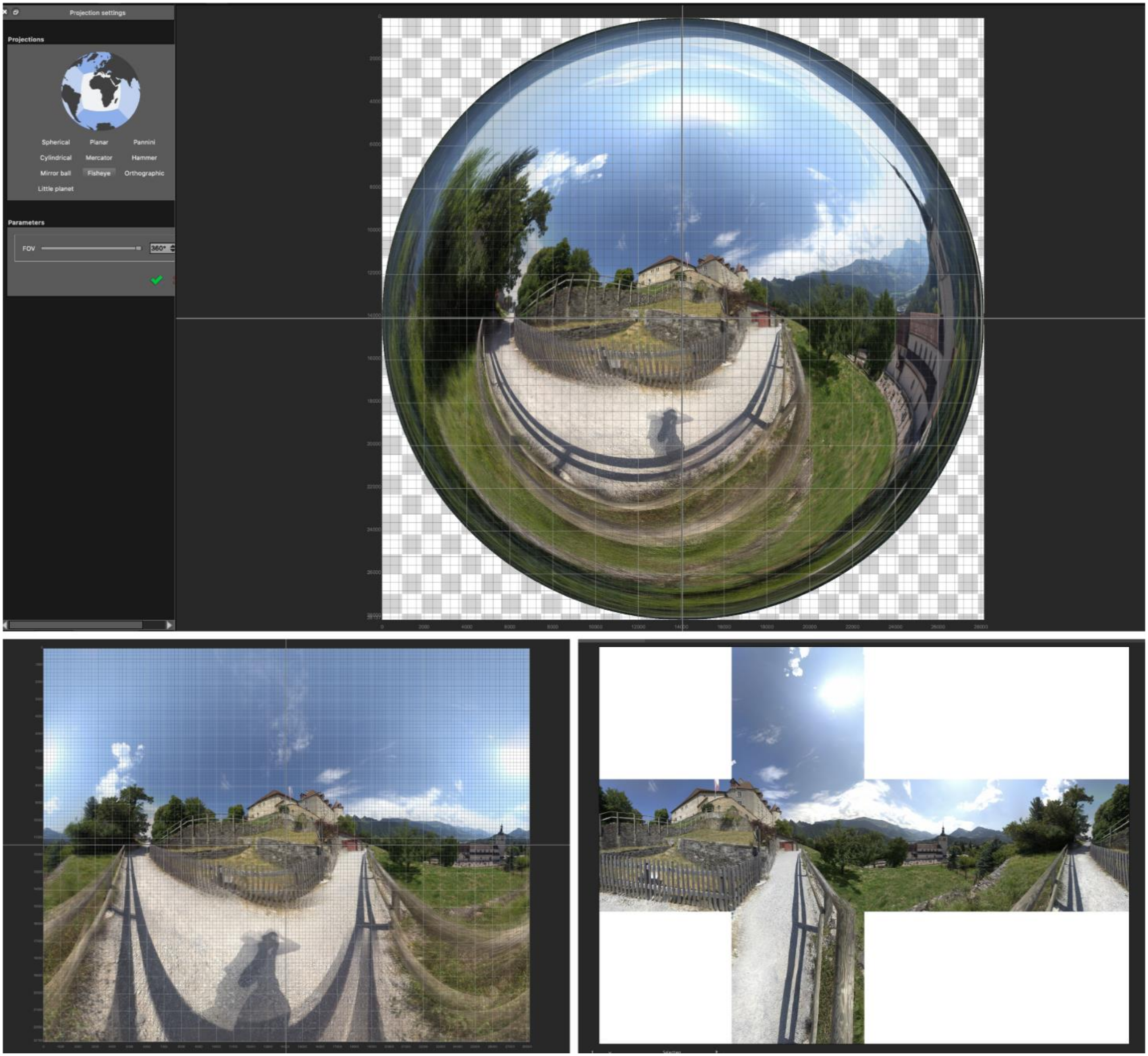


Figure 20 . Castle of Gruyere, Switzerland (2020). The same full panorama in different projections: azimuthal-equidistant (top) Mercator (down left) and cubic (down right).

A final spherical panorama[11] follows anamorphic principles: the environment is conically projected onto the sphere, thus followed by its flattening [93, pp. 1–6], [95], [96, p. 14]. Regarding the latter, there are many different cartographies of the sphere: all of them share the same graphic information but differ in leaving some 3D properties untouched and modifying others.

Thanks to the current dissemination of spherical perspectives in the field of photography, it is very easy to switch between these different cartographies using stitching software. To illustrate this, I used Autopano Giga[12] and converted the same panorama in azimuthal-equidistant (Figure 20 top), Mercator (Figure 20 down left), and equirectangular projection (Figure 21).

Note that it is also possible to export a cubical map (Figure 20 down right) since the morphism between cube and sphere has been largely studied in computer graphics after the introduction of the cubical environmental mapping in 1986 (see Second Part III.3) [98]–[101].

I.6.2 Drawing a spherical perspective

I mentioned in I.5.3 that a cylindrical panorama resulted from mixing several classical perspectives, then a projection onto the surface and a final tracing/painting. However, this a priori drawing is not strictly necessary. Instead, we can use the anamorphic principles (conical projection + flattening of the cylinder) to directly compose the drawing as it will be in the final destination surface. This latter procedure is the common one when dealing with spherical panoramas: one does the drawing thinking how the projections look after the flattening [96, p. 14].

[11] Understood in a contemporary way, see (**Error! Reference source not found.**)

[12] This program , a very versatile software from Kolor company, was deprecated recently. Nevertheless, the principles do not lose generality and any stitching software, such as Hugin [97], should be equally efficient in the conversion

In other words, the spherical drawing is defined in free vision (see I.4.4) accordingly to a convenient flat distribution, i.e., a cartography. Then to watch such an illustration in linked vision, one should fold it back to the spherical shape. Such a goal was very complex to achieve by physical means[13], but not anymore with the appearing of digital technology.

I.6.2.1 Drawing in equirectangular projection

The more standard output among spherical projections is the equirectangular [34, p. 17]. Such a map can be navigated in immersive modality with free desktop apps (e.g., FSP Viewer [102]), with online tools (e.g., RoundMe [103]), or used as input for the creation of virtual tours and collaborative spaces. Nevertheless, generally the equirectangular format is converted internally to cube mapping due to the technology used for the visualisation (i.e., tiles), which improves its rendering quality and speed of visualisation.

For cartography uses, there is a constant spacing among meridians which look in the flattened map like vertical straight lines. As well, there is the same constant spacing among circles of latitude (parallels) which look as horizontal straight lines [104]. This gives as a result a distribution with a highest distortion the closer one is from the poles (Figure 21).

In particular, if one wants to draw in equirectangular projection, meridians and parallels look like vertical and horizontal straight lines respectively, which do not represent such a big issue. Yet, a drawing composed by just meridians and parallels can capture only very simple architecture, and if one uses this kind of fixed grid method the task of plotting general scenes gets very complicated if not impossible.

Recent works of António B. Araújo [34], [96], [105] have resolved this limitation by providing simple ways to draw general geodesics segments (not just meridians and parallels)

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[13] My personal opinion is that this can be the main reason why Barre & Flocon never completed the perspective up to 360° in those years (what for?)

using simple descriptive geometry procedures. Since spatial lines always project as geodesic segments on the sphere, this allows one to draw any imaginable scene.

Architects and artists such as Gérard Michel, Bruno Sucurado, Michael Scherotter, Matthew Lopas, Jackie Lima, Arno Hartman, Chiara Masiero Sgrinzatto, Tom Lechner (among many others enthusiasts), use equirectangular perspective overcoming the limited field of view of classical perspective [91], [106]–[112]. They follow many different approaches from approximate trial-and-error procedures up to accurate descriptive geometry constructions. For example, they may draw guessing and understanding basic principles (Figure 24), attaching the content to a pre-formatted grid (Figure 22), tracing on top of an equirectangular picture (Figure 25) or constructing the spherical perspective using vanishing points and descriptive geometry (Figure 32).

Figure 21 . Castle of Gruyere, Switzerland (2020). Equirectangular projection

It is clear that there is not such a hard division among methods. Instead, one switches often between methods in the path of discovering and mastering the technique. For instance, one may use the grid and either construct the drawing simply guessing or using vanishing points. Therefore, I divide these approaches in two big groups: the first, includes those who can just create an equirectangular illustration; the second, contains those who can also explain the spherical perspective in descriptive geometry terms.

I.6.2.2 Drawing and guessing

The former group will act limited by a trial-and-error logic, that is, there is a self-experienced learning of basic principles by testing the pair drawing/VR visualization. For example, let us suppose that one knows anything about equirectangular projection, then draws a straight horizontal line in the equirectangular map and discovers that it looks like an arc of circle (or, in other words, it represents a circular geometry) when navigates the drawing in VR modality. As a consequence, the draughtsman sets mentally the rule: horizontal line (map) = arc of circle (VR).

Nowadays, essential principles like these abound on the net, opening equirectangular drawing to *any* person [106], [112]–[118]. Putting aside some of these tutorials' superficiality or commercial purposes, they help to spread the word and to raise the interest among designers. Maybe thanks to this position, away from the academical point of view, they constitute a first line of experimental practice without many protocols. They offer many times alternatives to illustrate outside the classical perspective and some of them, after many steps, may even be useful to open new research branches among scholars.

Nevertheless, one must be very careful since the disseminated information often does not guarantee that draughtsmen (both the one writing the entry and the one learning) really understand the representation system. In many of these examples, they operate through 3D software or plugin applications (see I.6.3). In other words, one may draw even fluently on the equirectangular map, but maybe never have learned spherical perspective.

Indeed, with the principle *horizontal line (map) = circle (VR),* it would have been easier to study the equirectangular projection and understand that following such a rule one was actually *drawing a parallel*. Deepening even a bit more we arrive to understand the projection in a basic grid already composed by essential points of views, i.e., frontal, left, right, back (Figure 22). In case like this, the draughtsman illustrates blindly and – at maximum - limited to the grid, without really being able to answer *Why does a straight line looks like an arc of circle?*

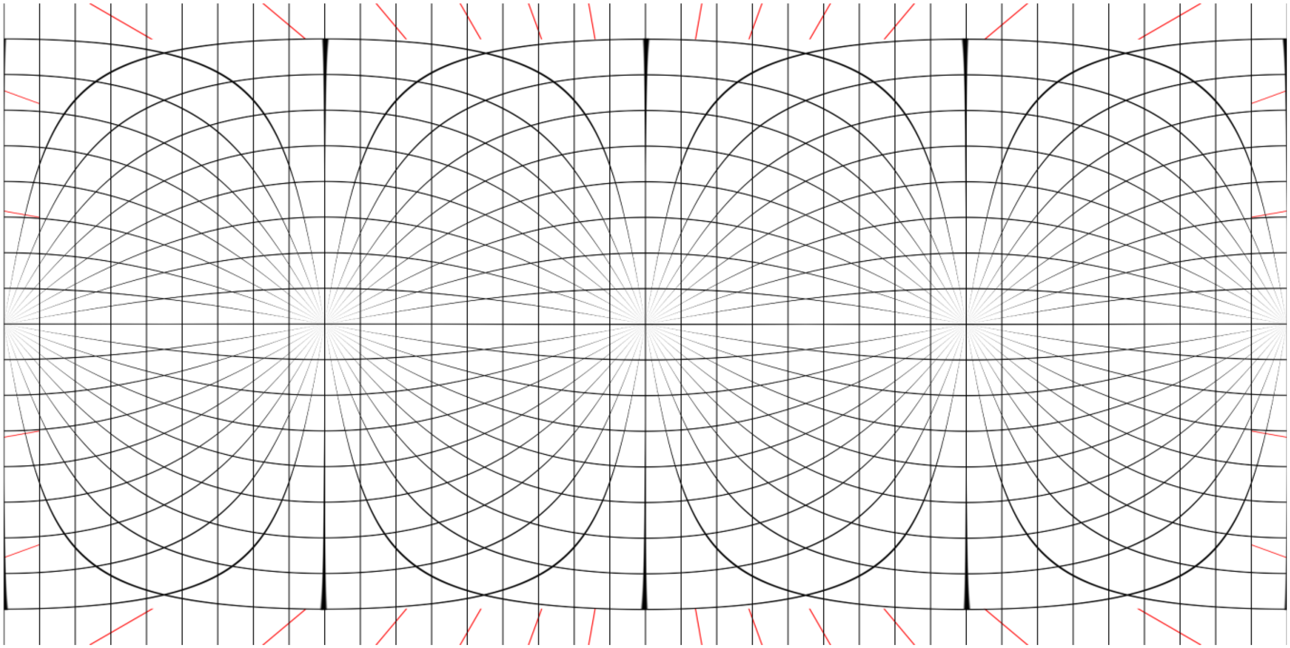


Figure 22 . Equirectangular grid from David Swart's website [106]

As an example of this approach, I quote my early equirectangular drawings, gropingly guided by these limited notions. I learnt the basic package of principles (e.g., the distribution of a basic grid such as the one from Figure 22) and found more “tips” by myself simply practicing. Then, I disseminated my experience with spherical drawings outside and inside university courses, spreading it mainly among architecture, design and engineering students [119], [120].

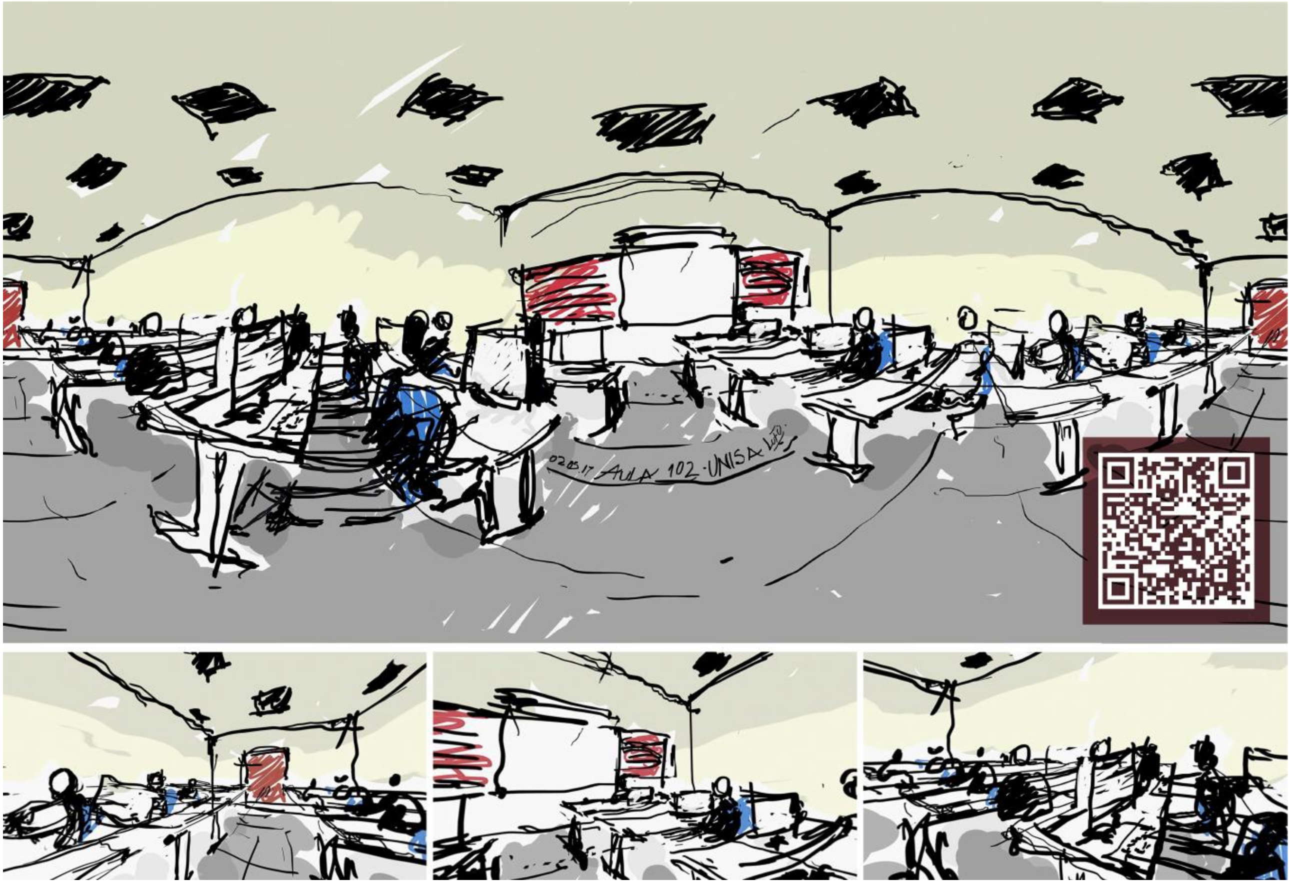


Figure 23 . Room 102 (2017). Equirectangular drawing and VR visualisation

Figure 23 and Figure 24 are examples composed with these few basic notions. Even if they both appear like proper spherical drawings at a first sight, they had some limitations: the former looks well during the VR visualization, but the arcs represented in the latter are wrong (Figure 24 down right). The difference between them is the represented geometry, which in the former case is simpler and limited to basic shapes, while the latter includes a circular geometry which I could not represent correctly for my limited comprehension of spherical perspective.

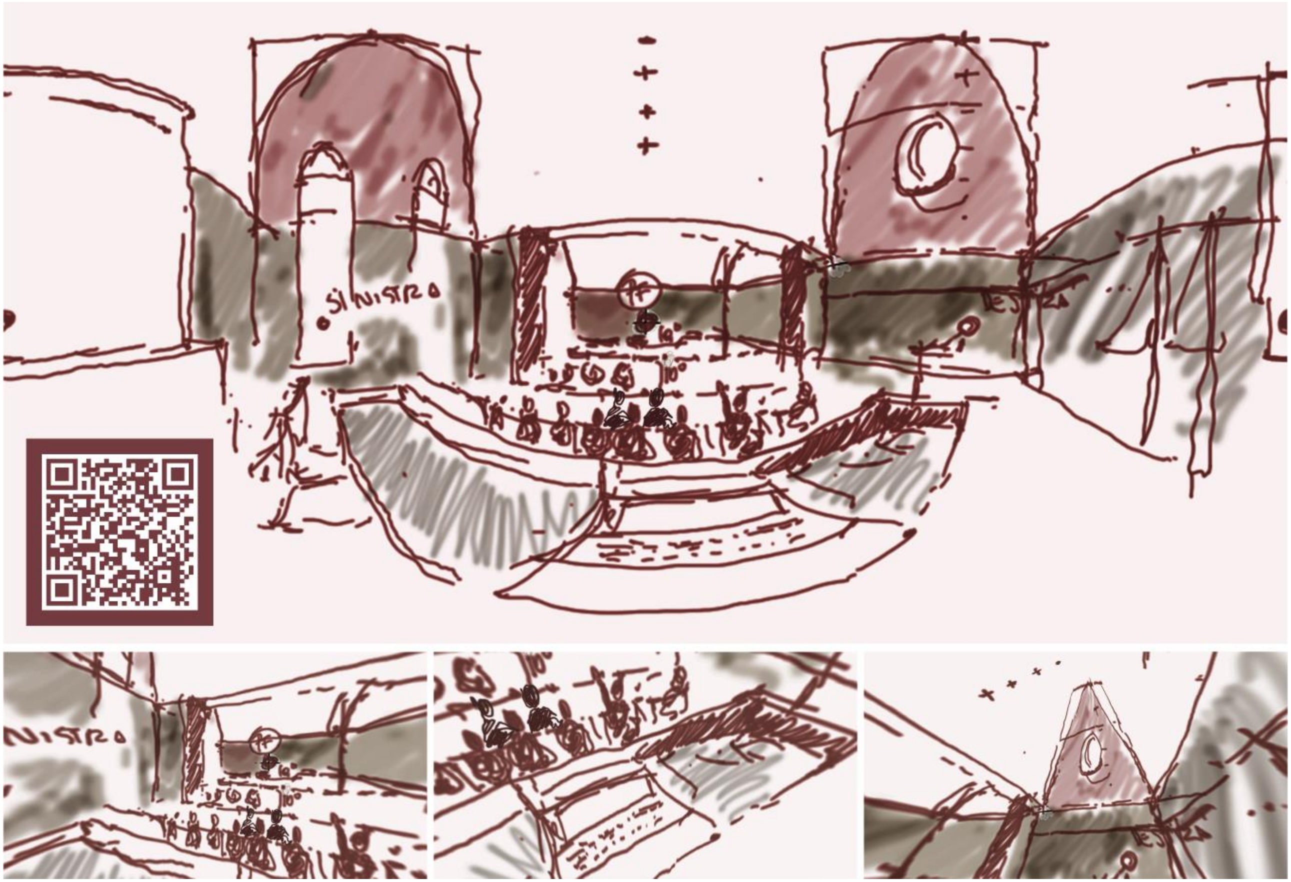


Figure 24 . Aula Magna Università della Campania (2017). Equirectangular panorama and VR visualisation

Examples like these give incoherent results, revealing the lack of a deep understanding and highlighting the weakest conceptual notions. In other words, the procedure may even be working up to some point, but that does not assure that is *correct* (see II.3.3).

I used to talk about these drawings as *guided by intuition*, but actually, that intuition was filled with my early notions of classical perspective. In effect, today I can see that I was naturally trying to transfer these concepts to the spherical case. Indeed, in the arcs of Figure 24 there is a construction based in proportions, right as I learnt to draw by direct observation using classical perspective. Hence, I was not failing to understand the geometry, but in understanding the representation at a macro level, i.e., the perspective at the anamorphosis level.

As a conclusion, drawings like these help the draughtsman to develop a reasoning by trial-and-error. Such a reasoning may be exhaustive for a correct final VR visualization, but it is limited in the way the designer does its work since it partially of fully ignore spherical perspective rules. The use of the grid may be a signal of start understanding vanishing sets, but the full understanding of the equirectangular projection is still incomplete.

I.6.2.3 Teasing rules

In a second stage in the path of mastering equirectangular perspective, I put some examples from designers that already understand much better the general system, although for specific purposes, they do not aim to an accurate delineated final illustration. Instead, they point to a more spontaneous expression, focusing into the quick impression captured either in surveying sketches made on-the-spot or in whimsical compositions. As well, they may or not use the grid and a proper construction.

For example, Architect **Bruno Sucurado** from Argentina used the grid to draw remotely Atrani, Italy, during a collaboration for Gigakahn project in 2016/17 [119]. He traced on top of the grid watching the original panorama previously shot on-the-spot. He then teased the traditional image of Amalfi's Coast, mixing his illustration with iconic works of Louis Kahn (Figure 25).

Figure 25 . Equirectangular drawing of Atrani, Italy (2017) by Bruno Sucurado. Traced on photography

When Bruno traced, he followed some criterion, that is, he did a personal selection among the many elements of the picture. That way, he understood the space in a personal key. Next, he played and manipulated vanishing points, hiding Kahn’s modern architecture among the traditional buildings and opening a *call to play* during the navigating of the panorama. His exercise had multiple results: he learnt about Amalfi’s traditional architecture, practiced spherical perspective and soften his hand.

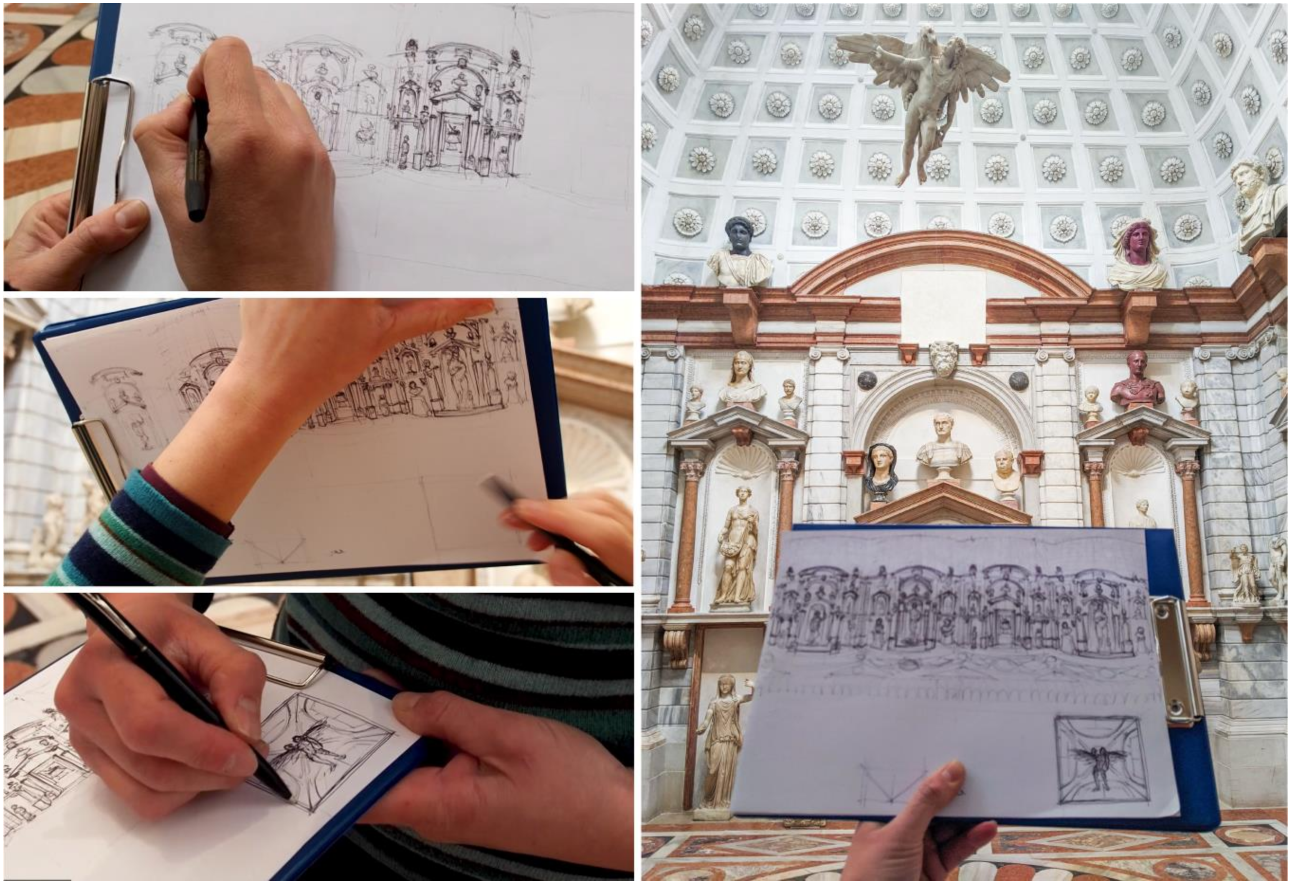


Figure 26 . Palazzo Grimani's Museum at Venice (2020) by Chiara Masiero Sgrinzatto. Mixed composition (equirectangular panorama + classical linear perspective)

Another example is given by architect and visual designer **Chiara Masiero Sgrinzatto**[14] [111] who applies the equirectangular format for surveying architecture on-the-fly and for

[14] Special thanks to Chiara Masiero Sgrinzatto, who told me about her methods and gave me all these details in an informal interview that neither her nor me realised to be doing

illustrating places from her memories. Since 2004, she creates VR contents, in the attempt of not limiting immersive media only for its descriptive purposes but also enhancing their narrative features [111, Sec. ‘about’].

Chiara is at the point of no needing the grid anymore, she freed herself from it, giving more flexibility to the final expression and drawing on-the-fly with her hand light as a feather. This is because she keeps the curves of the equirectangular projection in her mind after she got used to it (in part influenced by her works with photographic virtual tours). Yet, illustrations such as *“Bird’s Eye View of Venice”* demonstrates largely her mastering of the perspective [111, Pt. Venice, Bird’s Eye View].

Chiara likes to tease and play with the whole system, altering the reference system and dealing with poles” distortions in alternative ways. For instance, for surveying Palazzo Grimani’s Museum at Venice[15], she solves strategically the upper pole in a very simple way: she draws the top part in linear perspective and then stitches the illustration to the main panorama using software [111, Pt. Museo di Palazzo Grimani], [121]. In such a way, Chiara’s composition follows an equirectangular distribution for the main panoramic field of view (where distortions are simpler, almost as a cylindrical perspective) and a classical linear perspective for the upper part (where distortions are more complex) (Figure 26, Figure 27).

Another strategy of Chiara is in her Varisco’s street illustration: she applies the so-called “laundry machine” scheme, that is, a rotation of the reference system [111, Pt. Calle Varisco].. That way, Chiara puts the top vanishing point in the centre instead of the frontal one, concentrating the profiles of the alley in the less distorted zone of the equirectangular map (Figure 28). Once the drawing is finished, the reference system can be easily “put back” using software.

[15] Survey made with Luca Vascon by permission of the *Ministero per i beni e le attività culturali e per il turismo* and within the project *Tribuna Grimani VR* as part of the exhibition *Domus Grimani 1594-2019*

Chiara and Bruno's examples show how the designer can play with the spherical perspective on the fly, according to the circumstances and to its convenience. The mastering of the technique up here allows to have, on the one hand, the grid as a reference to define basic geometries, and, on the other hand, freer expressions to quickly capture an architectural hypothesis or a personal impression on-the-spot during a survey.

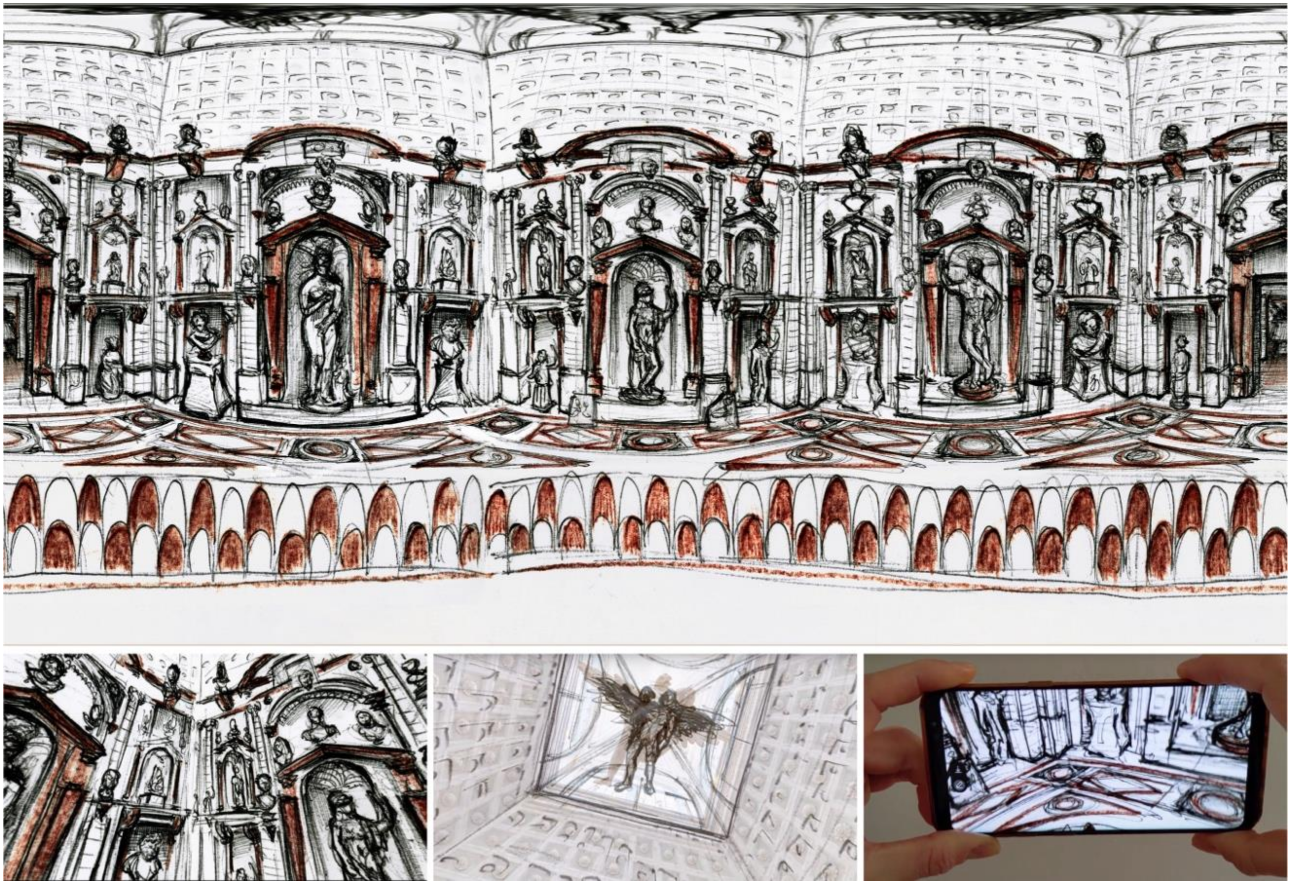


Figure 27 . Palazzo Grimani's Museum at Venice (2020) by Chiara Masiero Sgrinzatto. Final equirectangular composition and VR visualisation

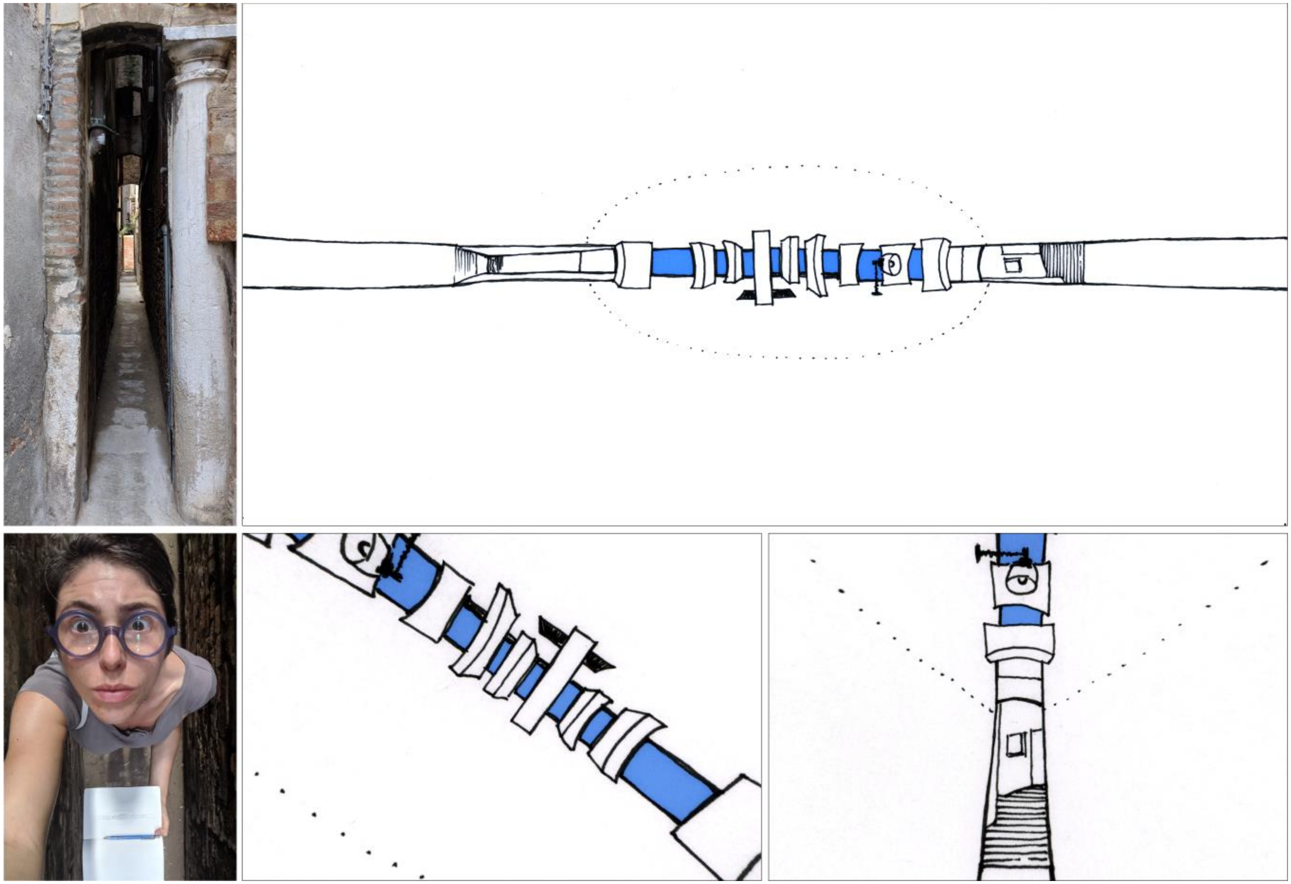


Figure 28 . Varisco's street, the narrower alley of Venice with just 53 centimetres wide (2019) by Chiara Masiero Sgrinzatto. Equirectangular panorama, VR navigation and pictures on-the-spot

I.6.2.4 Full constructions

Architect **Gérard Michel** draws always on-the-spot by direct observation. He creates grids accordingly to any specific need during the drawing process, i.e., he can construct it for any vanishing point and not just the essential ones (Figure 29). To such an end, he calculated the height of all the points crossing the vertical lines[16] [91], [122].

Gérard reaches a high level of accuracy in the final drawing, mastering perfectly every mechanism of the perspective (Figure 30). In effect, he managed to drew many details of his *Bureau* translating efficiently the descriptive geometry knowledge from the classical perspective to the spherical one, as can be clearly seen in the many possible accidental perspectives during the VR navigation (Figure 30) [123].

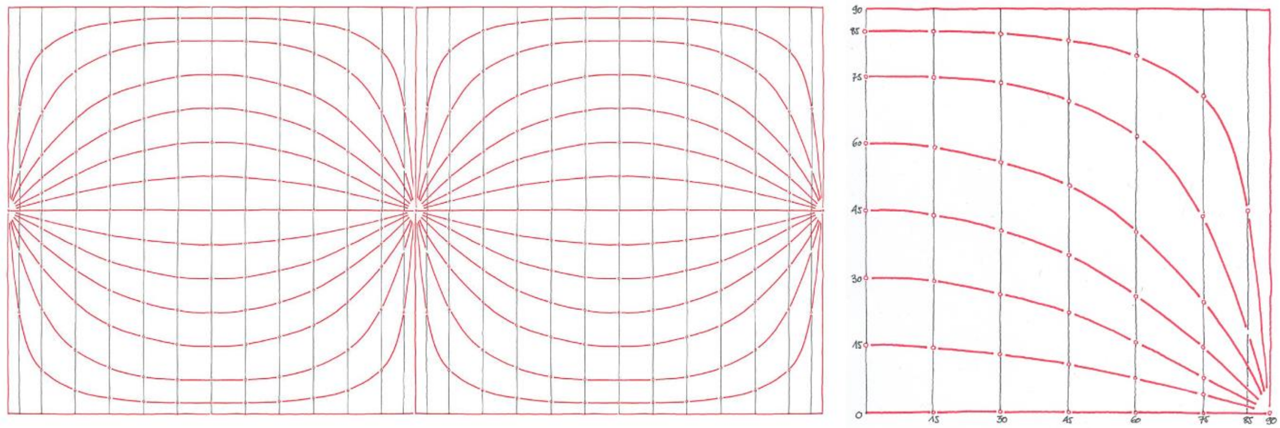


Figure 29 . Equirectangular grid and detail of the intersections (2018) by Gérard Michel

Another examples are *How to draw curvilinear perspective* of **Massimo Marrazzo** [124]**,** and *Extreme Perspective! For Artists* of **David Chelsea** [90]**.** In particular, the latter is a dynamic

[16] Special thanks to Gérard Michel, who kindly explained me these details through the *"Academical Facebook"*

and didactic book oriented for cartoonist-artistic purposes and it focus the content in practicing spherical drawing general concepts, not really reaching a deep theoretical development [90].

Marco Masetti from Bologna, professor at the Fine Arts Academy and graduated in Scenography, wrote a deeper theoretical and practical book, applying to both straight and curvilinear perspectives [19, pp. 127–172].

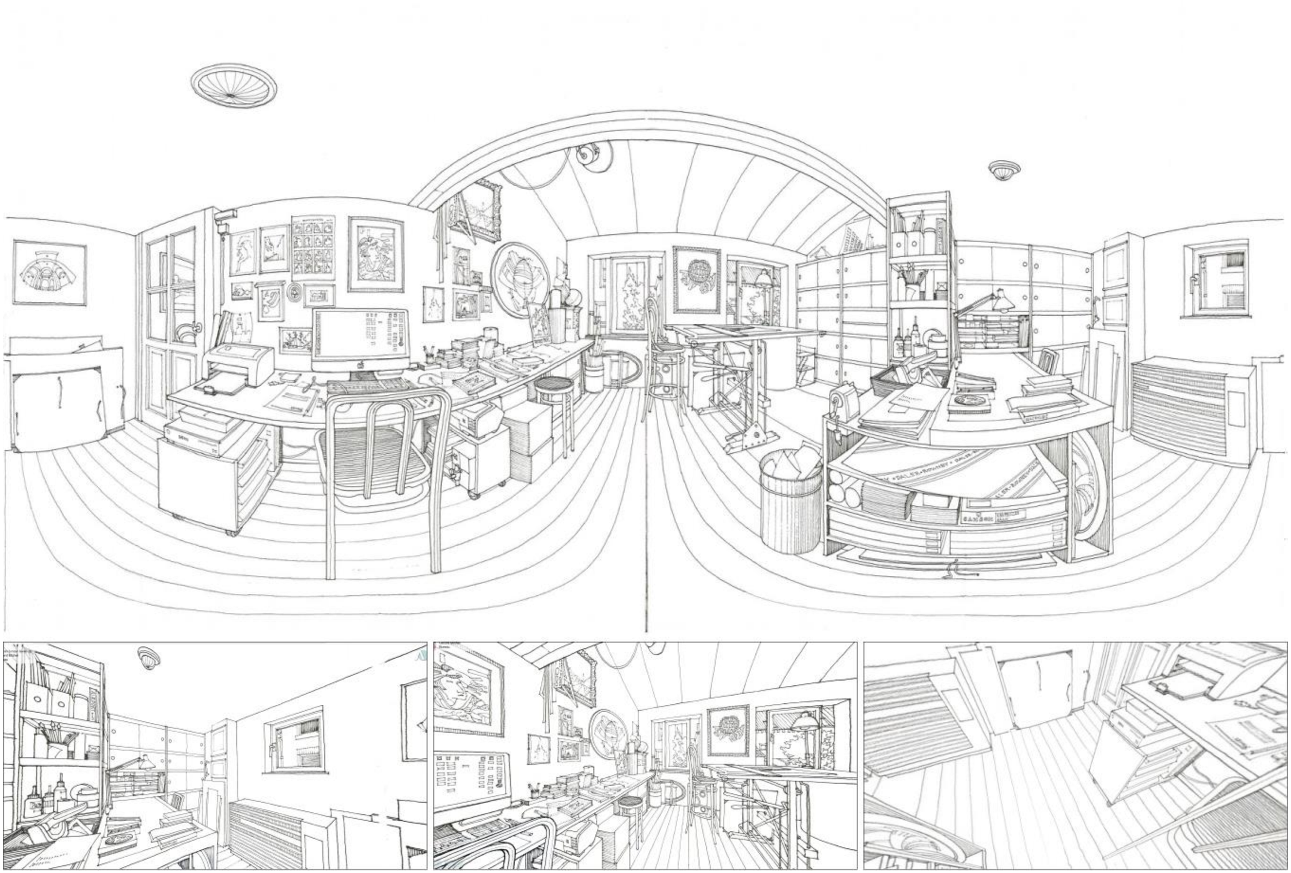


Figure 30 . Equirectangular panorama and VR navigation (2018) by Gérard Michel

António Bandeira Araújo solves spherical perspectives following the same general scheme from which all these methods derive as special cases [93, pp. 1–6], [95], [96, p. 14]. This scheme was first implemented for the azimuthal-equidistant case [7], [96, Sec. Azimuthal Equidistant Spherical Perspective], but it also applies to the equirectangular format [34], [96, Sec. Equirectangular Perspective] and any other spherical perspective [17]. It defines a spherical perspective as a two-step process: an anamorphosis onto a visual sphere around O, followed by a flattening of the sphere.

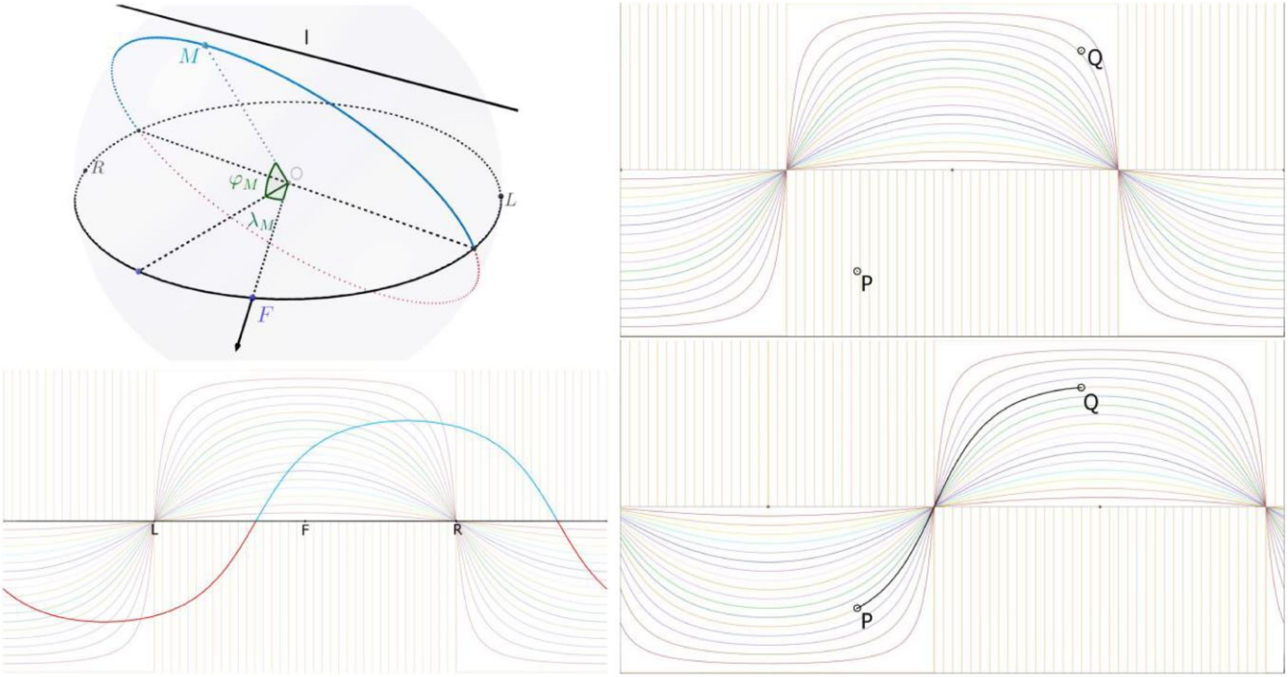


Figure 31 . A line l projects as a great circle (left). Given two random points in the equirectangular map, there is just one great circle passing through them(right). António Bandeira Araújo

[17] Indeed, it is also applied in the method considering the cube as a special case of spherical perspective (Second Part IV.5)

At the anamorphic step, each line *l* projects conically onto an arc of great circle (geodesic) of the visual sphere (Figure 31 left) [19, p. 127], [96, pp. 9–12], [125, p. 17]. Recall that any two points on the sphere define a single great circle trough them (Figure 31 right up). This anamorphic step is the same for any perspective, and defines vanishing points as intrinsic, spatial objects, each line projecting as exactly one half of a great circle, with always exactly two vanishing points, mutually antipodal on the sphere. Then on the second step, these geodesic arcs are flattened according to the rules of the specific perspective in question. So, for any given perspective (hence for equirectangular, in particular) one big issue is: ***How to plot the perspective image of a general geodesic from two given points?*** The way that a geodesic looks in the flattened map, adds a complexity since *"the plot looks sinusoidal up to around the* 45° *mark, and then grows squarish as* φ_0 *approaches* 90° *degrees"* [34, pp. 18–21].

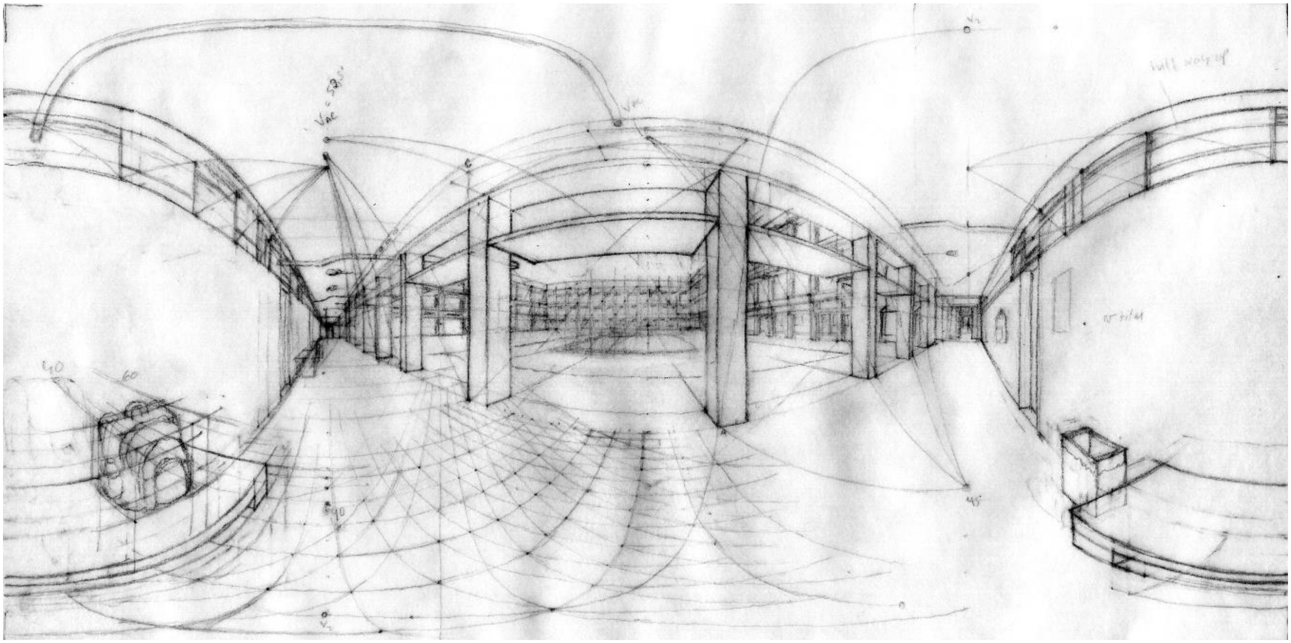


Figure 32 . Courtyard at ISEL school of engineering, Portugal (2018) by António Bandeira Araújo. Equirectangular construction using vanishing points and descriptive geometry

A. B. Araújo shows how to draw created a special grid of flattened great circles by ruler and compass and then how to make it into a dynamical grid: after drawing two essential points P, Q he shifts right/left such a grid until matching the unique great circle passing through them and traces the geodesic (Figure 31 right bottom) [34], [125]. This is shown to work for any line, not just horizontals and verticals.

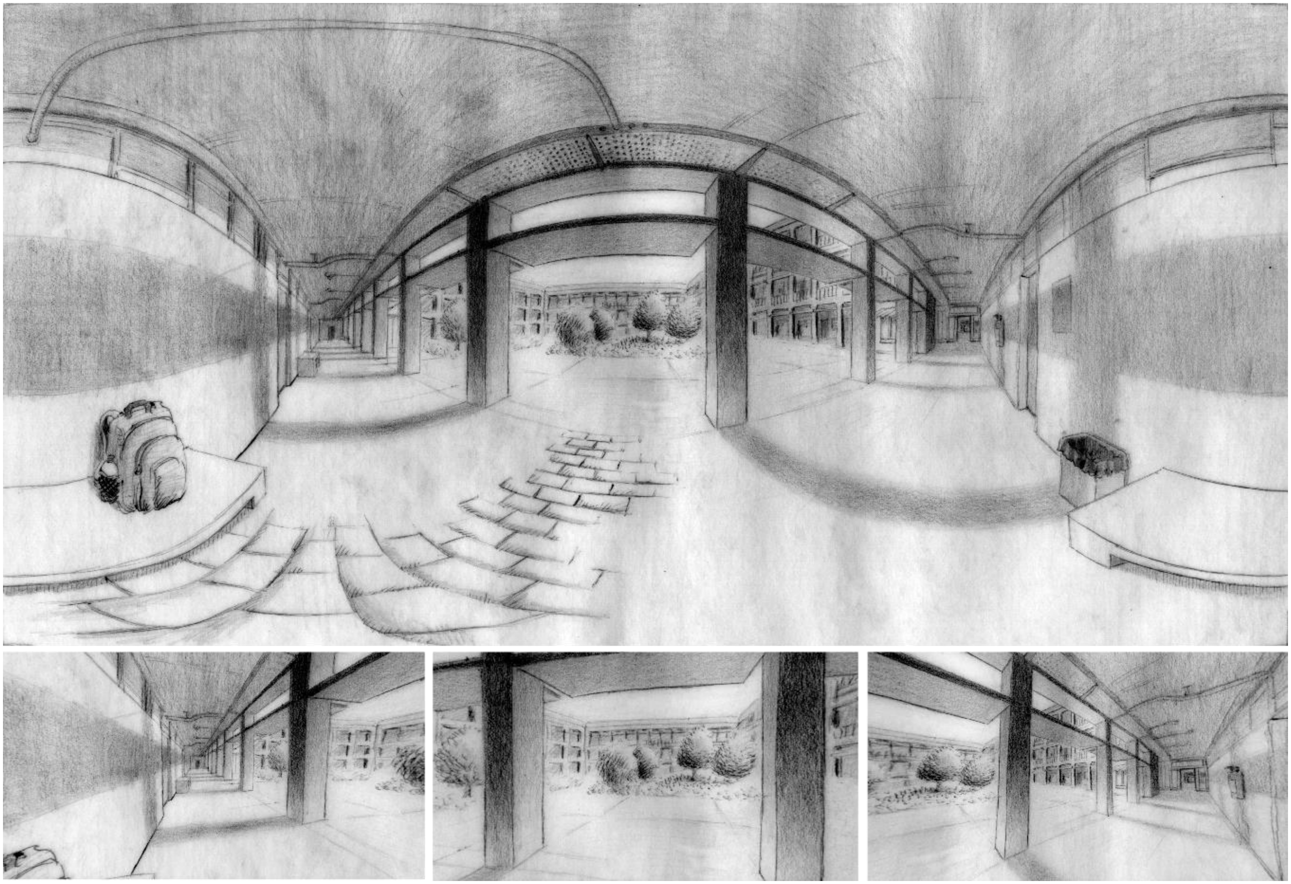


Figure 33 . Courtyard at ISEL school of engineering, Portugal (2018) by António Bandeira Araújo. Final panorama in equirectangular perspective and VR navigation

Such a system allows an accurate construction and an adaptation of methods from classical perspective such as the repetition of regular elements using vanishing points and diagonals (Figure 32). Furthermore, it is possible to reach a high level of details using geodesics even in a small-size drawing (e.g., Figure 33 is A4-size).

These last methods allow analytical creations that cover from any field of view up to the whole visual sphere, and fully use all the features of this representation system. Such procedures break down the whole perspective into simple constructions made with points, lines and planes, i.e., geometrical described methods. Only after understanding these simple elements one can give a full method that includes an integral solution.

As a conclusion, spherical perspectives drawing methods have an exhaustive material defined in scientific terms. In particular, they include procedures for tracing using simple tools (such as the ruler and compass), or, in other words they operate with classical geometry through straight lines and circles [126, p. 27].

I.6.3 Software

Some drawing applications were developed based in the theoretical definitions of spherical projections. Right as the different methods for drawing, there are also different approaches regarding the software setup: some may **adapt to the task of drawing** (even if they were born with other purposes, such as, editing panoramic photography) and some are **specifically optimised for equirectangular drawing**.

I.6.3.1 Drawing and guessing

Among the examples of the first group, Adobe included in the last versions of Photoshop [127] a tool to switch between the equirectangular flattened map (Figure 34 left) and its immersive vision or, as they call it, *SphericalMap viewport* (Figure 34 right). The instrument is meant to solve the editing of nadir and zenith zones of equirectangular photographic panoramas. Yet, it can adapt to the task of spherical drawing since Photoshop is also a raster-based drawing program. With this instrument, the user can edit the image either in the equirectangular map or in every classical perspective visualised at the viewport window [128]. The tool is accessible from the menu[18] and is useful for on-the-fly corrections or after-effect modifications of equirectangular panoramas.

The tool promotes more trial-and-error explorations than proper perspective constructions since the program itself focus on the impacting aesthetical experience than in the precision of its construction. As well, the guidelines are not optimised for managing vanishing points and the way the tool switches between one and another format is closed code.

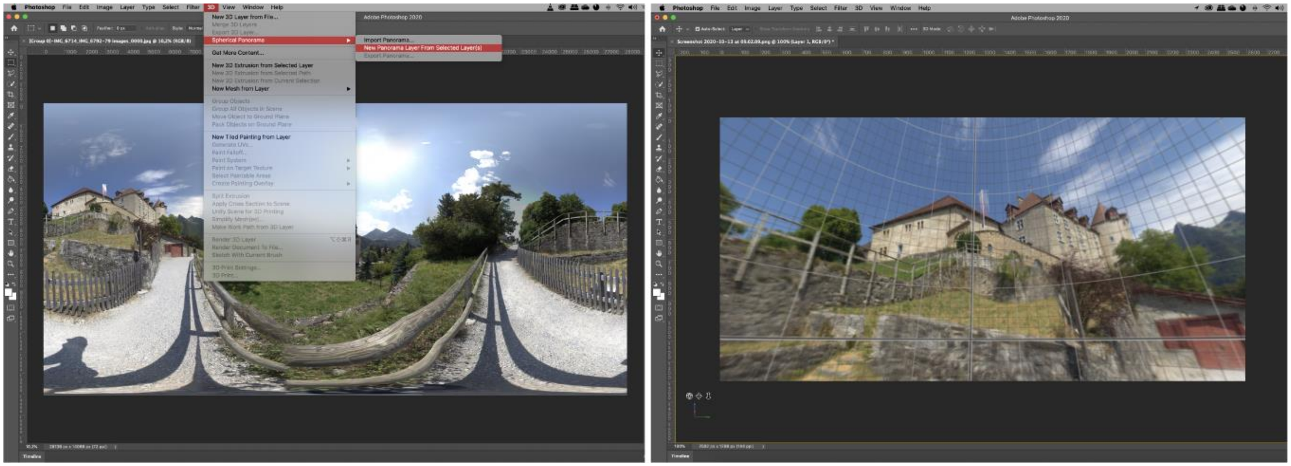


Figure 34 . Switching from equirectangular to VR viewport (left). Immersive view from the VR viewport (right). Adobe Photoshop v2021

[18] Selecting: 3D / spherical panorama / new panorama layer from selected layer

I.6.3.2 Full constructions

Among specific options for equirectangular drawing there are raster-based tools like ***Sketch 360*** by Michel Scherotter [110] and ***Eq A Sketch 360*** by Antonio Bandeira Araújo [105].

The former runs as a Windows-based application available for free at the Microsoft store. The user draws on top of the equirectangular grid and sees the results in VR modality in a parallel viewport (Figure 35). In the example, it is possible to see a very simple trace on top of the grid and its reconstruction within the immersive navigation (upper left corner).

By its part, ***Eq A Sketch 360*** is promoted as a *serious toy* aiming "*to develop sketching intuition regarding the structure of equirectangular drawing as proper perspective drawing, with its specific constructions of vanishing points, geodesic segments, line projections, antipodes, and grids*" [105, p. 1].

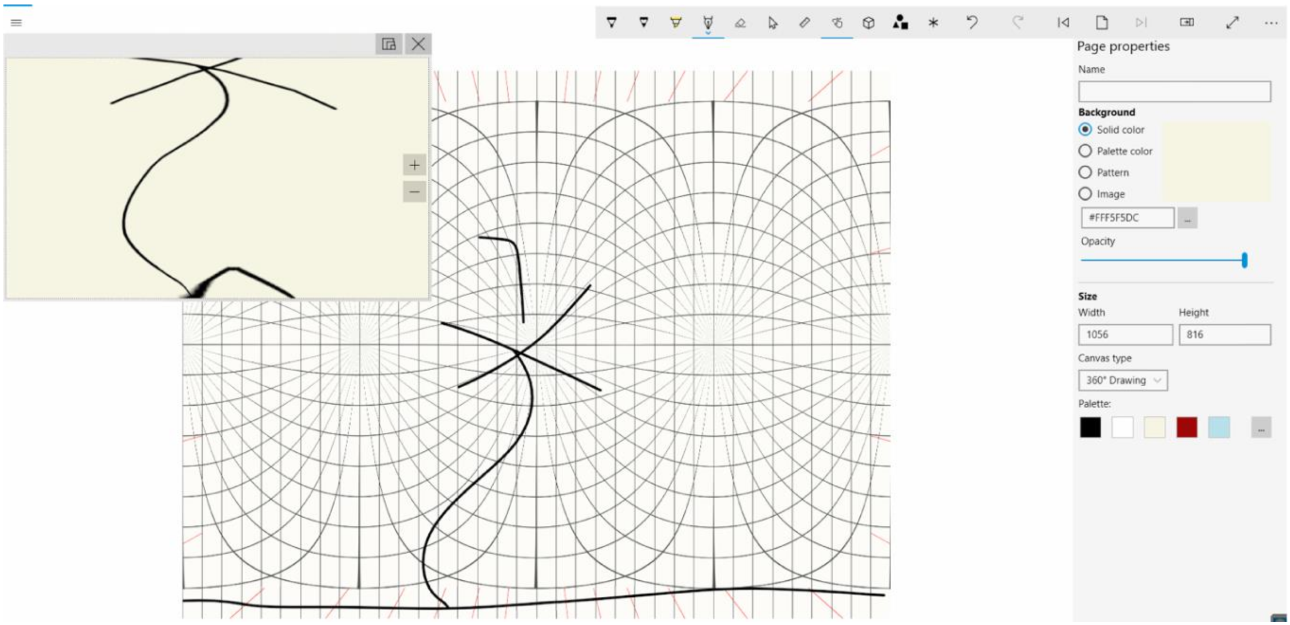


Figure 35 . Sketch 360 running on Windows 10

A preliminary version of the software was compiled using Processing and tested in both MacOS and Windows for quick sketches (Figure 36). The coding is still in development, but the current version has available commands to draw geodesics and vertical lines, erasing, saving and exporting the drawing. As in the previous case, the user can shift the background grid and see the correspondence between great circle and points.

The core function of *Eq A Sketch 360* consists in drawing the right geodesic from two given points: the user fixes two points and the brush follows the correspondent great circle among them.

On the one hand, *Eq A Sketch*'s code discourages the user from following the grid by hand, which takes away part of the sketching spontaneity (it allows for freehand drawings too, but freehand brush tool is poor and the focus is clearly on the snapping tool). On the other hand, it adds precision and allows the construction of a geometry in a linked way by fixing one of these points (e.g., using it as a vanishing point), placing the second in a new measured or wished position, and tracing again.

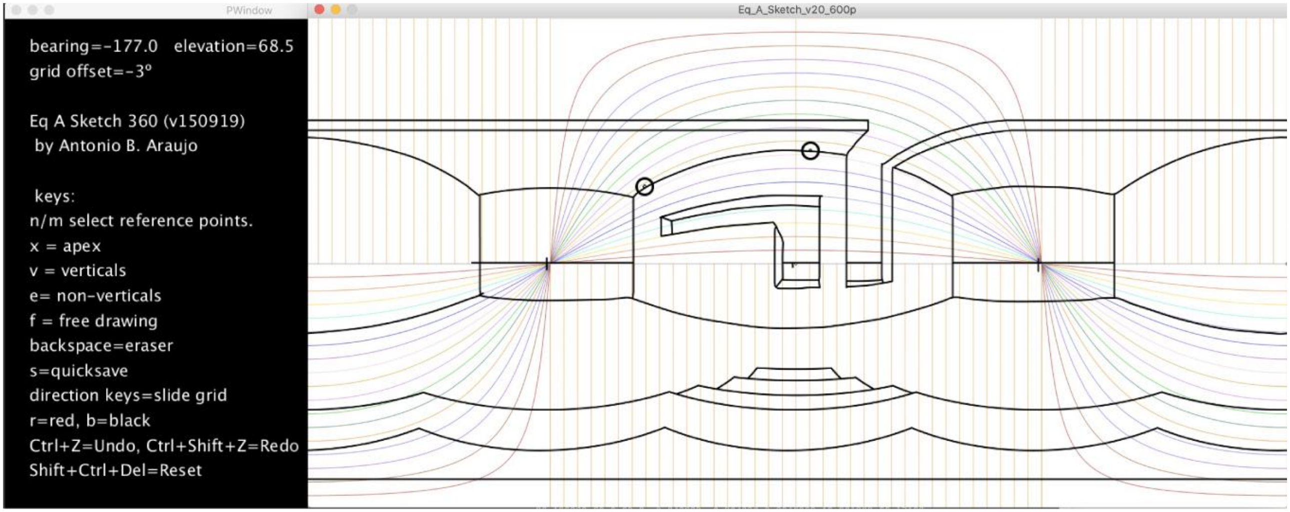


Figure 36 . Drawing using Eq A Sketch 360 and geodesic-based constructions

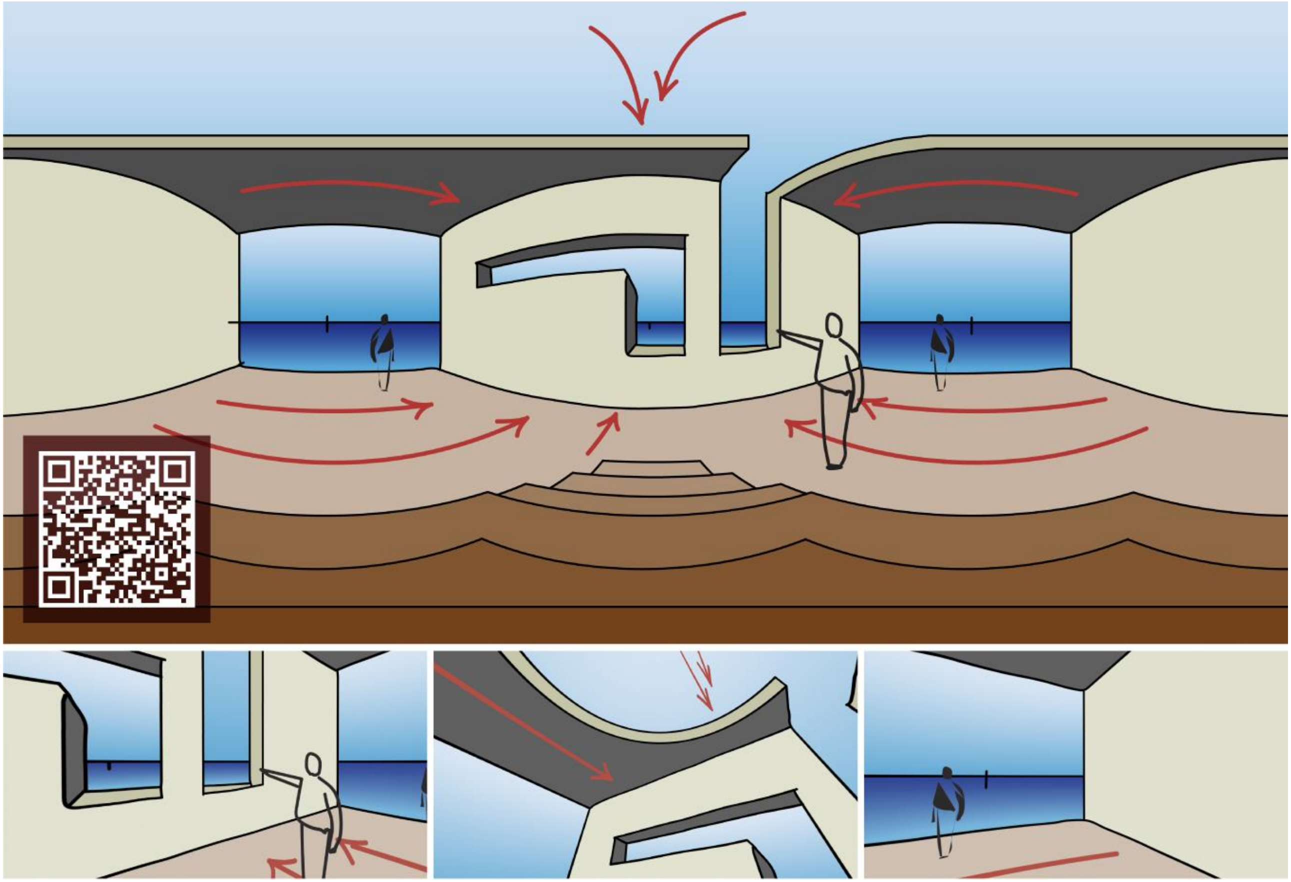


Figure 37 . Equirectangular drawing using Eq A Sketch and VR navigation

After this first raster-based option, the plans of the developer are to go for a vector-based alternative in the short time. The structure of the program and the gained experience with this initial programming, will surely be helpful for the new development. Still, there is also the possibility of exporting the drawing without the grid. Hence the illustration may be imported in any vector-based program, traced and filled with shadows or gradients (Figure 37).

In a recent collaboration with Microsoft, the equirectangular ruler tools and the sliding grid (a library called *Eq A Snap*) from *Eq A Sketch 360* have been integrated into the Microsoft's *Sketch 360* program described above. *Eq A Sketch*'s version of the tool remains the most dedicated to technical drawing, while *Sketch 360* is a more well-rounded program, fit for more causal sketching and integration with multimedia tools.

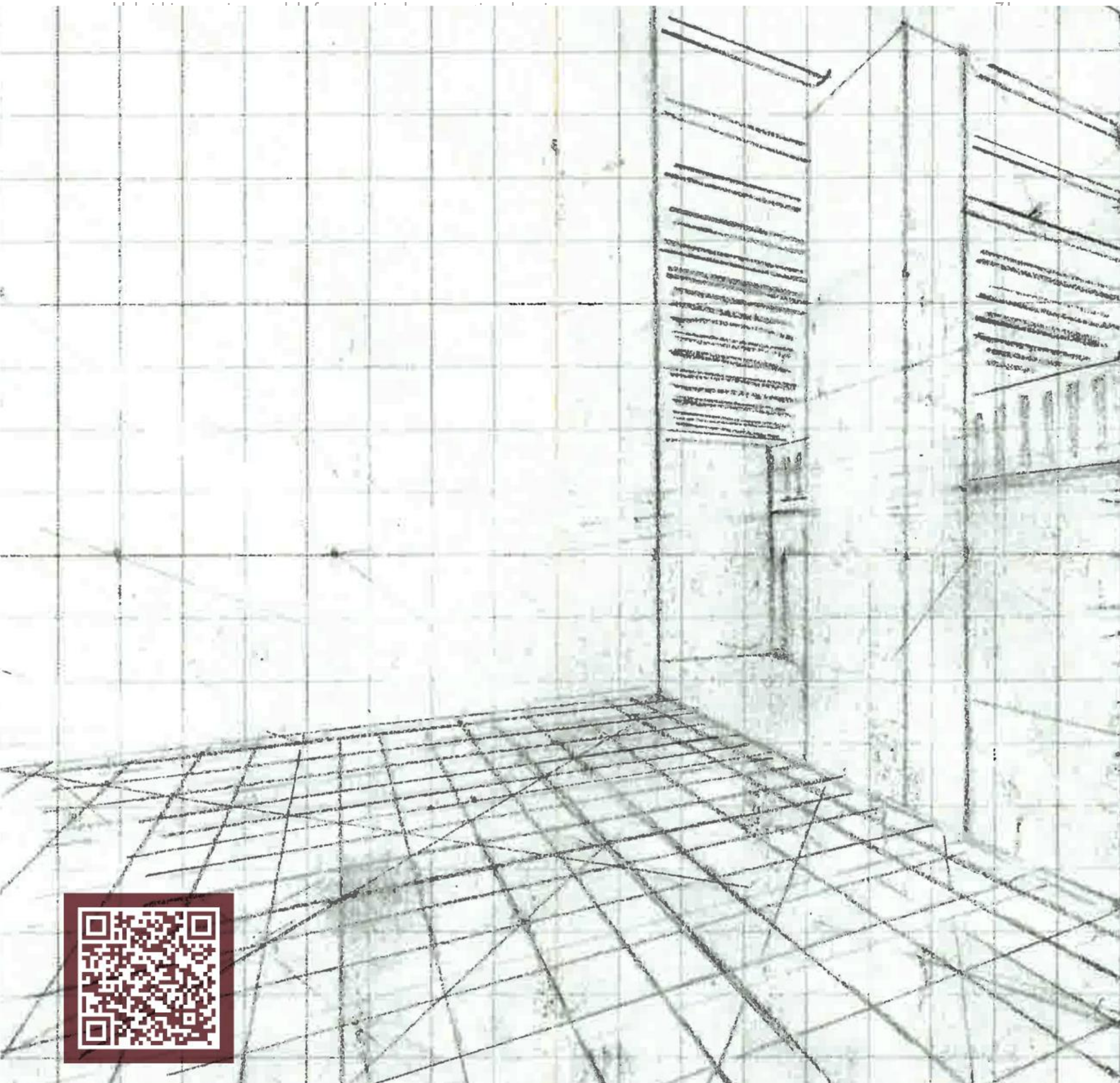
With all, drawing a spherical perspective allows architects and designers to explore the geometrical space and their project ideas without any limitation of field of view. In a further step, with digital technology and the aid of VR glasses, the user may not just watch but also *be inside* the artwork.

I.6.4 Projection centre and observation area

The immersive experience of the *Rotunda* led to nowadays projects such as M.O.D.E. and Pacific Dome [129], [130]. These immersive installations have an architecture that replies to an ordinary cinema format but closed with a dome instead a plane roof. The dome is actually the screen where the video gets projected. In terms of full immersion, the achieved result is even less effective than the old panoramas due to the physical limits of the dome and the distribution of the seats. The former restricts observers to a cone of view near 150° since they can just partially rotate their heads. The latter results in a false mimetic effect since the observer is not guaranteed to be at the area of observation[19]. Therefore, the result is a broken continuity of the full immersive illusion, which brings producers to cover the gap focusing (again) in the spectacle of a highly impressive show with sounds and dynamic content.

[19] Actually, just the projector is at the centre

In many of these installations (as also happens in the cubical case), the streamed content is the video projection of a pre-defined computer model thus projected using mirrors. This means that the content comes from a **dynamic and interactive model** (see II.2.2), far from a perspective construction. As a consequence, many of the problems faced for streaming the content are closely connected with the fact that designers do not deal with spherical perspective knowledge, but with trial-and-error procedures [5], [131], [132].



<------ a small poetic licence ------>

- Do you know how to draw a perspective?

- Yes, you just need to open this program, select this tool, click here, here and voilà!

<-------------------------------------->

This dramatized dialog comes from a real experience of A.B. Araújo. In front of this, I question myself: What happens if the software stops updating and eventually disappears? Do I really know how to draw? Do I know how to properly construct a perspective? Do we (that generation old as Autodesk) know what is descriptive geometry about?

In this philosophical questioning, I see the current situation as a possibility for transmitting the capabilities for reasoning and thinking. With the strategy of promoting the bases for cubical drawing and not just a software for it, such a knowledge can stay – literally - in our hands. From there, the role of a method for representing becomes clear: the rationalisation of the drawing process in terms of operable elements. Those elements allow the user to compose a drawing logically, and represent the key for the transition between means. The understanding of such components allows us to optimise resources and use each technique in their best moment within the design process. Hence, I aim to position the hybrid immersive model as a way of promoting the reflective actions thinking while drawing and drawing while thinking.

II.1 MODELS

Models have been studied since the days of Vitruvio's *De Architectura*. Marco Gaiani points out how Vitruvio emphasized the value of physical models (*maquettes*) as a safe way of showing the project to the final commission, that is, a way to show how the project will look like before its construction and after its theoretical definition. During Renaissance the *maquette* extended its function as a way of crystalizing a thought and anticipating its construction. *Maquettes* reappeared in the twentieth century, confirming their usefulness for shaping and defining the project, that is, *before* the final definition of the latter. With all, the *maquette* positioned itself as an essential tool for the inception of the project, a system to describe technological components, verify static behaviour and to previsualize a designed object, that is, for verifying formal, structural and functional hypotheses, for presenting the final result and for materialising thoughts amid the design process [133, p. 23], [134].

The similarity that a model wants to have with the materialised project impacts directly in the model itself. So, for example, while each *maquette* has its own conventions and figurative modalities, drawing models have a conventionalised figurative modality [133, p. 23]. Indeed, in drawing, the concept of similarity relies in the kind of projection used for creating it, that is (as introduced at I.1.1): central, parallel or double orthogonal projections" methods with perspective, axonometric views and associated floor plan and sections as respective applications.

Tomás Maldonado classifies models according to their similarity with the real project. He defines such a correspondence as: **homologue** (same structure, different shape and function), **analogue** (same structure and function but different shape) and **isomorphic** (same structure and shape and function being either the same or not) [133, p. 24], [135]. Gaiani highlights that such a classification allows the construction of a quick and efficient system able to link know and know-how [133, p. 24].

Maquettes and graphical models are among **iconic** models and they represent an **isomorphic** reality of the final project. On the other hand, a mathematical model, which is among the **non-iconic** ones, results an **analogue** reality. With the arrival of digital technology appeared 3D computer models (see II.2.2) which impacted and transformed deeply architectural drawing. Then, sciences of representation needed a major revision since **3D computer models are homologue, analogue and isomorphic at once**, i.e., they cover all vision's mechanisms with the functionalities of both iconic and non-iconic models [133, p. 24].

James Ackerman compares the technological innovation introduced by the computer with the one introduced by paper [136, p. 305]. In his study, he exposes the role of paper as something **not neutral** in drawing: *"(when paper was) introduced into the West in the fourteenth century, opened up the possibility of recording rapid impressions, of sketching, for the first time"*. Furthermore, the rectangular format (generally wider than taller) "*promotes a certain range of orientation in the drawing—in particular, the lining up of straight orthogonal lines parallel to the paper's edges*". As well, sheet's format is "*an analogue of the window through which an object is seen; there is an inevitable conformity between the technique of perspective projection described by Leon Battista Alberti in 1435, not long after the introduction of paper, and the format of the sheet*". Such influence arrived until the computer itself, affecting drawing boards first and consecutively CAD programs [136, pp. 294–295].

As paper, digital technology affected representation in a radical way: after an initial imitation of methods from traditional techniques, the computer filled (in just a few years) every task of the architect and the designer, opening new ways of thinking, doing and managing the project. Such a potential of the digital opened a first polarisation between who took the computer as a magical instrument and who resisted the change trying to keep the tradition of the pencil, that is, putting computer and pencil as antagonists. Nowadays, designer's positions are more flexible for integrating, on the one hand, the documented, studied and long dated traditional techniques; with the powerful, dynamic and interactive digital technology.

Nevertheless, digital technology conditions drawing right as the sheet of paper does. Before the computer, **hand** and **simple instruments** (e.g., ruler, compass) were the only two components for drawing. The former is connected to our sensitive, perceptive, cognitive and intellectual capacities, while the latter helps us to trace. In turn, with digital technology there is a new element to consider, i.e., the **software**, which is between the previous two. According to Gaiani, the software has an active and considerable role on drawing: on the one hand, it **conditions and limits** our actions; but on the other hand, it **introduces new assists and operativity** for every single task during project definition and the inspection of the final result. [133, p. 28].

One big strength and weakness of software is that one can define a new computer program in total freedom. Actually, even if is not always the best option, one can code a new language for performing internal tasks, e.g., based in *JavaScript*, *Geogebra* created the *"GeoGebra Script"* language for "simplifying" the scripting of known tasks [137], [138]. This means that every program can have its own structure for drawing, or, in other words, drawing becomes a dynamically defined system updated with every incoming version of the program. As a consequence, within the last years the transmission of the knowledge for drawing got progressively confused and fused with *learning a software*. Personally, during my career I saw many colleagues falling in the confusion of how to do *that* process without *that* software.

II.2 Immersive Models

The whole and laborious apparatus of old panoramas reached (decently, with the means of their time) a cropped semi-immersive vision. Over time, panoramas passed gradually from cylindrical to equirectangular projection, and from drawings to photography. Nowadays, panoramas reached the full immersion feature, i.e., covering the 360° around the vertical axis and 180° from zenith to nadir. All physical limitations (frame, building, lighting) are now controlled through digital technology. Indeed, it is possible to create full panoramas using a virtual sphere or a cube and placing the observer in the geometric centre of them. The result is scene explorable from the screen of the computer, using VR headsets or smartphones. The observers, with the aid of these devices can perceive scenes as if they were physically there, focusing on details, enlarging them or obtaining close ups with simple gestures.

The digital fruition of panoramas defines a model that allows the remote visit of existing places that for different reason may not be accessible. This characteristic makes it an important tool, for example, for cultural heritage documentation and promotion. Such digital models are to the base of informative and collaborative systems, and therefore one may enrich the experience of the visit with multimedia sets of documents, audio, videos, etc.

An observer standing in front of a classical perspective has a small degree of movement contained in an "area of observation", something that repeats also for anamorphosis and (old) panoramas (see I.1.1.2, I.4.4 and I.5.2 respectively). In those cases, the image is one single frame, and it has a limited field of vision as a result of using a linear perspective to the base of the first drawing.

In turn, an immersive model (IM) is a *digital* set that allows the observer to freely move when perceiving the image. Its definition joins "immersive" from Oliver Grau's selection: "*media that are the means whereby the eye is addressed with a totality of images [...] art that concentrate on immersive image spaces*" [52, p. 6]. In turn, "model" connects with definitions from the specific scientific field of drawing [139, pp. 8, 9].

An immersive model may have two degrees of freedom: either full rotation around the projection centre or full translation and rotation. The former is a static model made with immersive perspectives (e.g., a full panorama in equirectangular format). The latter instead is a dynamic model built through digital modelling, or what has been called an interactive and dynamic perspective (PDI)[20] (e.g., a digital *maquette*). In a scale of movement freedom there is linear classical perspective, immersive perspectives and interactive and dynamic perspectives.

The fruition of an immersive model must be digital for enjoying its maximum potential. In fact, it is enough to think how worth, possible and practical might be trying to build a physical device such that: it must have a perfect squarish or spherical shape; it must be able to increase/decrease such a shape adapting to the observer's height so to have its eyes right in the centre; it must have a perfect diffuse illumination; and finally, it must have no surfaces' joints. The complications of such an artefact are immense.

Indeed, thanks to digital means an IM can project a new linear perspective for any new position of the user's head in the screen of the digital device. Also, thanks to the digital fruition an immersive model has no limitations from physical frames, so it overcomes the limits of the traditional drawing media, i.e., the boundary of the paper or the canvas. Therefore, an IM is free of the "tyranny" of the single point of view and from the general constraints of perspective (paraphrasing Damisch) [9, p. XV].

Regarding the expressive means, an immersive model uses "computer graphics means", that is, a larger group of media *including* those of classical drawing, figurative art, photography, cinema and digital elaboration of images [140, p. 6].

[20] From the original *Prospettiva Dinamica Interattiva* [140]

II.2.1 Immersive Perspective

An immersive perspective is a graphical model (i.e., an iconic isomorphic representation), with enough information for fully exploring the visual sphere around one single spot. During the navigation, the computer does not *generate* new content but instead it *shows* a portion of a bigger picture previously composed, i.e., the screen displays just what the observer's visual cone focus within a full panorama. Hence, an immersive perspective is an XXI century digital *extended* panorama, or, in other words, an infinite set of classical perspectives.

An immersive perspective is one single raster image file, generally an equirectangular panorama, but may also be a 360 azimuthal equidistant, a Mercator or a cubic map (as shown in I.6.1). Such panoramas are currently generated with photographic shots, either in one-shot compact 360-degree cameras, or in many single shots took with a regular camera. In a following stage these shots are stitched and elaborated through software.

II.2.2 Interactive and Dynamic Perspective

An interactive and dynamic perspective is a computer model with mathematical definitions enough to *compute* and generate a new classical linear perspective in the screen for every position and view focused by the observer. This means that is an isomorphic, analogous and homologous representation with both iconic and non-iconic functionalities [133, p. 24]. This model was defined back in 2008 by Riccardo Migliari, who focused on the advantages that video games bring to architectural representation. In the homonymous book, he highlights the characteristics of a PDI along with many applicative cases of its implementation [140].

With a PDI, the user can explore infinite points of view, in contraposition to the unique one offered by the linear classical and the immersive perspectives. Indeed, a PDI is a set of real-time created classical linear perspectives projected in the screen and resulting from the inside navigation of a 3D model. The visitor can move freely in the three spatial axes and for every new position an algorithm will set up a new image projecting conically from the model's geometry to the camera position.

Therefore, the representation is automatically obtained no matter what the observer's position is, or the disposition of the model's surfaces regarding that point of view. To reach such a versatility, a PDI needs to be digital. In building the model, the designer uses either the mathematical or the numerical representation methods, defining relations, proportions and disposition among the model's elements [140, p. 6,7].

II.3 HYBRID IMMERSIVE MODEL

Art historian Oliver Grau divides immersive artefacts in two groups: "*large-scale spaces of illusion that fully integrate the human body (e.g., rooms with* 360° *frescoes, the panorama, Stereopticon, Cinéorama, planetarium, Omnimax and IMAX cinemas, or the CAVEs) and apparatuses that are positioned immediately in front of the eyes (e.g., peepshows, stereoscopes, stereoscopic television, Sensorama, or HMDs)* [52, p. 349]. A HIM takes place among the latter group of applications. A detailed story of how we reached the current virtual immersive reality may be followed in the works of Grau and computer engineer Jason Jerald [52, Ch. 4,5], [141, Ch. 2], [142].

II.3.1 Definition

A hybrid immersive model (HIM) is a digital, graphic and iconic model representing an isomorphic reality of the final project. It is also an immersive perspective, but it takes advantage of the fact that panoramas do not necessarily need to be generated with the computer. Indeed, its generation is not limited to any mean and therefore a HIM can be created from a drawing made with traditional means, even if the final character of the model is digital. Maybe, the digital character of current panoramas is more connected with common techniques used for their creation (i.e., digital photography and 3D models), than with their real nature. As a consequence of this mixing, a HIM results made with both traditional and digital techniques.

A HIM finds place in early conceptual stages, for instance for highlighting the modular laws from a critical analysis of spatial geometry' relations[21] (survey applications at Third Part V.3); representing a place that do not exist yet (project applications at Third Part V.2); and

[21] In Italian is often used the specific name "*ipotesi di conformazione*", where *conformazione* means the way an object is shaped, its structure and correlation with other objects [143]

testing hypotheses of reconstruction (restauration applications are not included in this research) [144]. In these initial phases, simple drawing is the key for concentrate the work between the hand doing and the brain thinking.

Indeed, a PDI is extremely more versatile than a HIM, but the elaboration of the computer model becomes unnecessarily laborious in early stages due to the high level of definition required for its construction. Needless to say, such a level of detail does not even exist yet during these first moments of the project. Therefore, a HIM is a variation of an immersive model where the panorama to the base has been created using human-centred traditional procedures, in contrast with the automatised computer-based nature of photographic panoramas and PDIs. The following paragraphs will try to clarify **why** the motion of creating panoramas with traditional techniques thus digitalising and exploring them with digital means.

II.3.2 The traditional side of an HIM

To start with them, let me clarify that I do not mean exclusively analogical drawing when I recall the use of traditional procedures. I actually mean to images constructed by an operator drawing line by line, using vanishing points and descriptive geometry, no matter if the board is the old drafting machine or the last version digital canvas of Adobe Illustrator. Note, indeed, that one can practice indistinctively perspective and anamorphosis with both analogical and digital means. This research promotes such traditional procedures since they are the more widespread way of externalising ideas in our days. In fact this is the approach that current adult generations learnt to draw and the way that universities still teach, a direct, simple, cheap and no-battery dependent way of connecting thought and schemes without any interface in between (i.e., Gaiani's **hand/brain** flow [133, p. 28], [145]). This essential setup is very welcome when one must learn how to translate a design from its mind to its materialisation, thinking in construction protocols (in the case of project) and understanding proportions and ratios among elements (in the case of survey).

On the other hand, who draws with a pencil can always emulate the same process with almost any drawing software, since what digital means give to them is another level of virtuality altering the relation between reality and imagination [133, p. 29]. Nevertheless, also because of this common character there is yet a part of that public "resisting" (maybe a bit melancholically) to *learn* new things after many years of specialisation. The traditional construction promoted is the result of a meditated human-made procedure and in that case the model is firstly stored inside the architect's mind. The versatility and efficiency of a PDI for changing perspectives has no comparison, particularly in a stage when the product is already defined or when the designer has access to direct 3D digital sketching (see below). Yet, the hybrid model does not even try to compete with that, it actually focuses into a representation made with the current technology *before* such a stage.

In effect, an HIM favours a process where every new perspective goes from the mental model to the paper, i.e., **is not calculated by any software but consciously made by the draughtsman**. In that way, traditional drawing stimulates the understanding of the relation among elements, the spatial geometry and, since we are architects, the elaboration of an own construction protocol thus potentially transferable to the final materialization.

Another aspect to consider, is that when the designer draws this kind of perspectives is (normally) already seeing the project/product from the user's point of view and contextually inserted. This approach puts the designer on a **human level/product design position**. By drawing at a human level, the designer may risk less to be detached from details such as equipment not fitting in a room or an "*entrance for elephants in a kindergarten*"[22]. A materialised effect of this is a natural tendency of the draughtsman including itself in the drawing, i.e., being part of the space/product and being active in the design (Figure 38).

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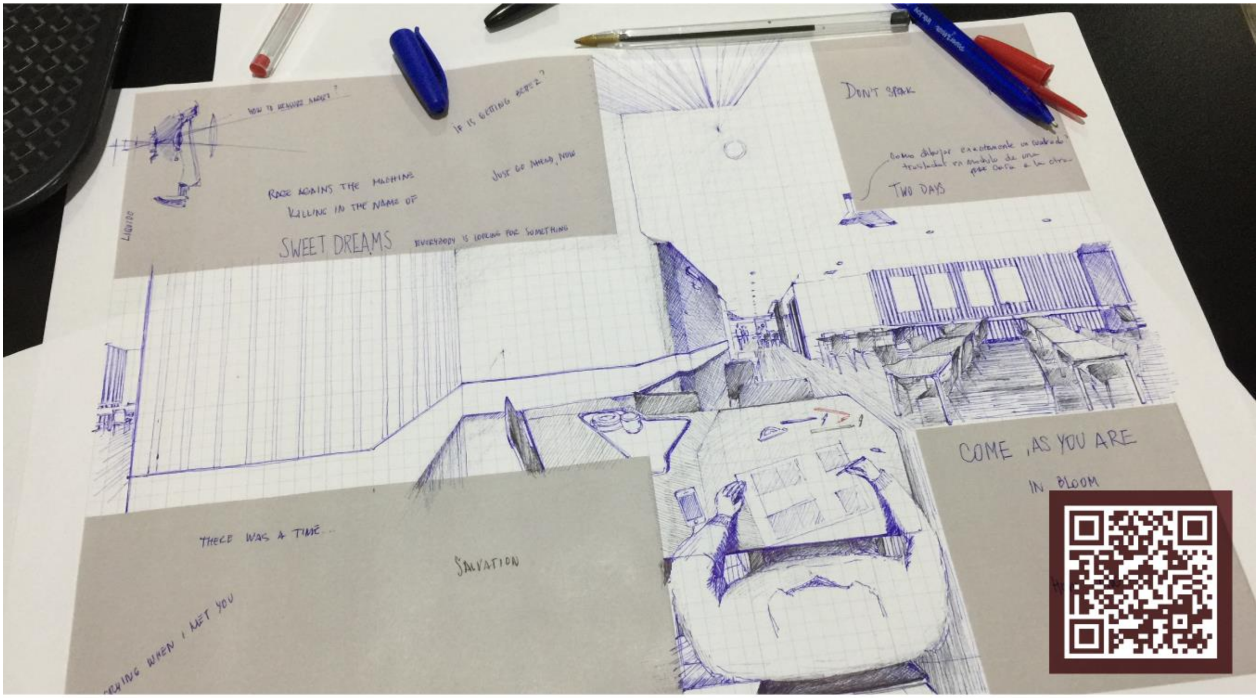
[22] Bibiana Oviedo is professor at FAUD, UNC, Córdoba, Argentina. I studied with her in 2008 and during that lessons she used to tell us to avoid drawing "*as an elephant or an ant would look in a bathtub*"



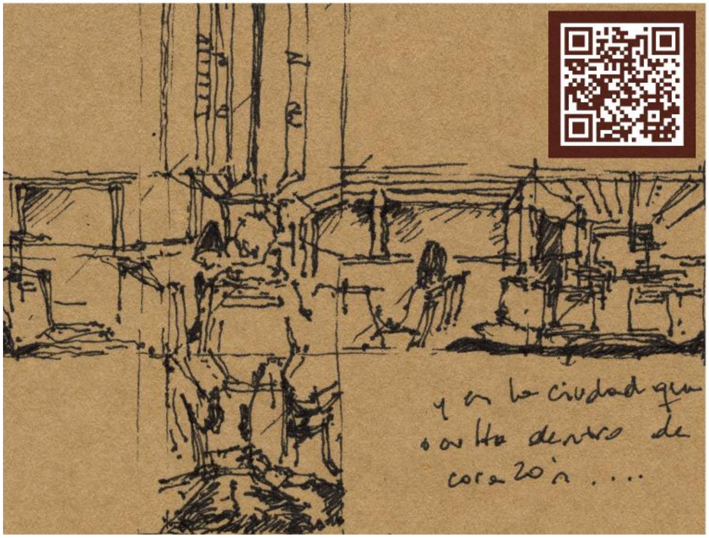


Figure 38 . Surveying existing spaces with HIM and including the draughtsman. Cubical panoramas (up and bottom left) by the author. Azimuthal equidistant perspective (bottom right) by António Bandeira Araújo

II.3.3 The digital side of an HIM

Digital techniques are opening new ways of thinking the project by achieving points never reached before, even if they do not have the connotation of tradition that analogical techniques enjoy. With them, virtual reality is quickly spreading in our society that is becoming more and more digital every new day[23].

A main goal pursued by virtual reality is to reach the highest *"suspension of disbelief"*, a term that Cruz-Neira taken from film criticism defined as "*the ability to give in to a simulation, to ignore its medium*" [146, p. 65]. The British poet Coleridge coined the phrase in 1817: "*The willing suspension of disbelief for the moment*" with reference to the audiences for literary works [147], [148, p. 281]. These oblique synonyms from other disciplines, point and mean the same that full immersion seeks, i.e., the perception of the spatial continuity without interference of the medium: "*Immersion arises when the artwork and technical apparatus, the message and medium of perception, converge into an inseparable whole*" [52, p. 348].

Ph.D. architect Marcela Rucq[24] affirmed back in 2010 that it was matter of time before a digital pencil could reach such a sensibility and speed so to match the analogical pencil. And, indeed, this technology already exists. The sensibility of drawing, a repetitive argument years ago, resulted to be just a technical/instrumental matter, not a core one. An HIM centres the question in the optic that both techniques are complementary, and therefore, it has no point to

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[23] Just to barely have an idea of the exponential growing of digital means, Oliver Grau expressed already in 2003: "*For decades now, the price of graphics hardware has reduced annually by a factor of 4, while performance increases 20 to 100-fold. For example, a supercomputer today can process one thousand million instructions per second (1000 MIPS). If a human were to read just one instruction per second, he or she would take 32 years, without sleeping or resting, for the same amount of data. The popular formula expressing this development is Moore's Law; in 1965, Gordon Moore predicted that the number of transistors per integrated circuit would double every 18 months*" [52, p. 170].

[24] Marcela Rucq taught me many things and among them to be very critic with my own ideas within academical research. I explored the idea of the "digital sensibility" with her in many of our talks within 2010 and 2012. Back then, I was stoically defending "analogical over digital" for my first papers, without really understanding why. Rest in peace dear Marcela

compare but to understand what it is possible to do with each of them. The two techniques serve different purposes, and the discredit of the one with the potentialities of the other is merely trivial. With the range of expression possibilities open, the designer is totally free to choose the one that fits it better for any task.

However, software *must* enter the game in some moment since the final visualisation ends there anyway: in an HIM one creates the panorama by traditional procedures, folds back the surface digitally and finally explores (also digitally) the full immersive model. To that end, one "cheats" the computer to "make it believe" that the original drawing is actually a panoramic photography (see Second Part IV.7). Hence, the object of recalling traditional drawing at the beginning is oriented to delay the introduction of the limitations/advantages of digital drawing (hand/brain/software).

Thus, a HIM seeks to avoid a more complex process (such as 3D modelling) where a mere drawing is more than enough. When the panorama is done, the project has already a deeper definition and digital tools help to verify the design hypothesis by reconstructing its third dimension. Already within the immersive view, the project can be understood and discussed with other designers or the client, up to its final definition and 3D modelling.

Something curious with digital drawing is the fact that from the moment zero we touch the screen with a pencil, the software starts computing pixels and transforming the impulse in binary information. Subsequently, we create a pixel-based model, and the drawing can be re-produced. There is no *original* in the traditional sense of the word, although paradoxically the computer can create countless copies and no human can create the same drawing again. Such a drawing has a unique defined structure, but it became a product that can be serially repeated without limits in quantity, size or colours. This paradigm of repeatability is not necessarily negative, but it is something we need to deal as society yet since is to the base of the cultural heritage conception [149]–[151].

II.3.3.1 Virtual and augmented reality

Virtual (VR) and augmented reality (AR) are increasingly becoming a common part in our daily life. They developed lately an innovative and independent language of expression, finding place among many different subjects. VR and AR applications cover a wide spectrum of possibilities from ordinary entertainment to medicine training, passing through mechanic maintenance, aeronautic simulations, digital arts, and so on.

This technology became so natural that with a minimal effort one can enter some website right now or download one of the many dedicated applications and interact in VR modality within games, information systems, collaborative spaces, artefacts, exhibitions using an ordinary mobile phone. Some everyday examples of VR/AR interaction are wide field of view pictures shot with our smartphones and shared on social medias, Google Street View navigation, virtual coffees with avatars, virtual visit to museums, playing video games and exploring multimedia content using virtual reality headsets such as the Oculus Rift, and many others.

In particular, VR and AR also reached the discipline of drawing, where they show their utility, versatility and powerfulness for architectural representation from real-time exploration of models to virtual visits past-reliving the cultural heritage [28], [152]–[154].

When digital technology appeared, there was first an imitation process: computer programs adapted techniques coming from the traditional world. An example is CAD (Computer-Aided Design) programs, that digitalised the traditional drawing board and added with the time some new functions. Still, the way of operating a CAD program is essentially the same than in the traditional board: one creates points, traces lines, calculates intersections with ruler and compass, or in other words, one is essentially repeating the traditional *construction/drawing* process [133, p. 29].

Nowadays, the digital world is an independent language that introduced many new operations that belong purely to the digital world. So, for example, one *extrudes geometries*, *stitches pictures* and *create dynamically a new perspective* (a PDI) in the screen for every new "step" inside a VR model. In turn, this research proposes a workflow in the other way around, that is, bringing resources from VR and AR and mixing them with traditional techniques and exploring hybrid techniques with features of both digital and traditional worlds.

II.3.4 Essentials skills for creating an HIM

A neophyte in cubical drawing will need to manage some essential skills with a double approach: on the one hand, it must know linear perspective and anamorphosis to compose the base drawing. On the digital part, it must know how to operate stitching and virtual tour software for the creation and fruition of the final immersive model.

A former notion of perspective is (or should be) given during the career of the designer, whereas anamorphosis can be understood operating the surface projection. For the software notions I present in Second Part IV.7 an open and free source guideline, with the intention to clarify the conceptual workflow more than promoting a specific software. In any case, the strongest theory to understand is the graphical one (perspective and anamorphosis) while the software part does not require skills in programming. For these reasons any average designer (or student from second year on) may be able to draw a cubical perspective.

While children may use pencils with the unique difficulty of how to sharpen it, the difficulties to operate a program for generating a PDI lie on the operator's computer skills [140, p. 7]. These latter competences imply the user focusing into learning *how to* operate the tool, something that change often with the winds of the program's updates and barely teach the reasoning of the algorithms behind such instruments.

On the other hand, when using plane drawing, the operator needs to have the descriptive geometry knowledge in its head to construct a proper perspective. To do so, the designer creates and follows mental algorithms, something that die with the operator itself and that can be reused in any new software after some producer goes bankrupt. This investigation pursues a reflective philosophy in design, where planning, thinking, constructing and representing should converge in the same unique act of capturing creativity.

Hence, the final product gathers the advantages of traditional techniques with the best way for creating a full immersion in our days. The fusion with the digital world adds a possibility rather impossible to reach with traditional means.

II.3.5 HIM and photography

Putting aside the versatility of PDIs, it may be also asked why one should construct a complex perspective when one could simply take a picture. Indeed, photography is very useful and even important for surveying and documenting an *existing* place[25]. But both photography and PDIs need a something made *a priori*, i.e., a physical building or a 3D model respectively. So, how to take a picture of a project that does not exist yet? Or, how to take a picture of an immaterialised project?

Thanks to traditional drawing, the hybrid model is a way to represent non-existing design or non-materialised criteria of such a design, for example, by tracing and composing essential elements on top of a picture (see Third Part V.3.2.1). Drawing helps to materialise, define the relations among elements and shape them in schemes that are neither a digital nor a physical model [145, p. 47]. An interesting work investigating how sketching contributes to design process may be followed at the PhD dissertation of Tran Luciani "*Designing for sketching to support concept exploration*" [132].

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[25] *Lo stato di fatto* in Italian

As a result, a HIM offers a possibility of using a versatile technique for exploring immaterial options with compact drawings thus magnified once digitalised (see I.6.2.4 and Figure 33). An HIM fuses advantages from both traditional and digital means in an extended way, being in itself a reflective exercise and an affordable way of creating full immersion. Consequently (and as mentioned above), there is no need for comparing techniques since each of them orients to a different purpose: while photography captures all the visual data of a building, drawing is useful to highlight what is not visible: ratios, proportions, relations among elements, project hypothesis, etc.

II.3.6 Open skills

A further consideration is that, historically, anamorphosis was a mere virtuoso's rare practice [53, p. 149]. This proposal instead, seeks a simpler insertion by promoting an open instrument reachable and possible to develop by *any* architect or designer. Then, thanks to anamorphosis and digital technology, one may mount a digital artefact in which these panoramas can reproduce the full immersive experience without the bulky dimensions of Barker's paintings nor needing Mitchell's building. Indeed, by combining just the small screen of a phone with lenses (VR glasses), it is possible to virtually enlarge the drawing. The visit of the panorama is thus improved by adding to the model the function of responding to the user's interaction and movements. Artefacts as such, follow the movement around the projection point without losing the observation area, updating on-the-fly the image in the screen by detecting the observer's head position thanks to the use of the gyroscope.

II.3.7 Summary

A hybrid immersive model allows the designer to communicate what has been conceived and elaborated during early stages. With conventional drawing to the base, this model takes the most of traditional techniques, “*a cumbersome legacy of processes based on the Vitruvian description and old more than two thousand years”* [133, p. 30]. Indeed, drawing defends its utility for itself, especially in the early conceptual stages where more than an instrument is a way to think through experimentation [4, p. 159], [5, p. 1491], [155, p. 1040].

It is very important to gather a big mass of information (e.g., through photography or laser scanner), but it is equally important to represent structural and visual information through drawing, highlighting the personal interpretation of the reality in the eyes of the draughtman, the conceptual model which “*if he is fortunate, will reflect the essentials of what he wants to understand about a given phenomenon*” and the “*exciting experience to bring about something visible that was not there before*” [145, pp. 2, 171].

On the other hand, a HIM increases the fruition and expands the graphic model of the panorama with the use of digital technology. The digital, indeed, “*presents interesting opportunities and is capable of offering new material closely connected with knowledge and know-how*” [133, p. 30]. With an HIM one creates a whole virtual scene from a small size compacted drawing thanks to the use of linear perspective, anamorphosis and digital technology. With all, it integrates different tools and methods in one model, thus let the designer to use them to guide, define and research spatial geometrical analysis and construction hypotheses.

II.4 A HOLISTIC APPROACH

During the Renaissance, painters and architects did not limit themselves to creating artworks, but they rather sought for a relationship with the new reality: using the new linear perspective, they tried to mathematically reconstruct reality through geometry. Virilio recalls this position inviting designers to a similar endeavour, but updated to our days, he called it a "*stereoscopic perspective*"[26]. This dual approach represents both the real space (like the paintings of the 14th century), and the current artificial reality in which "*the real is not given but always built*"[27] [156].

On the other hand, Luigi Prestinenza Pugliesi pointed out the necessary balance between physical and artificial realities, which are both integral parts of our daily existence. As an example, Pugliesi recalls Gordon Matta Clark's work, who mistook both realities in his artworks on purpose (Figure 39) [157], [158]. By following this path, prefigures Pugliesi, we will become aware like Leonardo and Michelangelo did before us that we live neither in an artificial world (ideal or platonic) nor in the physical one that can only be understood through immediate perception: instead, we live in a technical scientific system that mixes both worlds [159].

Moreover, Argan points out how human perceives through classical perspective as the mind perceives, or, in another words, how the "*mind's eye*" conveys the knowledge in the mental space of drawing [160]. In such a drawing space, the imagination connects the empiric space to the ideal one. This property keeps untouched independently the used perspective system, no matter if is a classical linear, an equirectangular or a cubical full immersive perspective since independently the election, there is always a linear perspective in the screen as final output.

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[26] The confusion of this name could not be higher in a thesis were both terms *perspective* and *stereoscopic* have a very clear and straight disciplinary meaning. Virilio instead, uses both terms obliquely, meaning with *perspective* the attitude towards something, and double, dual, parallel with *stereoscopic*

[27] Original quote: "*Il reale non è mai dato, ma viene sempre costruito"* [156]

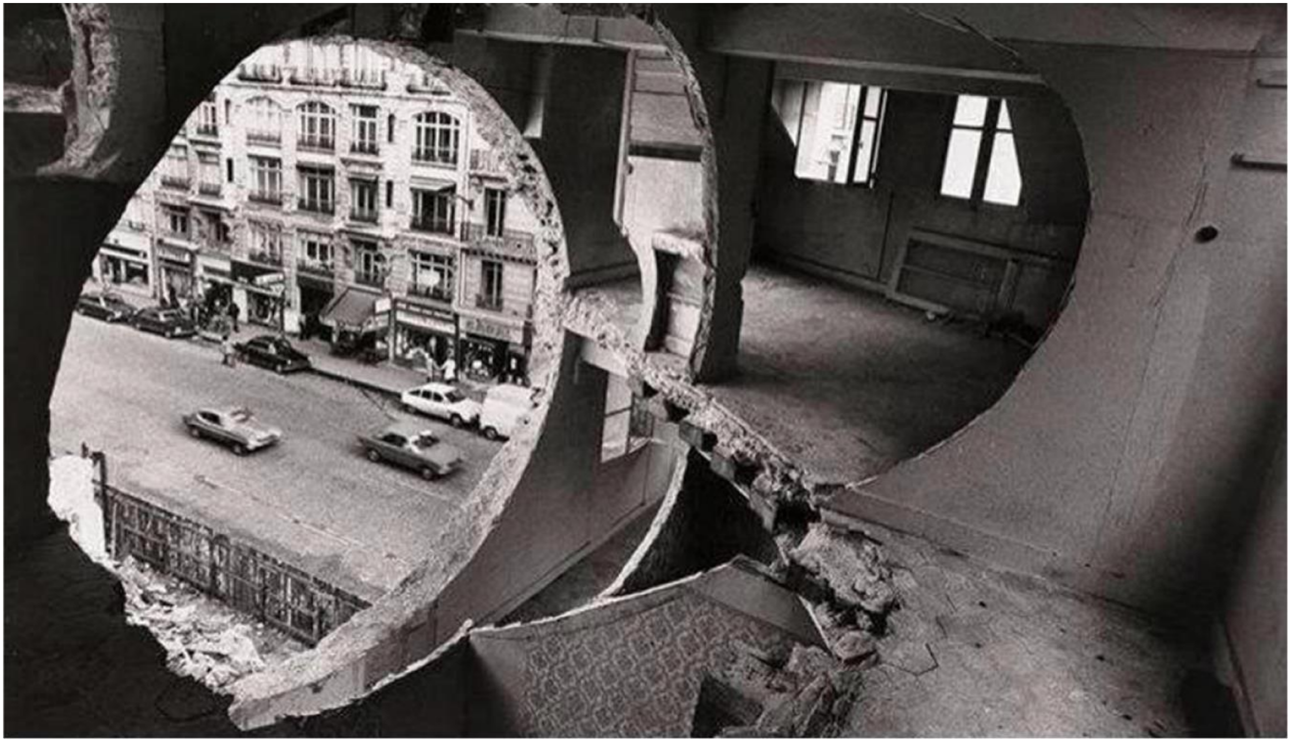
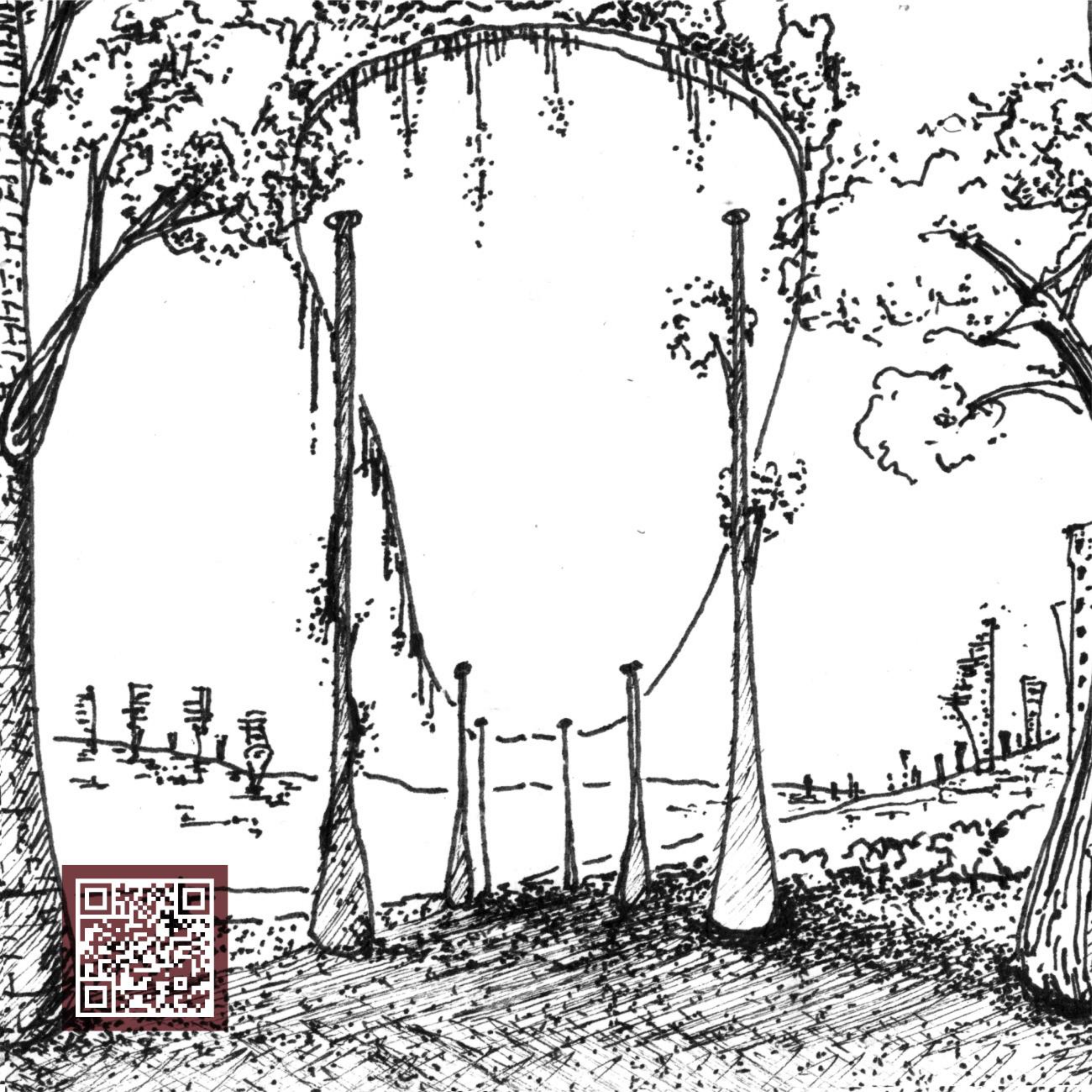


Figure 39 . Conical Intersection by Gordon Matta Clark (1975). Picture by Bianca Maggio (public licence) [161]

Summing up, the research promotes an instrument for an intellectual refoundation in the way that the designer sees both physical and virtual realities. That is, an approach pointing to a cognitive and geometrical reading of the same reality in a personal and different (new) key. The way to do it, is through the conscious construction and perception of a represented space, actions promoted (by both HIMs and the cubical perspective) via the co-existence of traditional/digital techniques and theoretical/empirical spaces (or virtual/real).

SECOND PART



CHAPTER III

STATE OF THE ART OF CUBICAL PROJECTIONS

III.1 INTRODUCTION

In the previous chapters I exposed the core role that panoramas and spherical perspectives play among immersive models. I seek to replace that sphere with a cube. The motivation behind such an exchange has several reasons that I will expose progressively along the development of this chapter. Yet, to start, let me remember that the fruition of a HIM is through **digital means**, which connect us automatically to the field of computer sciences. I may clarify that my use of digital technology is merely as an end-user, not as an expert. Nevertheless, I will try to read general principles of the relationship between VR rendering and the cube.

Among the different immersive perspectives, the cubical one presents some advantages in front of the task of exploring the geometry of a space or an early conceptual design. For example, the methods presented in IV.4 and IV.5 are associate with linear drawings methods, which recalls a familiarity among architects and designers in contrast with curvilinear systems such as the equirectangular one. Such a linearity makes handier the apparent distortions in every face of the cube, something not happening with the curved warping of spherical perspectives.

The current repertoire of cubical applications is mainly filled by technical applications in the CGI field (Computer Generated Imagery). There are also, as in the spherical case, some *drawing and guessing* procedures from the artistic field, and just a few examples of applications in the architectural field. Indeed, cubical perspective is a relatively recent system among immersive models based on central projections and the principles of linear perspective. The present study orders those principles in light of current knowledge and according to an autonomous theory.

Within the next chapters, I analyse a set of applications using methods and theories based on polyhedral projection surfaces as an alternative to the single plane of classical perspective. A natural consequence of using these surfaces is the fragmentation in the representation of a line l (Figure 40). In the specific case of the cube, this break-up can arrive up to four different segments (Figure 71 c). Therefore, I explore existing approaches from the CGI field and some historical examples such as the *Perspective Box*, so to see how they solved the problem of the continuity during the visualisation.

Thereby, I present a repertoire of examples extracted from the growing panorama of immersive applications, analysing them in three groups: Perspective Box, Cubic Environment Map and Cubical Map within Representation. I dive into their theories and methods, as well as the different relationships between the projection surface S and the observer O.

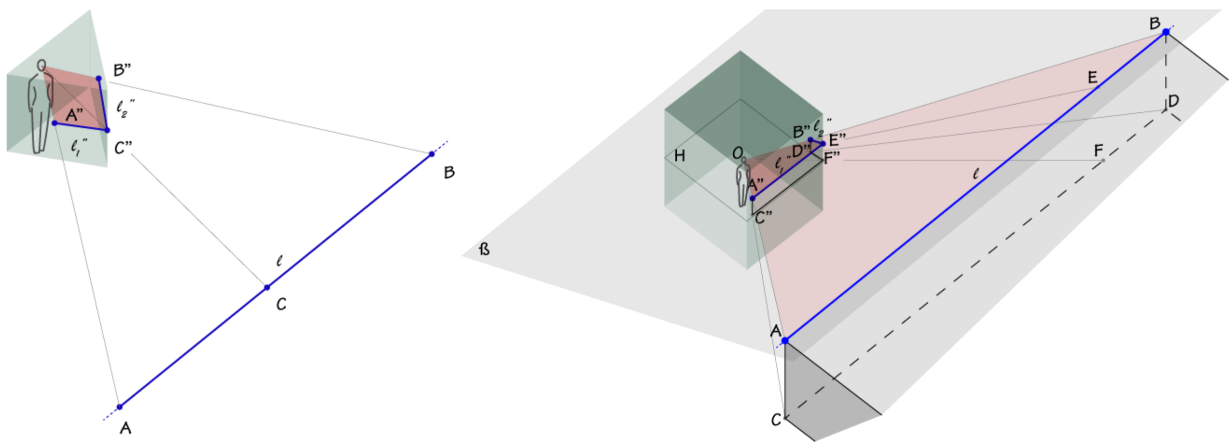


Figure 40 . Polyhedral projection surfaces, the representation of a line l is fragmented

III.2 PERSPECTIVE BOX

The *Perspective Box* is an old artefact that can be considered as ancestor of the cubical perspective. They were created by Dutch artists who joined the emerging interest for the use of perspective at the time [162, p. 35]. These contraptions used anamorphosis, mathematics, architecture and arts to explore and engage the first-person perception of pictorial images.

At the same time, they reduced drastically the size of the immersive artefact, actually miniaturised it if compared, for example, with the dimensions of a physical anamorphosis in a church. Indeed, a perspective box "*make a figure the size of your finger appear to be life-size*" [163, p. 4], [164].

They existed mainly during 17th century in parallel with the developments of the *Quadraturists* and the *tramp d'œil*. These devices travelled from Italy and Denmark to ancient Asia were motivated the born of the "*peeping karakuri*" and the "*nozoki megane peep-box*" in Japan [70], [162], [163], [165], [166]. These boxes' influence arrive up to our days, with examples recalling their utility for the exploration of interior architecture projects [70].

The interest behind these devices was the pictorial representation of interiors and everyday scenes, whereas there are no significative references mentioning the use of this boxes for design exploration [70, p. 1]. Artists looked for a connection between figures and environments, seeking the realistic representation of Dutch churches, landscapes, fields, towns and houses since it was there where their daily life was. They matched the iconicity of the represented objects with the representation of the activity itself [162, p. 35].

Figure 41 . A Peepshow: Views of the Interior of a Dutch House (1663) © National Gallery, London [167]–[169]

III.2.1 Drawing a Perspective Box

The setup of surfaces in a Perspective Box varied between parallelepiped, triangular and octahedral polyhedron shape [70, p. 9], [163, p. 4], [166, p. 47]. The specific reason behind the use of a polyhedron as support relates more with a self-structure having the function of keeping the observer at the right and precise distance from the projecting planes. Nothing seems to indicate a philosophical intention of such an election, but it rather looks as if the perspective boxes were polyhedral for their constructive simplicity.

The paintings were disposed in all the walls except in the upper plane, that is where the light came from. Such a disposition intended to create a diffuse distribution of light and cancel the limits of the drawing itself, negating any real physical reference to the observer. Such an abstraction involves physically the observer and the fruition becomes a psycho-physical experiment, i.e., the same engagement aimed by Brunelleschi's experimentation. With this, perspective boxes are "*the beginning of a line of development that complements the immersive spaces that envelop the full body, where the illusionistic effect results from bringing the images up very close to the eyes of the observer*" [52, p. 52]

The use of multiple surfaces of projection raised the question of how to solve the intersection among planes. To deal with such a question, linear perspective and anamorphosis knowledge would have been very useful, but the used method was more a manual technique with arrangements on-the-fly, almost an ad-hoc procedure for every new device.

III.2.1.1 Projection centre and observation area

To solve the fragmentation, artists considered the "*Euclidean theorem that if two straight lines meet at an angle, they appear to be continuous if viewed on the same level*" [52, p. 52]. Consequently, they solved the problem following this principle but dealing with a practical solution more than a theoretical one.

For example, Oliver Grau points out that the inside illustration of the perspective box "*Views of the Interior of a Dutch House*" ascribed to Samuel van Hoogstraten (Figure 41), followed a practical method consisting in a sort of Alberti's window and Pozzo's grid adaptation. Van Hoogstraten used a traditional flat drawing in classical perspective and punctured through it with a needle coming from the peephole. The needle was used as "ray of light" to pinpricks in the walls of the device the reference points [52, p. 52]. Hence, Samuel van Hoogstraten was materially creating a radial occlusion.

The Perspective Box (usually tall as the observer for a direct observation), works analogously to Brunelleschi's *tavolette*, that is, placing a peephole from where the scene is watched and offering a magnified perception of the image. This peephole is not randomly located but previously positioned in front of the image, yet not necessarily at the geometrical centre of the device and may even exist many of them for one same device: (indeed the quoted example of Samuel van Hoogstraten has three different holes). Some theories explored the possible existence of external lenses attached to the observation point used for a correct reconstruction of the anamorphosis [168, p. 3].

The peephole is both the projection centre and the area of observation at once. Indeed, the drawing is constructed from the peephole and one cannot see more than the physical hole allows to observe.

III.2.1.2 About the surface

It is clear that nowadays the practical method of using needles would not be plausible for the cubical case. Still, the importance of recalling it is to remark that Perspective Boxes were constructed as mini-anamorphosis, i.e., considering the surface already in its final shape (the box), and not as a flattened map.

The limited perspective knowledge of the artists confirming the hypothesis: the main structure of the drawing was built with the "needle system", while later objects such as people, animals or various objects were added to cover conflictive zones where the drawing had errors or weaknesses in the construction. As well, these latter *repoussoirs* did not follow clear perspective structures, even looking sometimes outside of place and breaking the illusion more than enhance it [52, p. 52].

III.2.1.3 Summary

As panoramas, both cumbersome and miniature cases of anamorphosis required a specialised hand. Perspective Boxes were used to show ideal images among just a few: "*They were prestige showpieces, owned by members of the upper classes of society. Their main attraction was the voyeuristic element, their direction of one's gaze through a peephole toward something that is inaccessible to others*" [52, p. 50].

In any case, these boxes represented new geometric challenges and in particular in the field of anamorphosis that was in full vogue at the time. The theory of linear perspective influenced directly their implementation, and artists and mathematicians expanded the potential of the method to create with these artefacts surprising visual deceptions: "*The perspective box not only participated in dialogue with the discipline of perspective, but reified its tenets, albeit in a highly entertaining and captivating format*" [163, p. 1].

III.3 CUBIC ENVIRONMENTAL MAPPING

This section explores the so-called “environmental mapping” or “reflection mapping” for computer rendering purposes. This group gathers current CGI applications that use a cube as a cognitive and expressive modality of representation.

Originally conceived for technical and scientific engineering applications in computer sciences, the cube (no longer a generic parallelepiped) started to be used as a surface of projection with the emerging digital technologies in the 80’s when programmers and computer designers explored its utility to represent digital *worlds* and *skyboxes* [170]. Currently, the applications of this group use the geometry of the cube for the creation of ludic-artistic scenes, e.g. videogames [171]. The variant of using cube came from Ned Greene, who proposed the *Environmental Cube Mapping* as an alternative to the existing possibilities for graphic rendering at the time, i.e., using spherical mapping [98].

Nevertheless, the hardware limitations back then delayed the introduction of this application since graphical cards could not process six textures at the same time instead of the unique spherical one. When the cubical texturing was finally possible, it helped to create better reflections and performed more efficiently than the spherical mapping, rendering better quality graphics with less computer resources [172].

After Ned Greene, many more methods were created and explored to speed up and simplify the correspondence between cube and sphere, such as *Isocube* [173] or *Continuous Cube Mapping* [174]. In these regard, the studies of Dimitrijević, Lambers and Rančić expose pros and cons of cubical-spherical mapping methods: tangential, adjusted, outerra, COBE quadrilateralized, cartesian and Hierarchical Equal Area isoLatitude Pixelization (HEALPix) [100], [101], [175].

Applications of this group have a strong influence on this research’s developments for several reasons: first, it is among the currently available technology; second, the final visualisation of panoramas is mostly using a cubical map; third, cubical perspective would not have had strong reasons to exist *before* digital age (see IV.1); and finally (as mentioned in First Part I.6.1), because the relation between the sphere and the cube is something well known in the CGI field. This latter condition implies a biunivocal correspondence among spherical and cubical maps, and therefore studying one it is possible to transfer analogously the same reasoning to the other. Such reason is at the base of the second method for drawing a cubical perspective (see IV.5).

In architecture, cubical mapping is used during survey for minor editions of photographic panoramas, such as deleting the tripod [176, p. 698] or for point cloud coloration [177, p. 356]. These activities use cubical mapping for a last aesthetical intervention in post-production but not for the early stages of project.

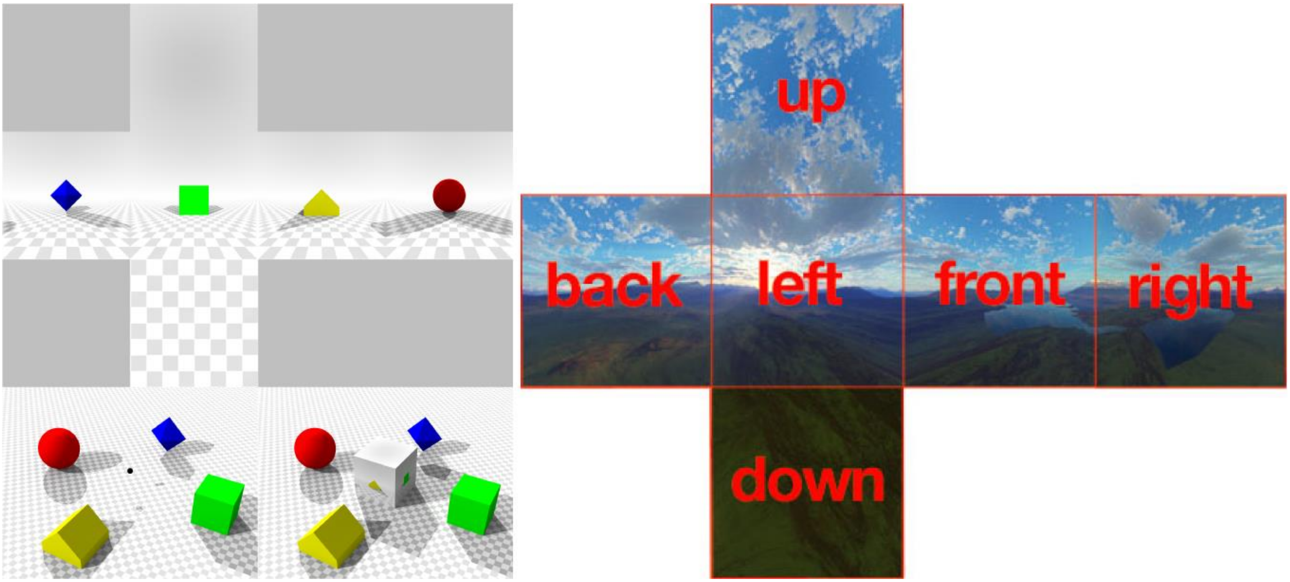


Figure 42 . Cube mapped panoramic image (2009) by Michael Horvath (left). Skybox and faces' alignment (2011) by Arieee (right). Both images under Creative Commons License [178], [179]

III.3.1 Characteristics

The spherical surface is more fluid for computing, yet it also causes inconveniences such as significant storage and radial blur in the rendered image (due to poles' high distortion). Consequently, a reasonable approach towards optimisation is to subdivide the sphere, thus projecting each region into different faces of a polyhedral surface and render just the one focused by the observer. Nevertheless, subdividing the sphere into too many parts will increase the number of faces and interruptions and go against the optimisation.

At this point, the cube represents an interesting alternative: *"For paper maps, interrupts make visual discontinuities, while for electronic maps (i.e. textures) they may require separate data sets for each region. The number of data sets may have a direct impact on the memory usage. Therefore, it should be minimized if possible. For the purposes of computer graphics, the projection to the faces of a cube (as a special form of a hexahedron) is of particular interest because each face is rectangular and thus allows straightforward storage of map data in common 2D file formats, as well as management of rendering data in common 2D texture formats"* [101, p. 260].

Some other advantages of cubic rendering are a naturally easier representation of the surroundings (for example, due to the fact that each face has a known classical perspective, and also to the fixed position of the observer at the geometric centre of the cube), and a lower buffer required for rendering each scene due to the smaller dimensions of each image. On the contrary, an important issue is the discontinuity for lighting conditions between the centre and the edges of each face [174, p. 29].

III.3.2 Applications

Nowadays, many years after Greene's proposal, the potentiality of the cube has been extended to every area were virtual reality and scenes rendering are required, such as in videogames for entertainment purposes or in education and collaborative spaces for culture consumers [170], [171], [174], [180].

In the field of digital arts, probably as a handy way to avoid dealing with spherical perspective's weird curves, the cube started to be explored as a support for graphic artists and comic designers. The flattened cube indeed, allow them to draw VR environments with less apparent distortions.

In the field of architecture and cultural heritage in particular, it has been used during virtual tour's generation for example, allowing high quality visualisation during the navigation of surveyed buildings [28], [29], [181]–[184].

III.3.3 HLOD visualisation

When navigating an IM, the texture displayed in the screen is previously mapped from the original model and stored accordingly to different resolutions. What one sees in the screen during the navigation is a rendering of this data, accessed accordingly the position and direction of the observer [101, p. 259].

The visualisation of interactive panoramas has adapted to a lighter visualisation of virtual environments to cover our daily interaction with web browser interfaces. In this sense, it is very common the use of tiles and Hierarchical Level of Detail (HLOD) algorithms, which allow an increasing of the visualisation performance and the reduction of resources used by the computer whit big data models [185].

Such a technology was used, for example, with Google Earth and is nowadays the web-based technology for visualisation high detail data using WebGL [186]. These algorithms map the content, fragment it in small pieces, and project in the screen just the image corresponding the observer's visual cone. Only after having fulfil this partial vision, the remaining pieces around the cone start to load [185].

As the spherical case, cubical mapping finds the observer always at the spatial centre of the cube, which implies a constant and fixed position of the observer regarding the projecting planes, a relation that will give us particular advantages by the moment of drawing.

III.3.4 CAVE projects for 3D sketching

Much of the current VR evolution started in the military field with projects for training using augmented and virtual reality, but such a beginning vanished when the technology moved to the entertainment and culture consumption fields [52, p. 169], [140, p. 6]. Not many years after these first investigations, it was already possible to fully navigate inside 3D models and enhance the virtual reality experiences with headsets, e.g., the art project *Osmose* by Char Davies [187] and the paradigmatical *Cave Automatic Virtual Environment* (CAVE) of Cruz-Neira [146], [188] both introduced in the 90's. This latter system has been largely used since its first appearing and recommended, for example, as a medium to revive the cultural heritage [183] or for engineering purposes such as "*touring a virtual nuclear reactor, train staff, orient subcontractors and to consider new designs" (*Figure 43) [189].

The CAVE system "*consists of a room whose walls, celling and floor surround a viewer with projected images […] is a cube with display screen faces surrounding a viewer*". As an immersive system, a CAVE tries to reach an effective virtual reality, i.e., when one can "*ignore the interface and concentrate on the application"* [146, pp. 65–67]. The cubical map in this case, matches the cubical shape of the artefact, facilitating the correspondent projection in each face.

As well, some specific projects use the CAVE for 3D sketching. For example, *CavePainting* from Daniel Keefe [190] and *Sketching in Space* from Habakuk Israel [6], [191] have been exploring CAVEs implementation within early conceptual design. These proposals explore a paper-detached sketching tool analysing the impact of drawing inside the 3D environment with ad-hoc tools. The workflow goes directly from the designer's thoughts to its digital model (PDI): "*3D sketching has the potential to develop towards a new tool that supports creative work and extends the human understanding of the expressive potential in digital space*" [6, p. 11].

The proposal is presented as a viewer-centred perspective artefact, which implies "*a mythical camera positioned along an axis extended perpendicular from the center of the screen. Viewer-centered perspective simulates the perspective view from the location of the viewer. To maintain correct perspective, a sensor that continuously reports the viewer's position to the simulation is commonly used*" [146, p. 65]. The characteristic of recreating the right perspective *from* the simulation accordingly to the observer's position, highlights either the pre-existence of a PDI or an automated computing of the image. These examples show the usefulness of digital interfaces for binding two stages of design, i.e., sketching and 3D modelling. These CAVE-based devices capture early conceptual design ideas and build its digital model on-the-fly, i.e., the designer is creating and literally being immersed in a PDI at once [6], [190], [191].

In other words, these artefacts consider 3D space as a different and more direct medium for exteriorising thoughts, a mean in which modelling happens automatically and there is no need for projecting anything, neither in paper nor at all [6, p. 4]. This technology is in its way to become more natural and common among architects and designers. Still, their current availability for surveying, analysing an existing place, and massive education for example, is still reduced. These innovative devices will need some time to get assimilated as an open and massive option among designers and in teaching classrooms. Still, an analogical option will always keep its safe place as a necessary alternative for those who think and construct in other terms.

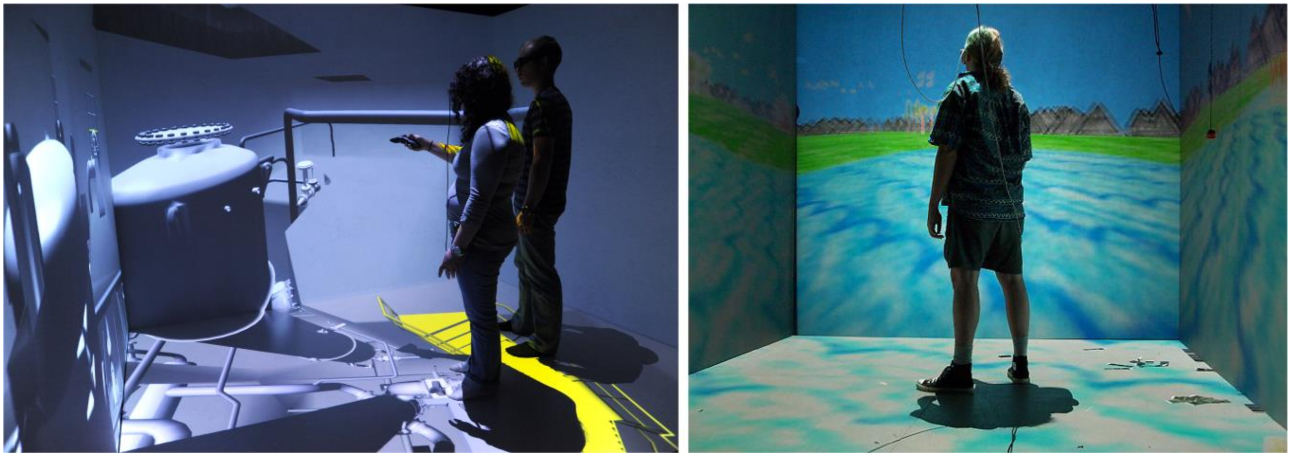


Figure 43 . Engineering teams using a CAVE at INL's Center for Advanced Energy Studies (left). A VR cave installation at EVL, University of Illinois (right). Images with Creative Commons License [189], [192]

III.4 REPRESENTATION WITH THE CUBICAL MAP

This third group of applications collects examples strictly connected to this research's goals. I analyse cases and extract from them theoretical and practical concepts, evincing where and how the methods for cubical drawing developed at IV.4 and IV.5 nourish the field of representation.

III.4.1 Drawing a cubical map

By drawing in cubical perspective, I intend to match the 2D drawing with the digital technology used for the immersive rendering, i.e., the cubical environmental mapping. Therefore, a HIM made with cubical perspective joins the previous group, keeping the same properties, e.g., the observer is always in the geometric centre of the cube.

Among the examples there are *drawing and guessing* procedures, or, in other words, verified by mere visual proving, as exposed with spherical case (see First Part I.6.2.2 and I.6.3.1). Yet, in contrast with the latter, there were no full methods based in descriptive geometry constructions for cubical perspective[28].

By a full method, I mean a biunivocal system between geometrical characteristics of an object or building (e.g., in floorplan and section), and its perspective image in such a map. During the studies, it emerged that the cube was being limitedly used just to trace on top of it, guessing how line fragments divide by repeated trial and error.

[28] See Introduction

III.4.1.1 Drawing and guessing

In parallel with spherical perspective examples, there are cubical drawings following an approximative procedure. In many cases, they use grids *converted* from the equirectangular perspective, not actually producing them. Then, such instances use the known workflow for switching among projections for navigating the panorama (as in the example at First Part I.6.1).

For example **Theng Seng Lai**, founder of *Studio Behind 90* wrote the tutorial "*4 Steps to create a 360 VR illustration/painting in Photoshop*" explaining how he deal with the cubical map using very simple graphical pieces [116]. The tutorial[29] explains principles to draw in a cubic map: first converting an equirectangular grid to the cubical format, then sketching the first lines on top of it (Figure 44), and finally filling and painting details (Figure 45).

Tutorials and guidelines like this example are useful for having a first idea that *something* is happening every time one crosses an edge. This is the most basic way to understand how cubical perspective works, and certainly one of the first things we needed to deal with: How does the representation of a line *break*?

The construction of Theng Seng Lai follows the following principle: "*create a guide at the edge of each cube. Every time you draw something across an individual Cubemap*[30]*, re-imagine your perspective into a new single point perspective. Always see every Individual Cubemap as a new single-point-perspective-grid*" (Figure 44) [116].

Explanations and tutorials like this point to a similar setup to David Chelsea's book, they focus into solve the "*how-to*" in its minimal expression. There is not a theoretical framework, an analysis of the projection from a descriptive geometry point of view, a classification of its

[29] The same Then Sen Lai deprecated recently the content of this blog entry, choosing instead the above-mentioned tool of Photoshop [128]

[30] *Cubemap* must be understood as *face of the cube*

elements and characteristics. The reason behind this is at first sight cubical perspective does not look more complicated than solve this only issue, but actually replying that question takes a bit more than one blog entry.

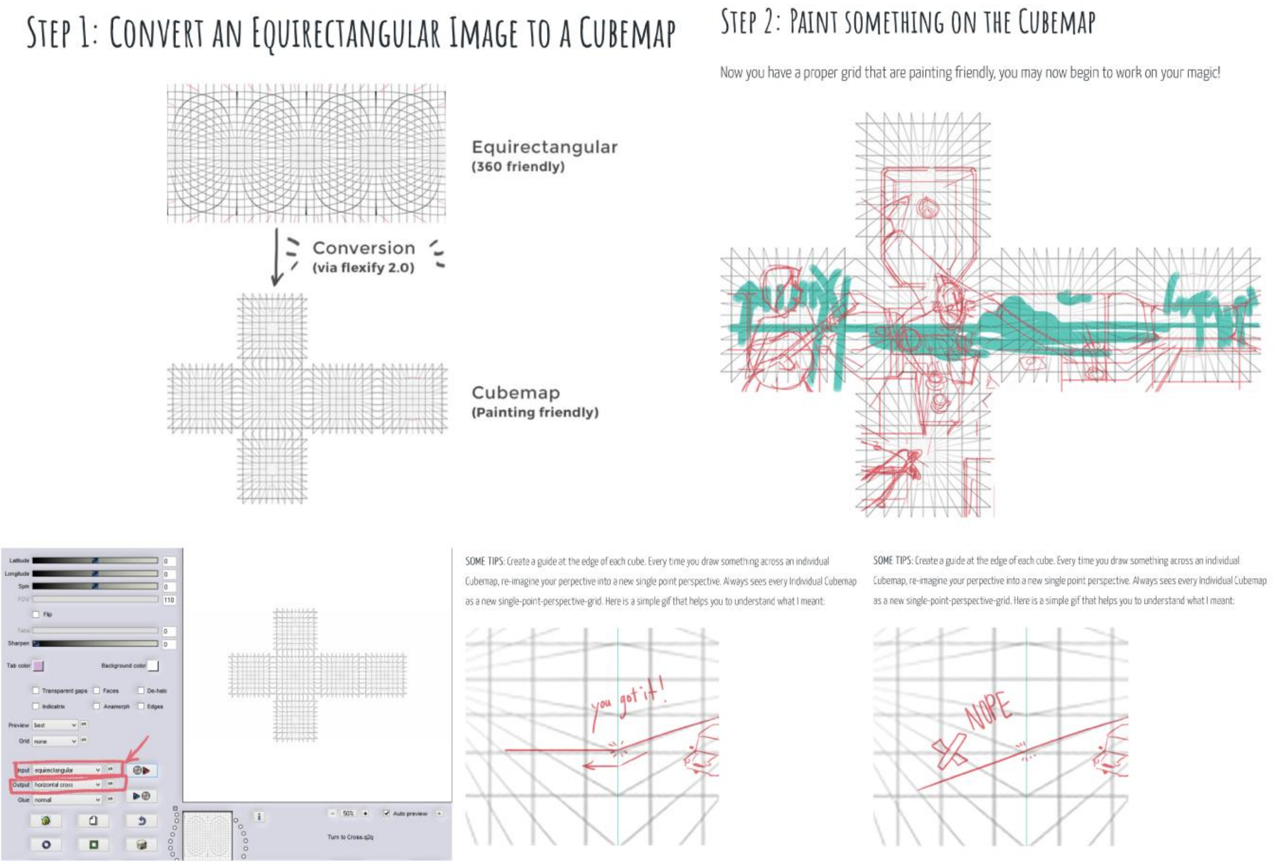


Figure 44 . Tutorial for constructing a cubical drawing (2016) © Lai Theng Seng. Converting from equitectangular to cubemap using Flexify (left). Preparatory draft (up right). Graphic explanations for crossing edges (bottom centre and left) [116]

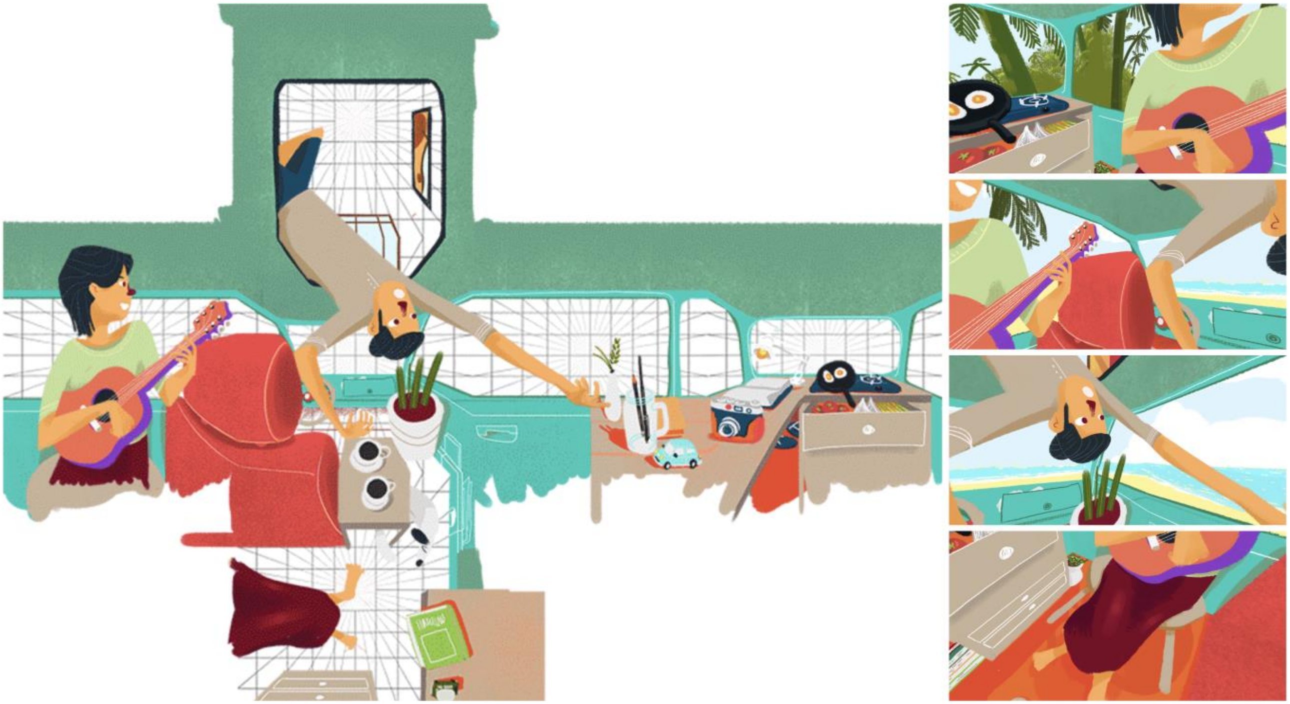


Figure 45 . Final coloured illustration and VR navigation (2016) © Lai Theng Seng [116]

Personally, I also started with cubical drawing following this kind of explorations, opening and unfolding the cube in different ways and proving the different effects with handmade drawings (Figure 46).

In short, it is possible to easily draw on top of the cubical map converting the equirectangular grid into the cubical format and then reverting to the equirectangular format again. Nevertheless, as mentioned in First Part I.6.2.2, the draughtsman may or may not be really conscious of what it is happening at a higher level of definitions. In fact, as explored in First Part I.3, the depth effect of perspective can also be the result of an approximative trial-and-error procedure (Figure 47).

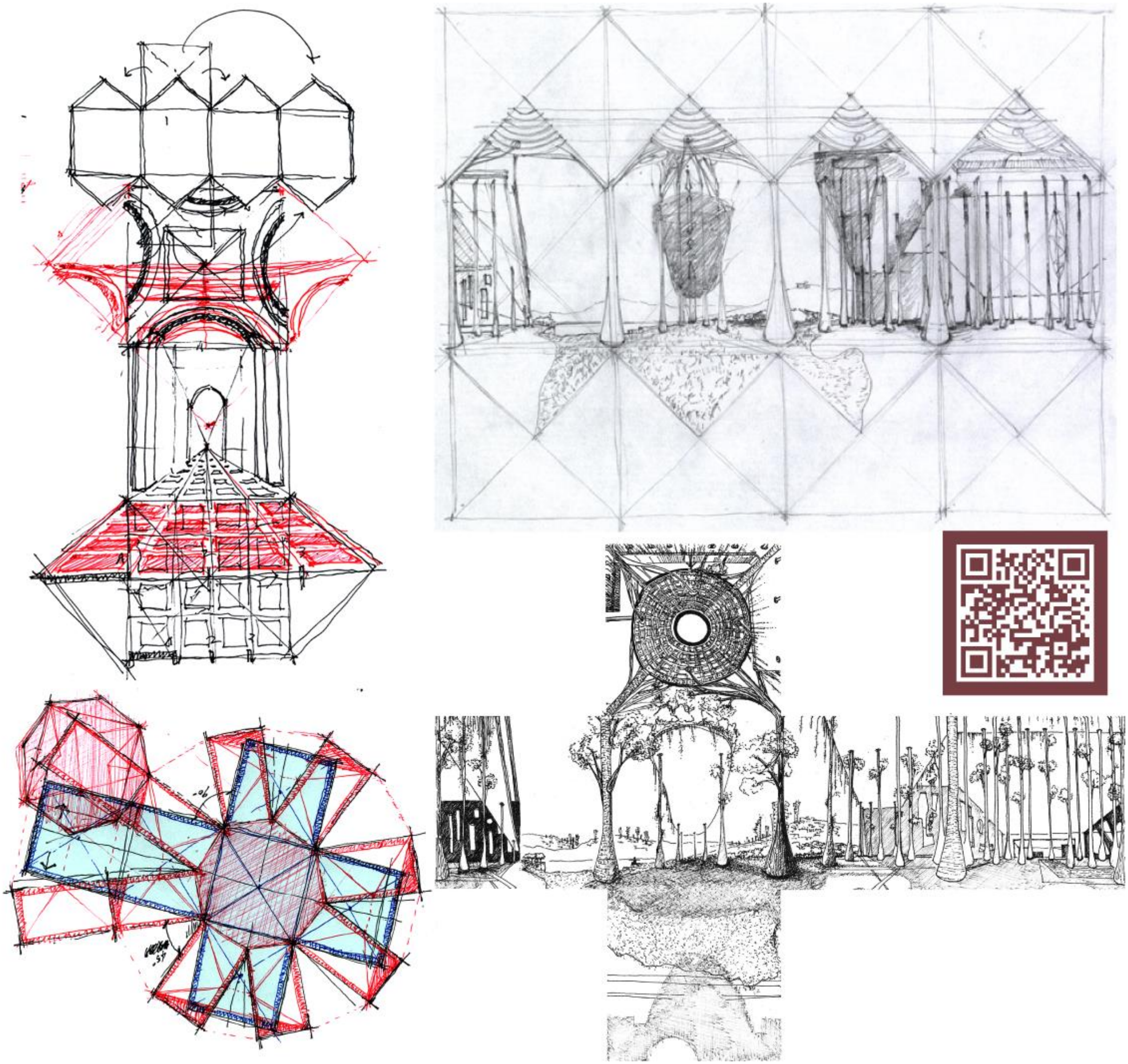


Figure 46 . Discontinuities and other possible surfaces. Variations to open the cube. Right: Minerva (2017)

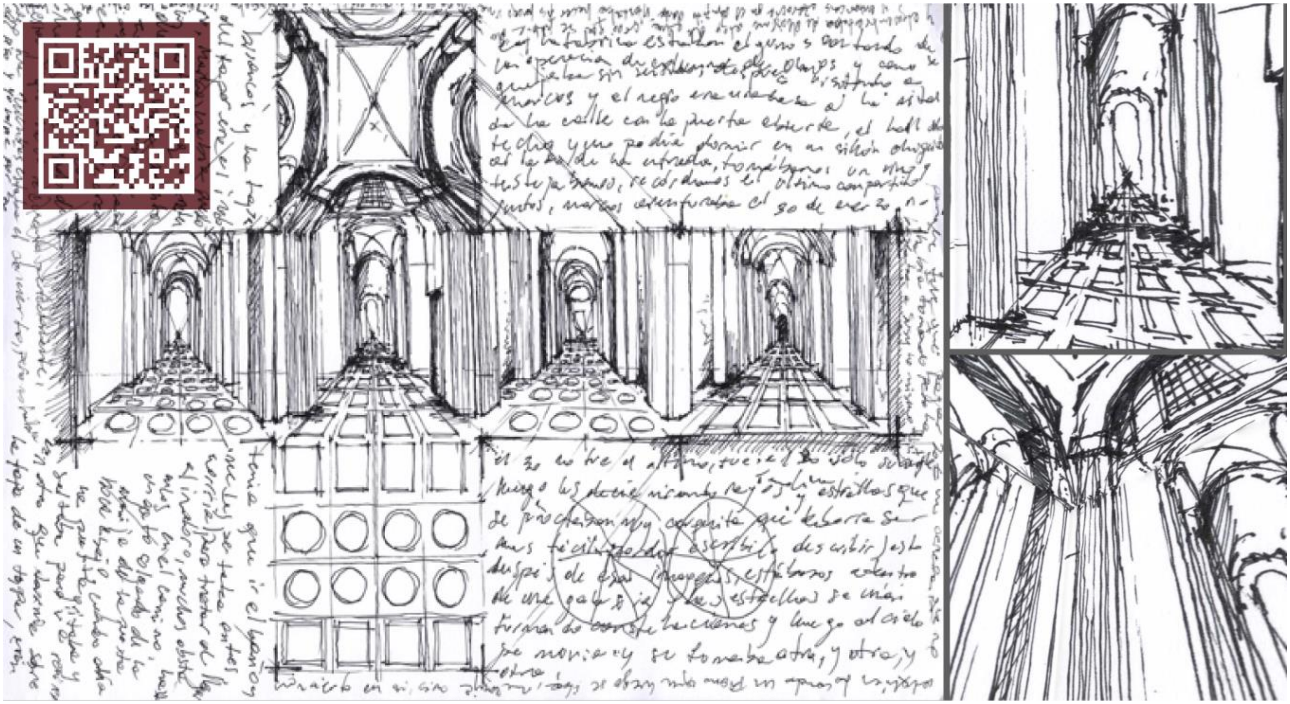


Figure 47 . Requiem (2017). Trying basic setups in cubical perspective

These methods manage to cover more or less well the *how-to* procedure, but there is not focus (at all) in the scientific precision of the drawing. Once more, and as happened with panoramas (First Part I.5) and Perspective Boxes (III.2), entertainment purposes have a strong presence in the conception and use of immersive illustrations, seeking the goal of increasing users' emotional impact: *"Immersion can be an intellectually stimulating process; however (…) in most cases immersion is (…) a passage from one mental state to another. It is characterized by diminishing critical distance to what is shown and increasing emotional involvement in what is happening"* [52, p. 13].

III.4.1.2 Full constructions (or the lack of)

There were not full constructions for cubical drawing by the moment this research started in 2017[31]. It is worth to clarify that many existing methods may have been *adapted* so to give a full solution, such as the circle of distances[32]. Nevertheless, there is a big confusion with *cubical perspective* term since the cube is largely and often used as a canonical shape for representing basic geometries, that is, as the object to be represented. Many times, this gives as a result the discussion of a cube's perspective, while instead I understand the use of the cube merely as a *surface of projection*.

Some of the studied methods were taken as a very valuable starting point because they may solve the problem (partially or fully). The issue is that they solve the case as a special one of classical linear perspective, which was a good influence and help for the definitions of the first method. Some of them are *Prisma T-homólogo de un Cubo* used in *Homologías entre figuras de tercera categoría* [193, p. 2], and a method with points and circles of distance used to draw polyhedral structures inside another polyhedral geometry (per case, a cube) by following grids projected in the walls and the ceiling [194, pp. 10–12].

Perhaps, one of the reasons for lacking a proper scientific background, is that representation sciences may had not enough strong motivations for cubical perspective to exist until now (see IV.1) In any case, these methods do not consider cubical perspective as an independent perspective, but they solve the drawing projecting or calculating a classical linear perspective made *a priori* (as happened with the first cases of anamorphosis and panoramas). Two indicators of such a logic are the aleatory position of the observer regarding the cube and the projection in several planes but one-by-one. In the former case, the observer's position is

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[31] See Introduction
[32] *Cerchio delle distanze* in Italian

always fixed at the geometric centre of the cube within CGI cubical mapping and therefore would constitute a further special case of this application. Regarding the latter, working one single plane at a time implies to be solving partially the immersive view with several classical perspectives.

This research focuses the problem as an alternative construction where cubical perspective has its own theory as a full and independent case. In such a case, is possible to draw the whole scene (i.e., the six planes) at once, as it happens with the equirectangular map.

III.4.1.3 Summary

The current state of art shows the presence of *drawing and guessing* methods. Consequently, the results orient to exhibition and entertainment purposes, leaving a high degree of uncertainty about the degree of knowledge of their authors, and with a suspicious or inexistent scientific character. These drawings are rather the output of enthusiasts virtuously operating preset software and tools. The limitation of the constructions is evident in the same results usually limited to lines parallel to the faces of the cube. There is not precision for representing organic objects or architecture elements with exactness regardless lines' direction. All these signs highlight the lack of a holistic solution for drawing in the cubical map.

Nevertheless, the state of art also testifies a growing interest in those techniques that unify traditional constructions in a cubical map and VR technology. In 2017/18, there was a limited but growing propagation of cubical drawing with procedures leading more to a *"black box doing"* than a full thinking for discovering the potential of the perspective [34, p. 16].

III.4.2 **Software**

As in the spherical case, I analyse in this section workflows that may adapt for the cubical drawing and those directly thought for cubical drawing and previsualisation of the results:

III.4.2.1 Drawing and guessing

Predisposing and using a cubical map is not an issue at all with any drawing software. Indeed, within the experiences presented in some operators used indistinctively raster-based (Figure 48) and vector-based options (Figure 49).

An important problem to solve is how to visualise in immersive modality the content with certain fluidity. The conversion steps from the cubical to the equirectangular map complicate the task, especially if one needs to verify every new line because is guessing on-the-fly how a line may continue after passing an edge.

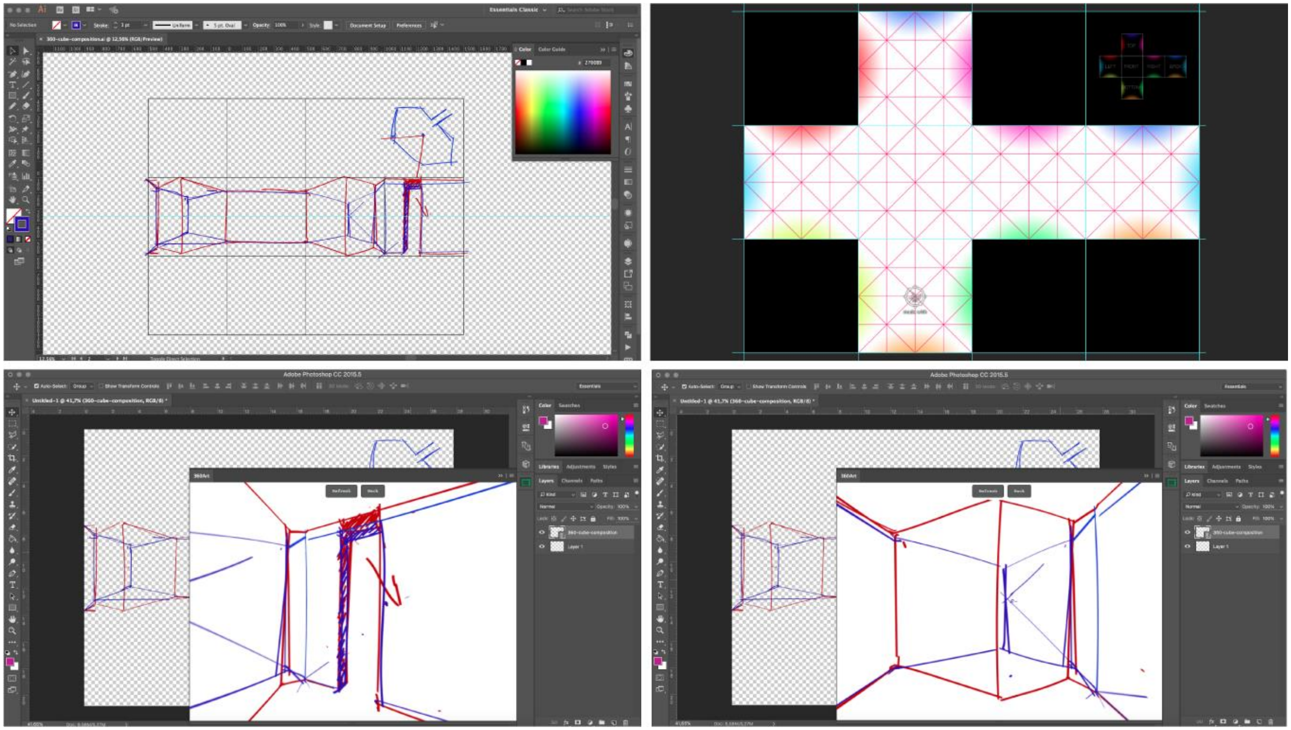


Figure 48 . Drawing in the cubical map with illustrator (top left). Oniride 3D Art plugin interface in Photoshop 2015.5 (top right). Using Oniride 3D Art for navigating the first drawing in immersive modality (linking the Illustrator file to Photoshop) (bottom)

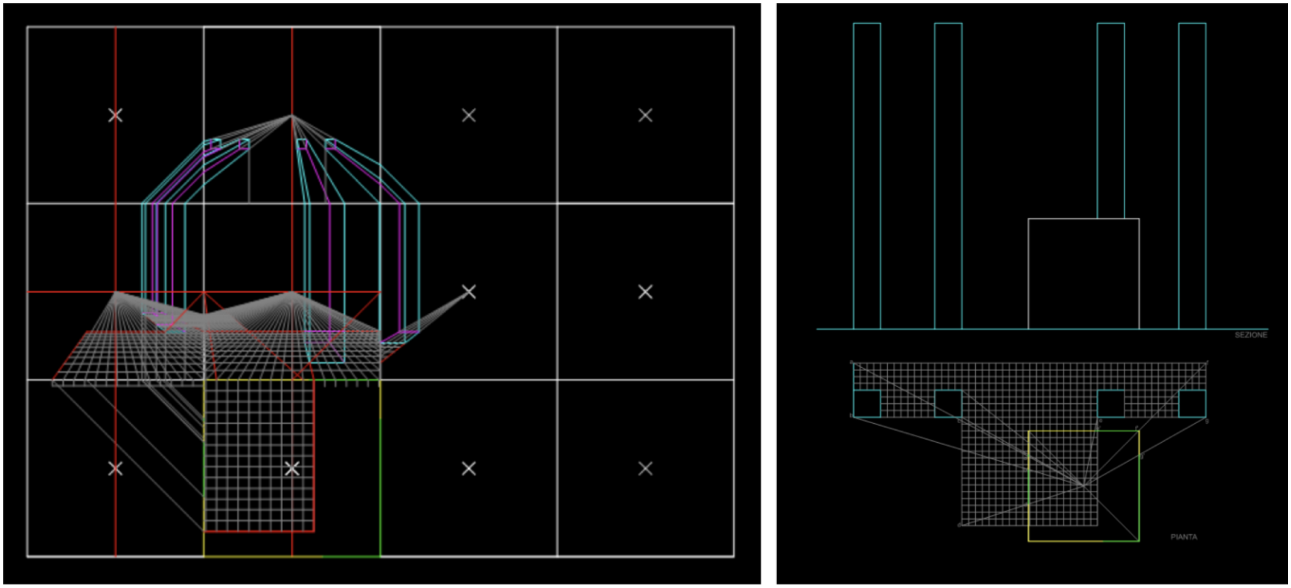


Figure 49 . Drawing a cubical map with AutoCAD (2020) by Teresa di Palma

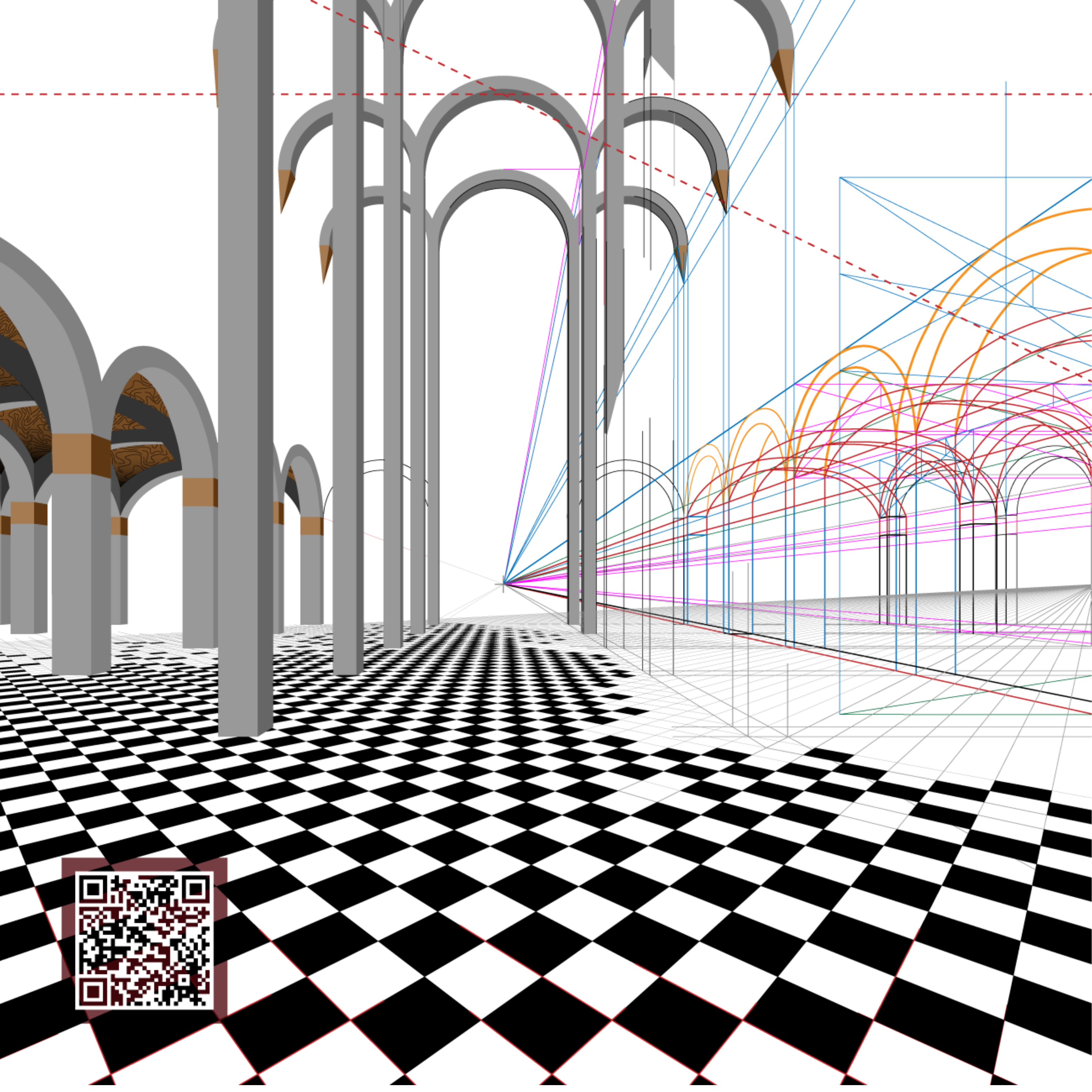
To help in such a task, there are direct and indirect options. Among the former, *Oniride 3D Art* is the unique software I found so long. Along with *Sketch 360* (see First Part I.6.3.2), this plugin allows to draw in the flattened map using the raster-based tools of Photoshop (Figure 48 bottom), and to visualise the results in a separate viewport (Figure 48 top) [195]. Nevertheless, the VR window does not update automatically, and the solution works as a dependent plugin of Photoshop 2015.5 and since that version they have not updated the plugin anymore becoming currently deprecated. The results of this workflow are validated only by visual appreciation, without a certain parameter of correctness.

Another option is presented by **Massimo Marrazzo** who uses the SketchUp-based plugin *CubicPano Out* created by John Wehby and Bixorama 360° photo software [196]–[198]. This is not a cubical drawing option, but it opens the door to a possible workflow for it: from a 3D model, Marrazzo gets the six faces of the cube using *CubicPano Out* plugin. The images are automatically processed and generated. Then, with Bixorama he switches among different maps: cubical, equirectangular, azimuthal-equidistant, etc.

Marrazzo's approach could be useful, for example, after surveying an existing architecture, getting its model from a point cloud, thus applying the plugin for getting the cubical faces, tracing on top of them and stitch them back in an equirectangular format. However, the PDI needs to be *created a priori*, which bring us again to the same question made at 0: How to create a cubical perspective when the project does not exist yet, when there is no model or material reality created *before*?

III.4.2.2 Full constructions

I have found no specific software for drawing in cubical perspective, that is, nothing equivalent to *Eq A Sketch* for equirectangular drawing (see First Part I.6.3.2). This lack of software reflects the scarce theoretical developments for cubical drawing as well. Indeed, up here, the explored experiences were not exhaustive to entirely solve the problem of drawing in a cubical map.



CHAPTER IV

CUBICAL PERSPECTIVE

IV.1 INTRODUCTION: HIM MADE WITH CUBICAL PERSPECTIVE

Cubical perspective would have had not many reasons to exist until these days due to its own genesis, that is, the application within the digital world. In fact, the presence of digital technology and the cubical environmental mapping is one of the strongest reasons for cubical perspective to exist. Before that, there was not real reason for using a cube to project, What for?

Indeed, some *Perspective Boxes* were *parallelepipedal* but none of them actually *cubical* [70, p. 9], [163, p. 4], [166, p. 47]. In times of Barker's panoramas, a cubical physical artefact may have been practical to overcome the physical limitation of the cylinder in the upper and the bottom parts, but maybe more complicated for placing the observer in its geometric centre and for having a good diffuse lighting. Indeed, using a cylinder solved many problems such as the fluidity while drawing, the lighting and people circulation.

Already in our days, the historically pursued goal of the full immersive illusion reached one of its highest levels of credibility thanks to digital technology [141, p. 15]. With it, the cube acquired more importance, becoming nowadays a natural medium for immersive representation [119], [199]. Nevertheless, as exposed in the previous chapter, methods for drawing a cubical perspective have a very small footprint in literature, despite the current general interest in immersive drawing.

With the arrival of the digital, the problem of placing the viewer in the geometric centre is solved and, on the other hand, the general definition of anamorphosis presented at First Part I.4.1, opens the possibility for gathering information within the whole visual sphere.

Yet, for reaching the full immersion illusion (i.e., *suspension of disbelief*), one of the conditions is that the boundaries of the flattened cube should not be distinguishable when the cube is folded back during the VR navigation. If so, the perception and recognition of the represented space is maximum, thus senses get mixed between what they are perceiving and the material reality [52, pp. 13–14].

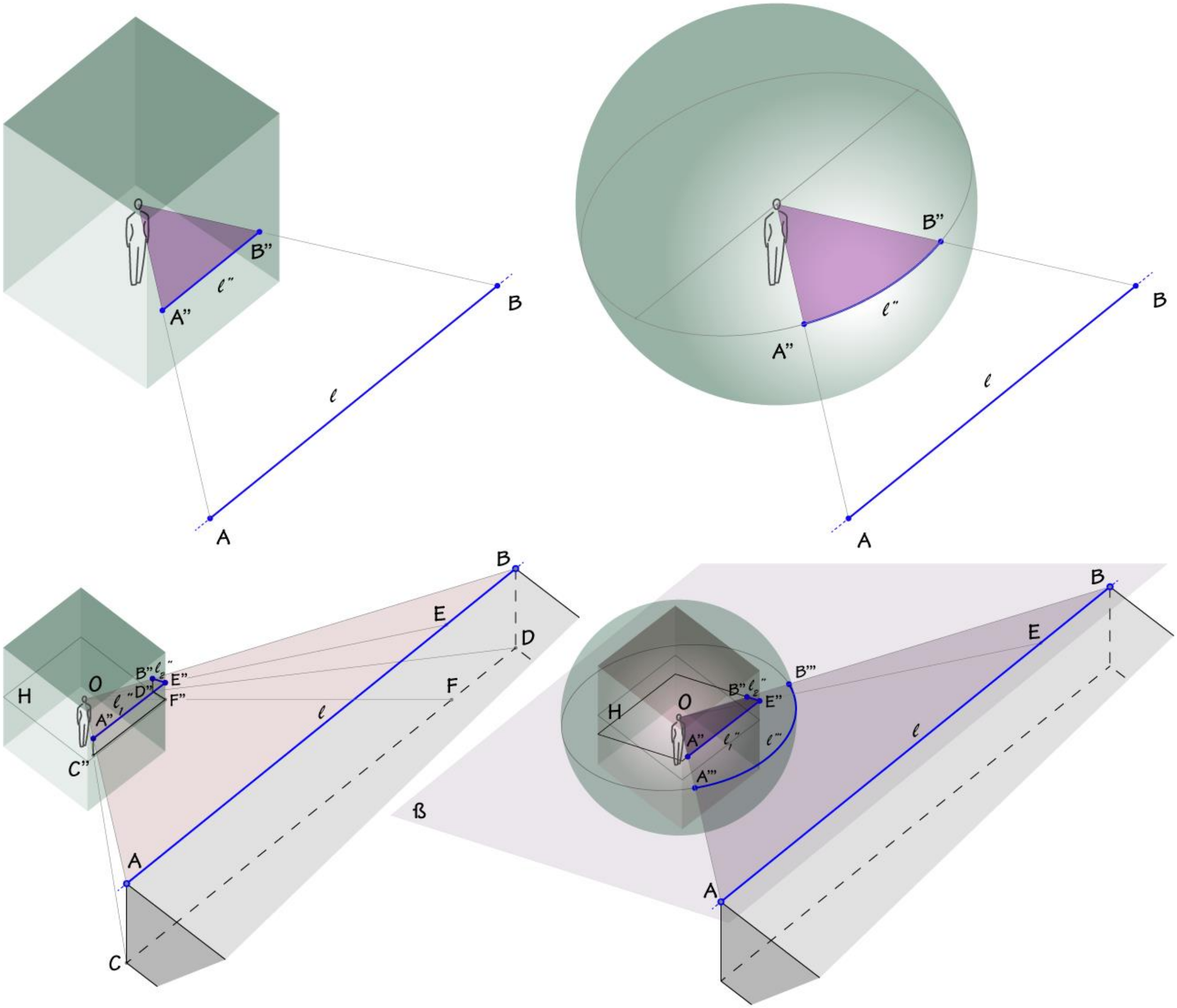


Figure 50 . The plane of projection is substituted with a cube (first method) and with a sphere (second)

Therefore, the precision of the drawing has an important role since any missed correspondence among edges will result in a discontinuity in the VR representation. Any full method should be a construction maximising the correspondence among edges, so to guarantee the continuity during the visualisation.

Going beyond the existing limitations, I substituted the single plane with a cube (Figure 50 left), and with a spherical surface thus corresponded with an equally centred cube (Figure 50 right). From there derivate two different methods that start to cover the lack of full approaches[33]. The first procedure conceives the cubical image as a set of classical perspectives and takes reference from floorplan and section as Piero della Francesca's second method (see IV.4). In turn, the second method joins the general scheme for spherical perspectives (introduced at First Part I.6.1 and I.6.2.4), gives a classification of lines, defines vanishing points and grids (see IV.5).

Both methods look for solutions contained within the flattened map and consider cubical perspective as an independent theory. These general solutions move cubical drawing from "*it should work more or less in this way*" to "*it works in this way*", thus offering and full tool for architects, engineers, artists and graphic designers creating a HIM.

IV.1.1 About the surface

These two surfaces are both a recognisable medium for the further digital interaction, but also, they are far more appropriated for a better manipulation of the gathered visual data: "*the sphere and not the projective plane is the natural manifold of visual data up to an anamorphic equivalence*" [57, p. 1]. In particular, an anamorphosis defined with a spherical surface fulfils

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[33] Certainly, a team-work result, led by the council of both advisor and international advisor

both linear and curvilinear perspectives, including perspectives for larger field of views (e.g., cylindrical) and "*sets it at the foundation of both classical and curvilinear perspectives*" [57, p. 1].

Once exchanged the surface, I project conically the environment to the observer, gathering the graphical information. In a further step, I flatten the surfaces, obtaining the map of the content in a plane. Studying the correspondences between the spatial geometries and their representation in the map, it is also possible to invert the process, creating first the map following descriptive geometry rules thus digitally mounting the surface and getting the VR environment. With the latter option, one can draw what it does not exist yet and see virtually its third dimension.

IV.1.2 Projective centre and perception area

The position of the observer in cubical perspective coincides with the projection centre since this is a step accomplished digitally, i.e., as a cubic environmental mapping. Therefore, their correlation guarantees an observer watching always from the rightest point of view. From this, some applications calculate a deviation for creating a fake stereoscopy (see IV.7.4). At this point, I leave the door open for those who want to experiment with accelerated or delayed perspectives (such a development is not object of this research).

IV.2 THE BASIC PRINCIPLE BEHIND CUBICAL PERSPECTIVE

Cubical perspective joins the full immersive representation with linear drawings methods. The latter recalls a familiarity among architects and designers with a known apparent deformation in every face and a handy distortion. Right as in a classical perspective, the projection l' of a line l is within a plane surface in the cubical case. This means that l' should be a rectilinear segment in both perspectives. Nevertheless, the faces of the cube represent not a unique plane of projection but six of them and with different orientations. Note that since l changes its condition regarding the different faces, it is possible for example, that $l \parallel F, B$ while intersecting faces L, R, U, D. Every different relation between l and some face, gives a different segment in each face and with a different orientation. Understanding the behaviour of this fragmentation is the key to learn cubical perspective. To illustrate this principle, let me get the image l' of l in both classical and cubical perspective:

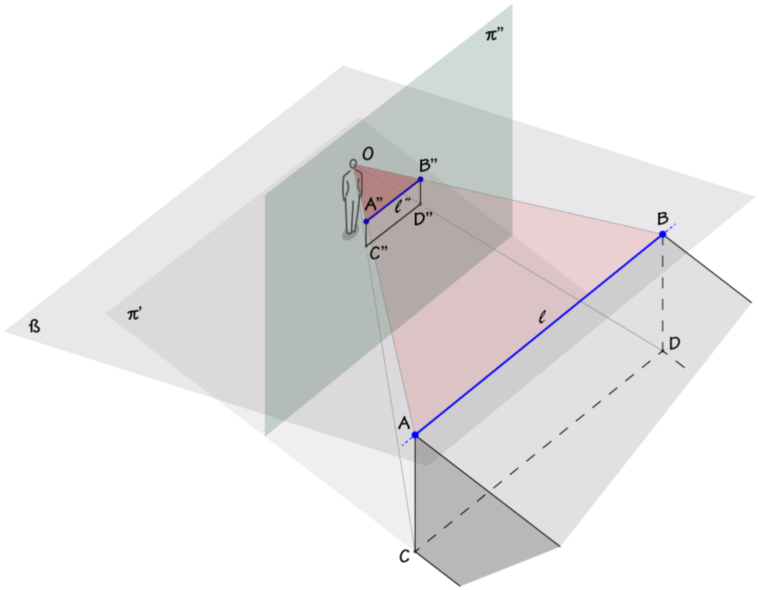


Figure 51 . Representation of segment $\overline{AB}$ in linear classical perspective, S = plane π"

In the **classical linear perspective case** (Figure 51): let A and B be two points of l. Between l and the observer O there is a projection surface S. Let S be plane $\pi"$ and $\overline{A"B"}$ the image of segment $\overline{AB}$ on S. To build $\overline{A"B"}$ we join O with A and B. Rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ intersect S in two points $A^{"}, B^{"}$ such that the searched segment is found joining $\overline{A"B"}$.

In the case of cubical perspective, there is a substitution of S changing plane $\pi"$ with a cubic surface. Let that surface be $Cube$. Let l be such that $l \parallel H, F \notin H$ and that belongs to the half-space in front of the observer when this is watching towards F. Let A', B' be the images of A, B on S.

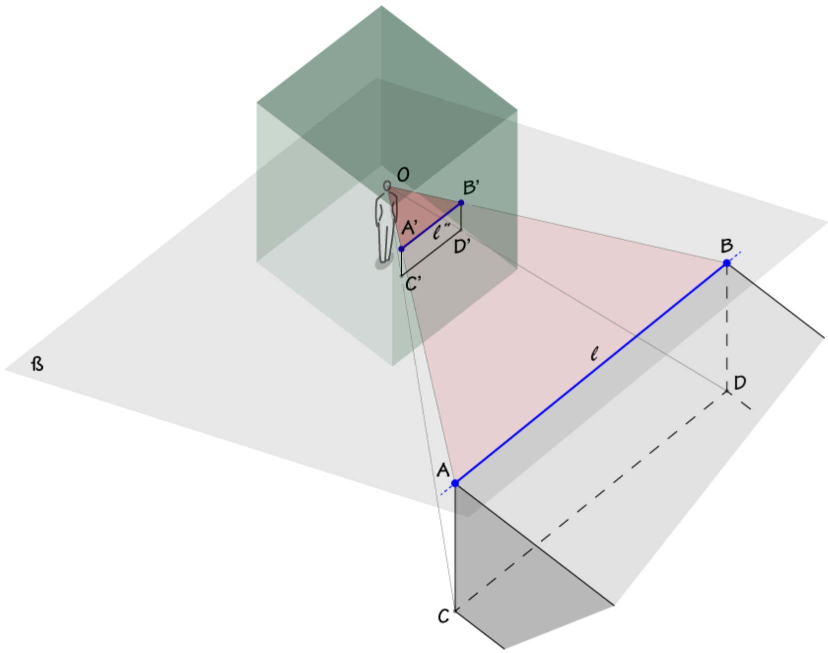


Figure 52 . Representation of segment $\overline{AB}$ with $S = Cube$. First case: $\overline{A'B'} \in F$

There are two possibilities: either $A', B' \in F$ or one of these points belong to L or R. In the first case, $\overline{A'B'} \in F$ and the solution reduces to the previous case (Figure 52). In the second case, $\overline{A'B'}$ projects in more than one face (Figure 53). This implies a third point E between A and B such that its projection E' is located in one of the vertical edges of F. Let suppose $B' \in L$ and $E' \in FL$ without loss of generality. The image of AB is made by segments $l_1 = \overline{A'E'} \in F$ and $l_2 = \overline{E'B'} \in L$. $\overline{A'E'}$ can be found as the first case. To find $\overline{E'B'}$ instead, we must consider that face L is rotated 90° regarding F. Because $l \parallel F$ results $l \perp L$, then its vanishing point is exactly in O_L, the centre of L. Therefore, $\overline{E'B'}$ is a fragment continuing $\overline{A'E'}$ but vanishing to O_L.

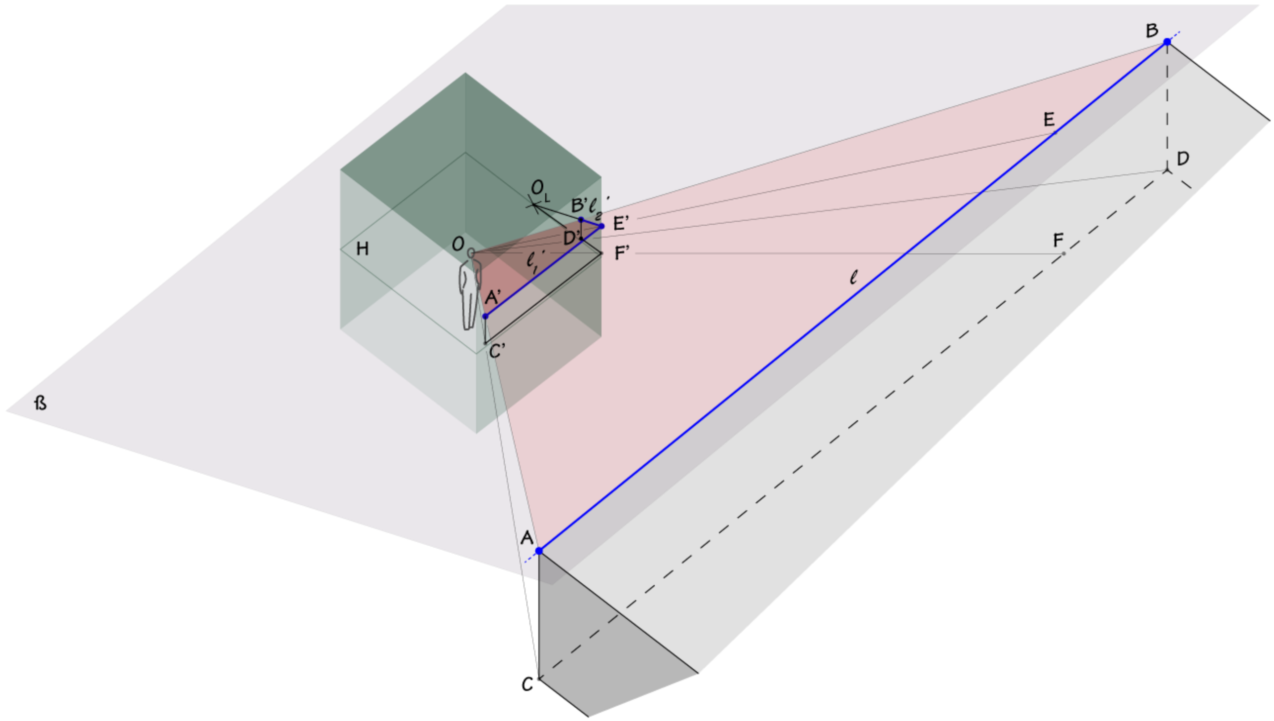


Figure 53 . Representation of segment $\overline{AB}$ with $S = Cube$. Second case: $\overline{A'B'}$ projects in more than one face

Generalising, if A', B' are in different faces, its representation in the cube is composed by more than one segment. Segments are interconnected sharing one extreme, but each of them follows a different vanishing point. Considering all possible orientations and inclinations, l can project up to four adjacent faces (Figure 54). In such case, to construct l' using classical methods there will be necessary five points plus three in the edges which supposes a double problem of visualization and efficiency. And after all, if a line in the space is defined by just two points, then such condition should also be sufficient to draw l' in any perspective system.

IV.2.1 The problem of representing with the cube

Consider a station point O (the observer's eye) and place a cube around it with centre O. Project the 3D environment conically towards O and mark where each ray hits the cube. Now cut and flatten the cube and you get a picture like Figure 61. Locally, on each face, it looks like a classical perspective: every line project to a line segment and sets of parallels have at most one vanishing point; globally, however, lines are (sometimes disconnected) unions of segments, and all families of parallels have exactly two vanishing points. It was pointed out in [17, p. 33] that it is quite hard, when a line crosses an edge of the cube, to know what angle it will make with the edge where it reappears.

The obvious solution is just to solve the individual classical perspectives of the cube faces the line projection touches. These are four at the most. This would require at least three points measurements for each pair of faces (Figure 54 a, b). This is not only inefficient, but it also creates a consistency problem when drawing from direct observation (unlike from plan and elevation), as each measurement of the visual angles (azimuth and elevation) will come with an independent random measurement error. The resulting object will not be a line but a union of segments, which will visibly change direction when seen in VR (Figure 54c).

A further analysing of the spatial situation (Figure 55), shows that all the represented points ($O, A, B, C, A', B'C'$) can be joined in a unique plane π. If a sphere is placed centred in O, the intersection of O with π is a geodesic. Thanks to this, can be followed the general statements already defined for other spherical perspectives and used in the second method (see IV.5). Then finding l' in the cubical map is a matter of understanding how plane π intersects the cube, more than forced reasoning of finding segment by segment.

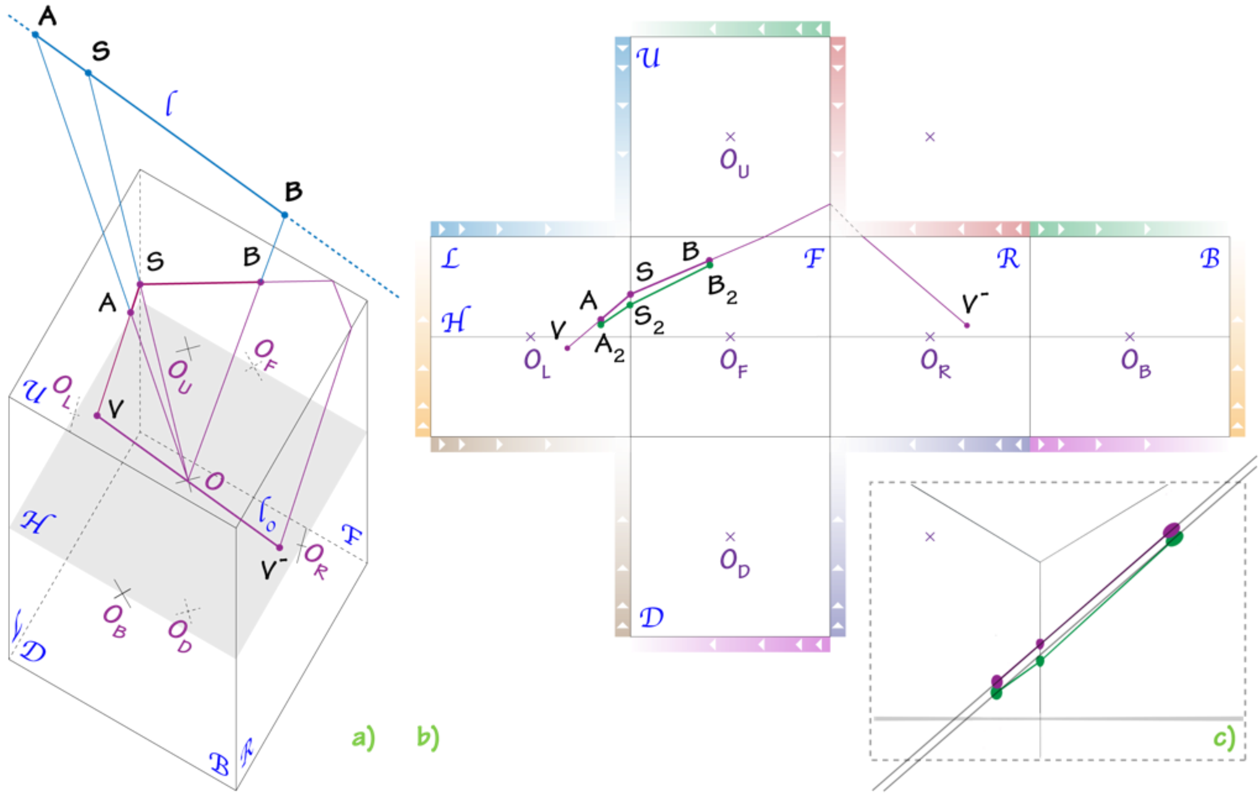


Figure 54 . Plotting a line l. The green line is obtained from independent measurements of three points A, B, S. In the purple line only A and B are measured and S is calculated from A and B by the method presented ahead. c) In the VR visualization, the two green segments do not align. The purple segments, though still affected by measurement errors in A and B, will always align in the VR view

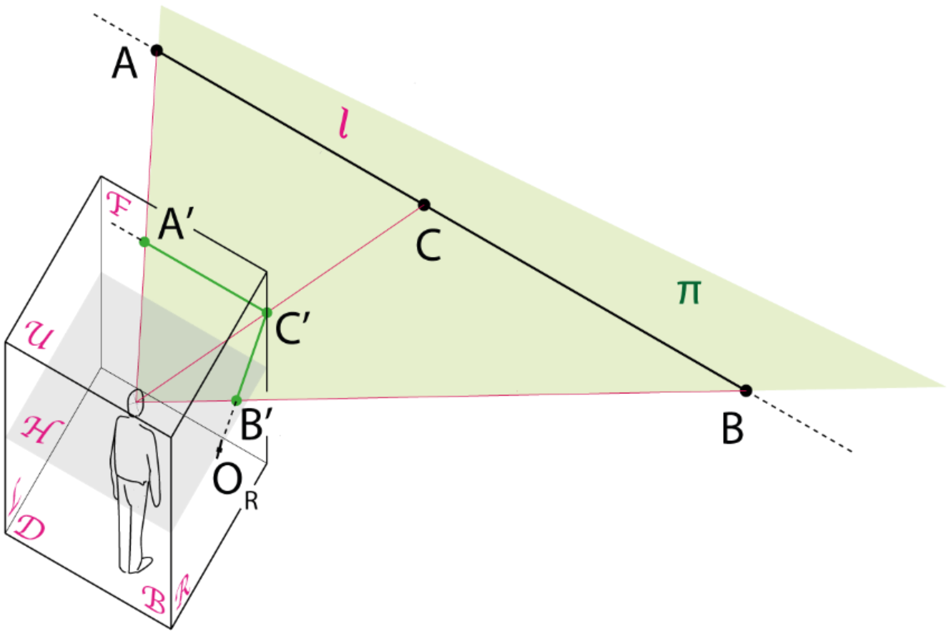


Figure 55 . Representation of segment $\overline{AB}$ with $S = Cube$

IV.2.2 Artefacts

For a better comprehension of cubical perspective and anamorphosis, I experimented with a very simple handmade device (Figure 56). The artefact consists into a piece of paper that can be folded to emulate two faces of the cube and the diagonal trough them. In the interior side of the two faces there are some lines drew following the previously presented basic principle. In the centre of the diagonal there is a hole: the observer point of view. Once closed, this peephole emulates the centre of the cube, where the right point of view for the anamorphosis lies and where the "deformed" lines will look straight (Figure 56c).

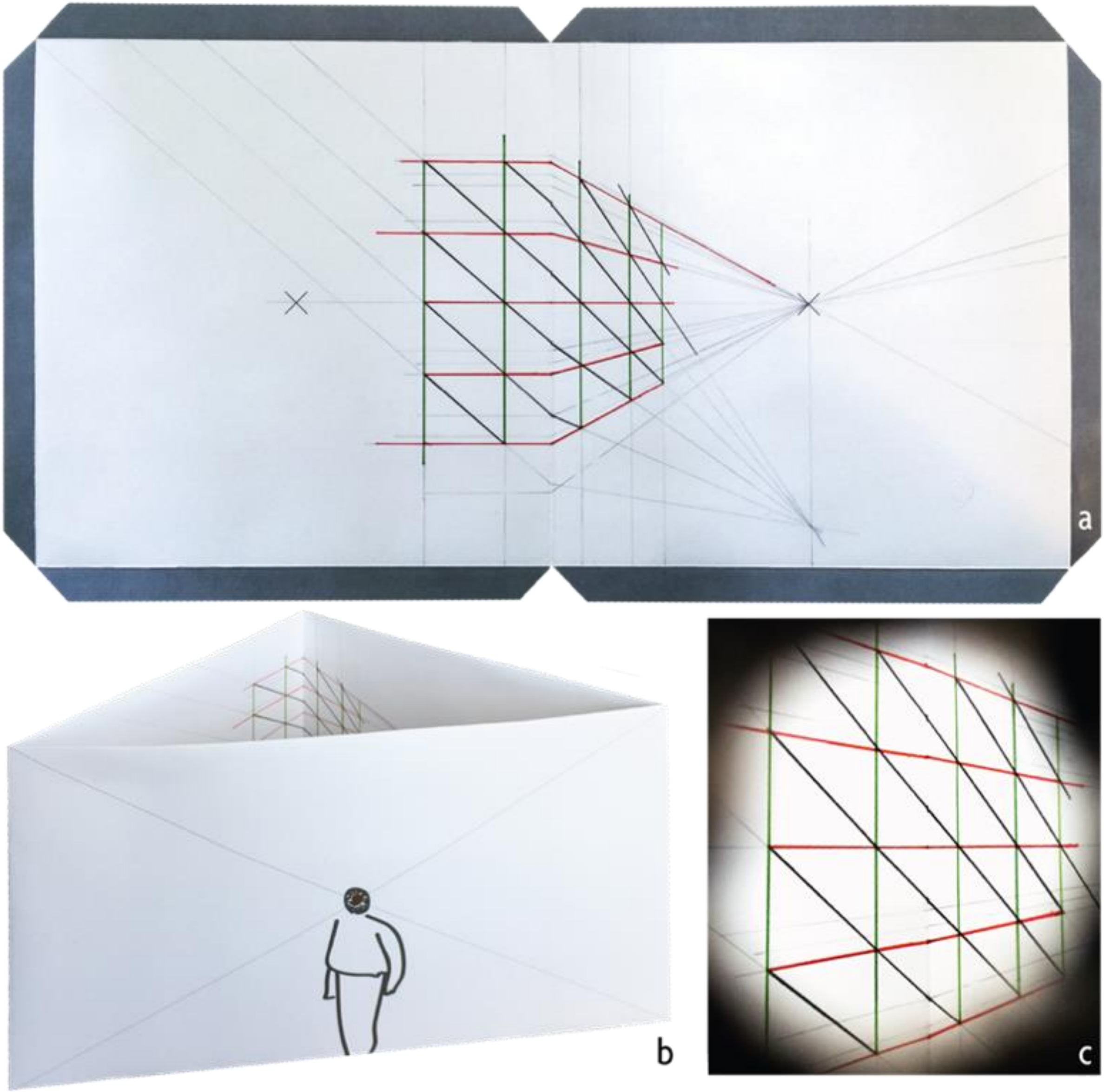


Figure 56 . Artefact to test the anamorphosis on cubical perspective

Thanks to the similarity between cubical perspective and the Perspective Box, if one restricts the attention to a half-space defined by a plane through O, it is possible to get a Perspective Box with especially simple symmetry (Figure 57).

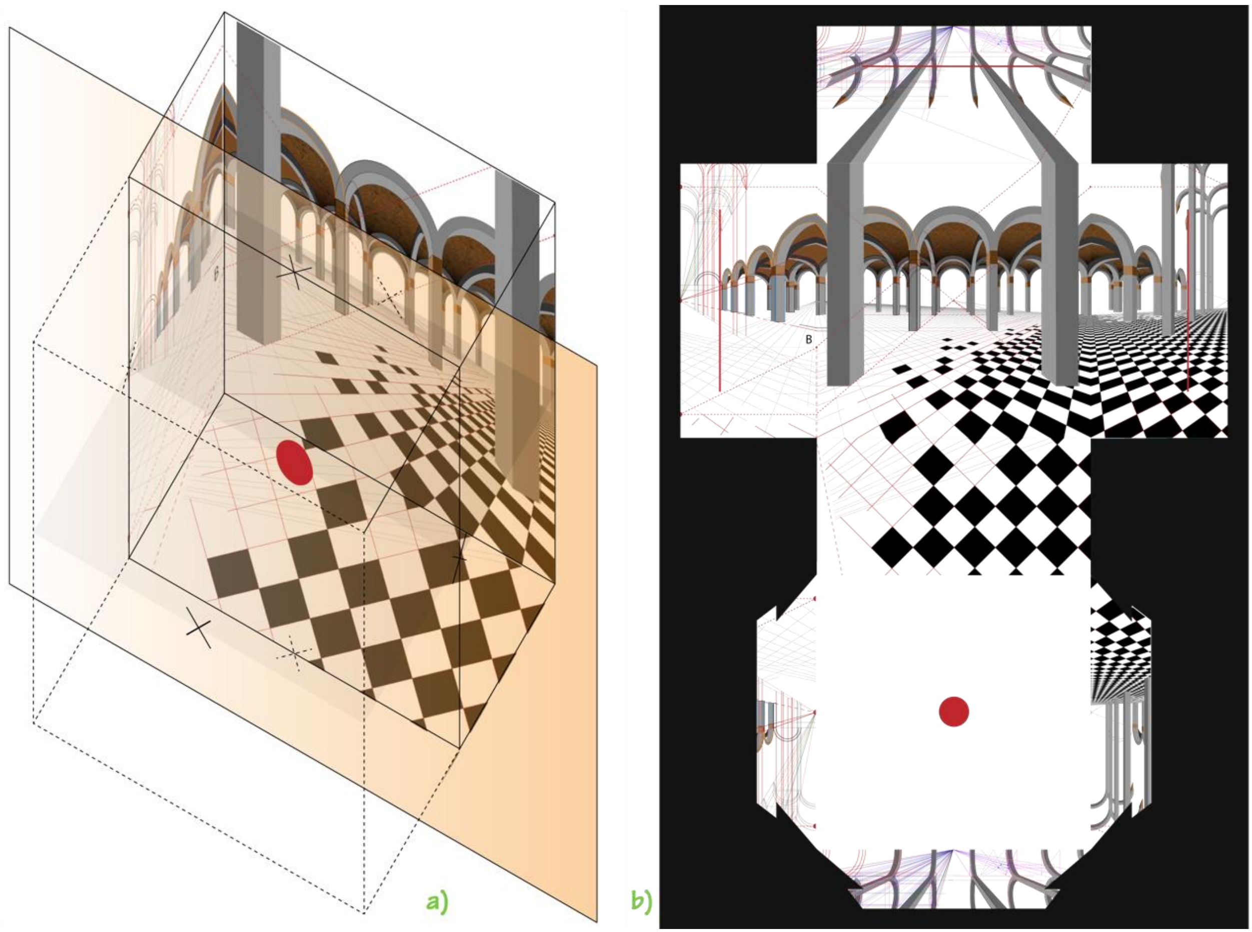


Figure 57 . a) Uniform grid on a perspective box, made up of half a cube with the observer at the centre. b) Cut-out based on a modern perspective box example from [166, p. 61]

IV.3 DEFINITION OF CUBICAL PERSPECTIVE

To solve the fragmentation of the method it was necessary to have a precise definition of the cubical perspective. Therefore, I introduce such definition used for both methods:

Given a point O in the 3-dimensional Euclidean space R^3, a *cubical perspective* with regard to O is a map from $R^3 \setminus O$ to a compact (i.e., bounded and closed) subset of the plane R^3 obtained in two steps: a conical projection towards the centre O of the cube, followed by a flattening of the cube onto a plane (Figure 58).

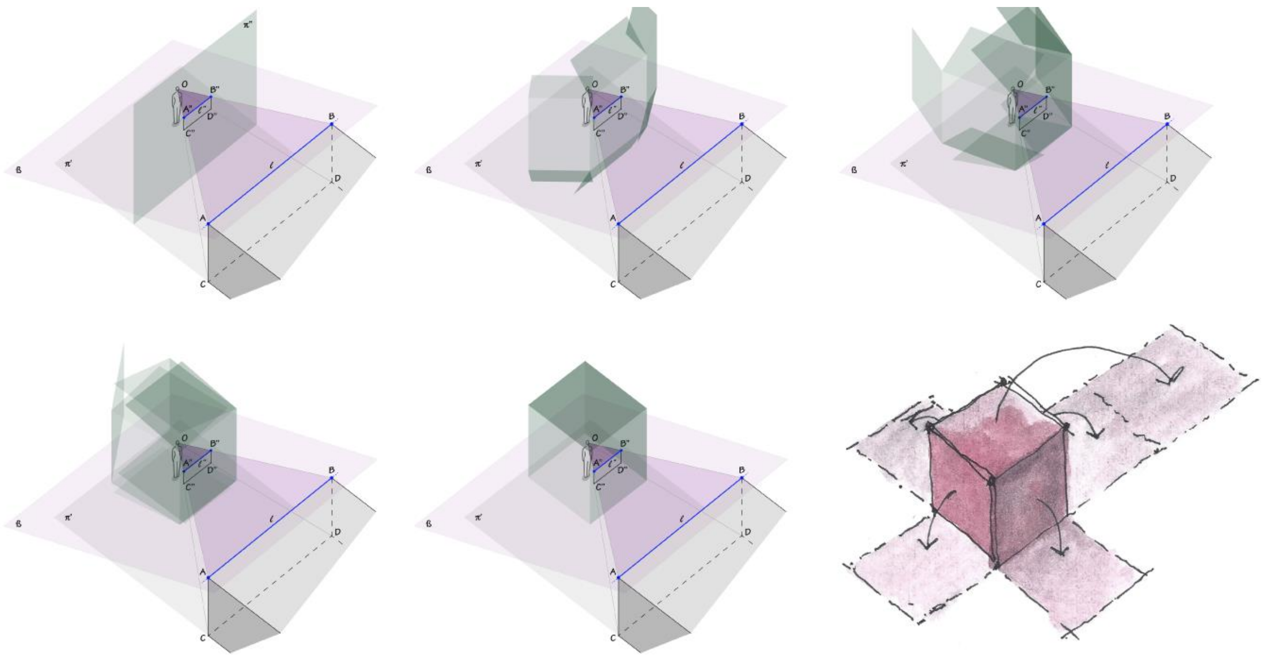


Figure 58 . Exchanging the projecting surface and fattening the cube

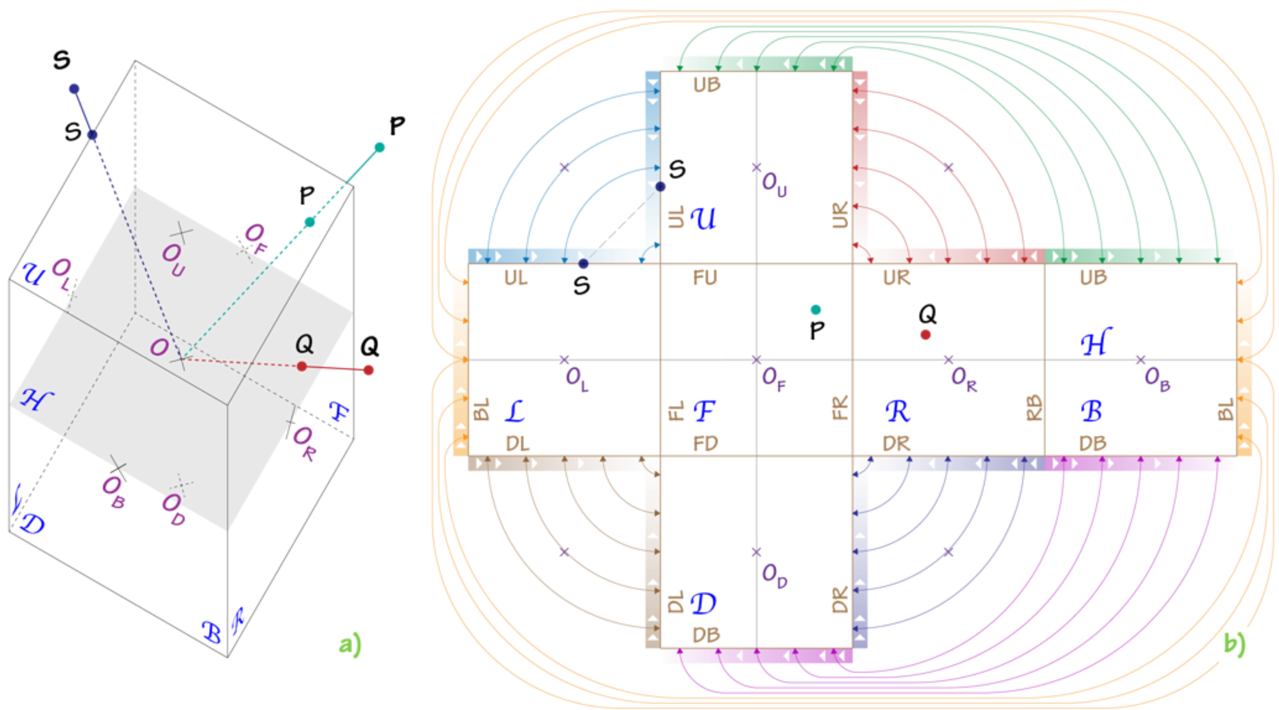


Figure 59 . a) Conical projection of three points P, Q and S onto the cube b) Flattening of the cube. Notice that S has a double projection as it lies on two segments that are identified as the double image of the same edge. Colours denote edge identification

Given a spatial point P, its conic projection is the intersection of ray $\overrightarrow{OP}$ with the cube's surface (Figure 59a). We obtain the perspective image of P by flattening the cube. This flattening consists in cutting seven edges and rotating the faces around the remaining edges in such a way as to bring all faces onto the same plane. The projection is defined up to the choice of the cuts. We specify one such flattening: choose two arbitrary adjacent faces, denoted by F (for *Front*) and R (for *Right*). Denote the other faces L (left), B (back), U (up), D (down), in the order implied by this choice of relative directions.

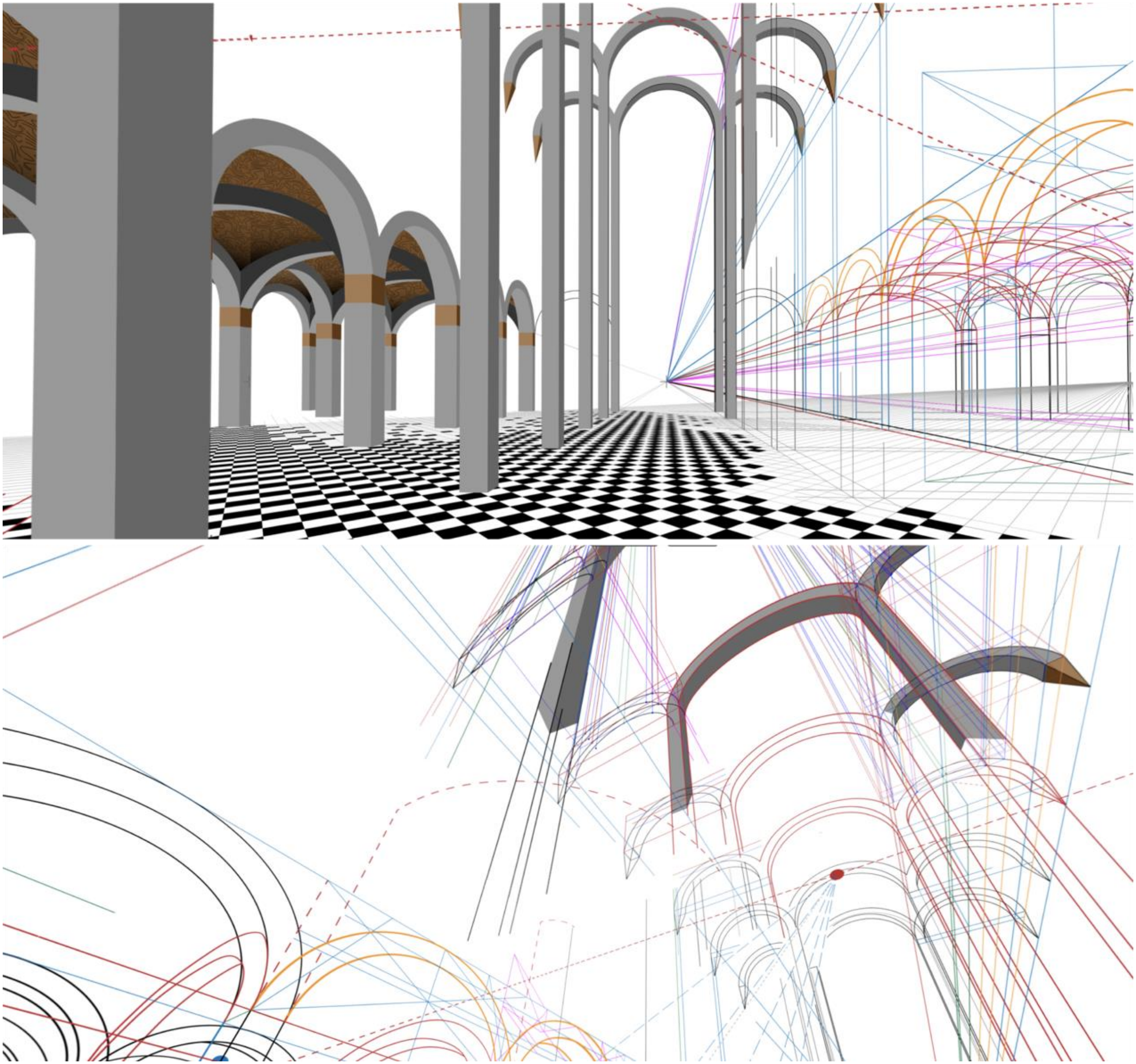


Figure 60 . Immersive visualization of Figure 61

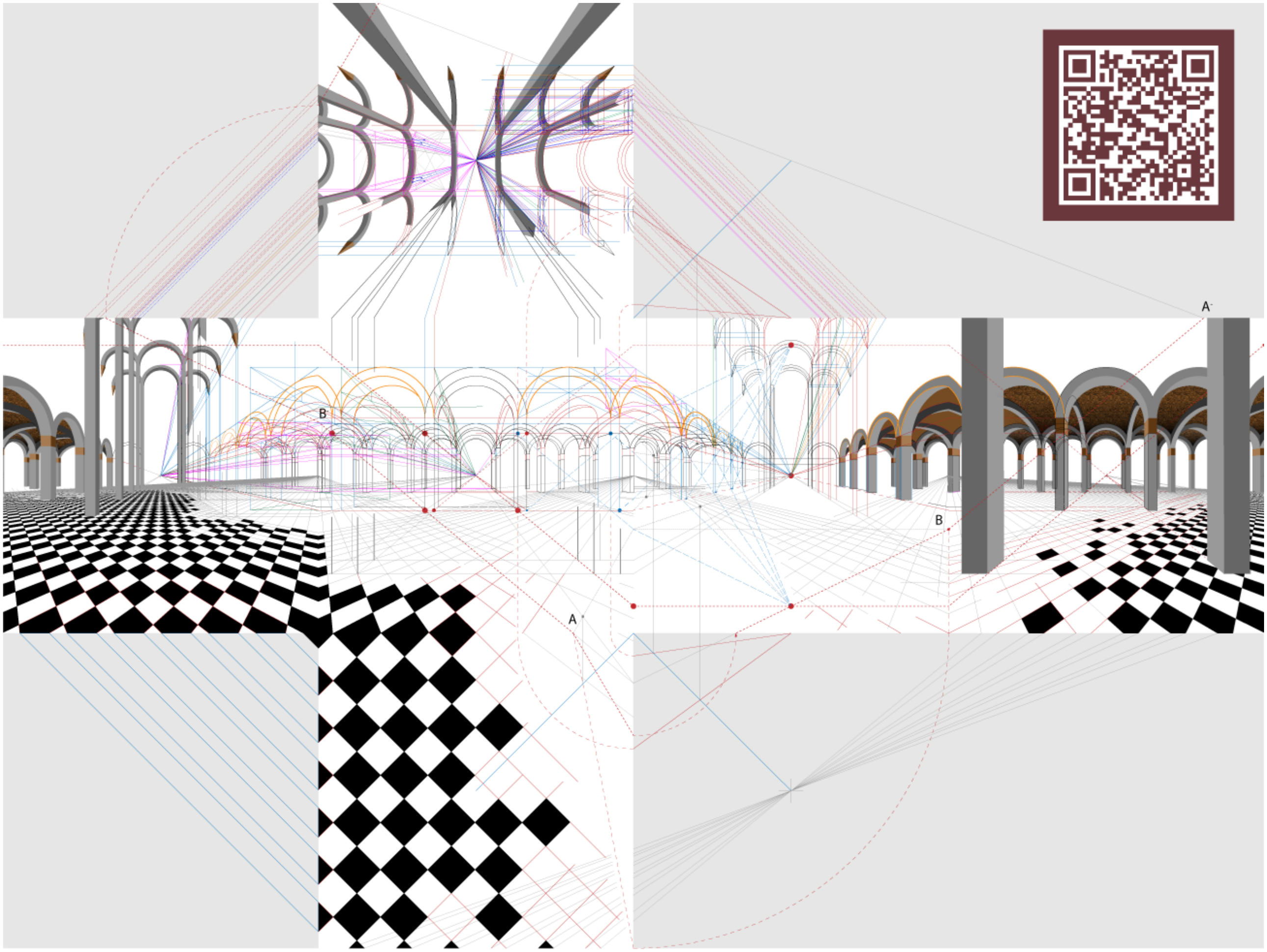


Figure 61 . Cubical perspective of an ideal architectural model

Name the edges by the faces they separate, so for instance FR is the edge between faces F and R. Then cutting edges $UL, UB, UR, DL, DB, DR, BL$ we obtain the flattening of Figure 59b. Edges that are cut in the flattening appear twice in the drawing, so for instance point S appears on the edge UL on both faces U and L.

This procedure defines a perspective that behaves like a classical perspective in each projected face of the cube, but on the whole obtains a full 360-degree view of the environment around O with the interesting property that **each line will have exactly two vanishing points**. Note that the conical projection onto the cube creates an anamorphosis when seen from O. That is, an observer at O, looking from inside the cube at the conical projection of a spatial scene painted on the cube's surface would have the impression of seeing the actual spatial scene.

This anamorphic effect can be reconstructed from a given cubical perspective by folding it back into a 3D cube. This is just what happens with VR visualization: the planar image is folded onto a virtual cube and the viewer interactively observes a flat anamorphosis (against the plane of the monitor) of the cubic anamorphosis specified by the perspective drawing. This allows us to go from an imaginary or observed flat drawing to an immersive environment (Figure 60).

IV.4 FIRST METHOD: CUBICAL PERSPECTIVE AS SIX UNLINK PERSPECTIVES

IV.4.1 Introduction

The goal of the method is to solve the fragmentation problem of a line *l* in the cube. In Figure 62, I recall a generic conical projection following a method that suggests finding the correspondent position of each point in the perspective (bottom part of the figure) by defining rays from C_1 to C_4 to the observer *O* and intersecting the projecting plane *DP*. In particular, C_3 and C_4 result in real dimension in *H* because they are in contact with *DP*.

It is possible to proceed at least in two ways to find the depth of elements that are not in contact with *DP* (such as C_1 and C_2): in a first procedure, I use C_{1DP} and C_{2DP}, which are the intersections of rays C_1O and C_2O with *DP*. We find C_1 and C_2 in the bottom construction corresponding intersections C_{1DP} and C_{2DP}. C_3 and C_4 have a direct correspondence.

Another procedure is to project the searched point with an angle of 45° towards the projecting plane, i.e., an overturning centred in the point's projection in the projecting plane. That way, it is possible to use known vanishing points: those corresponding to 45° lines. For example, in C_2, the diagonals intersect *DP* in C_{2DP45} and C_4. Then, we place the real height of the object h_c, in *H* in the extension of C_{2DP45} (or C_4). From there, C_{2DP45} is projected to the vanishing point I_1, (or to I_2 if we use C_4). Any of the two diagonals give the same result.

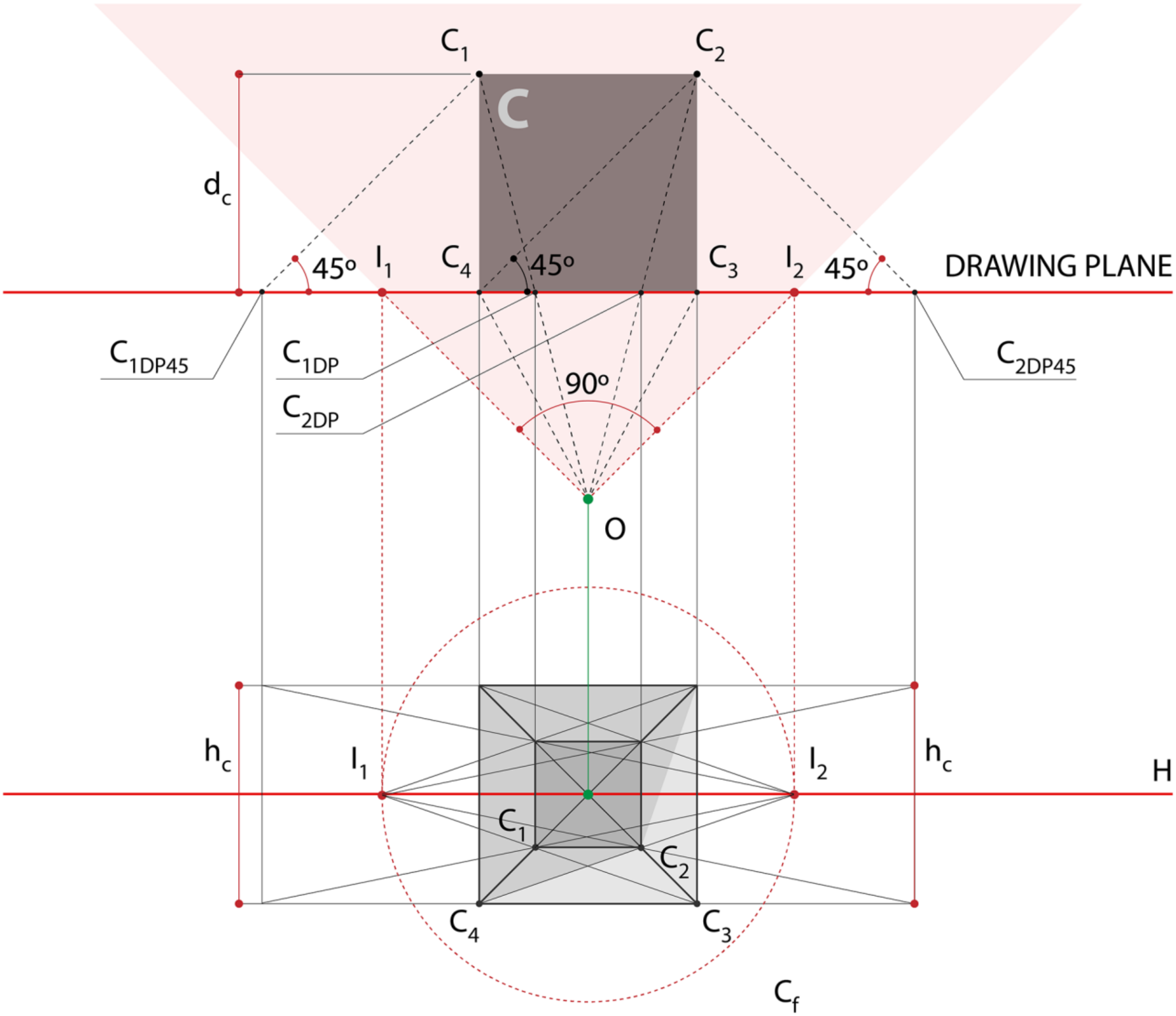


Figure 62 . Single conical projection

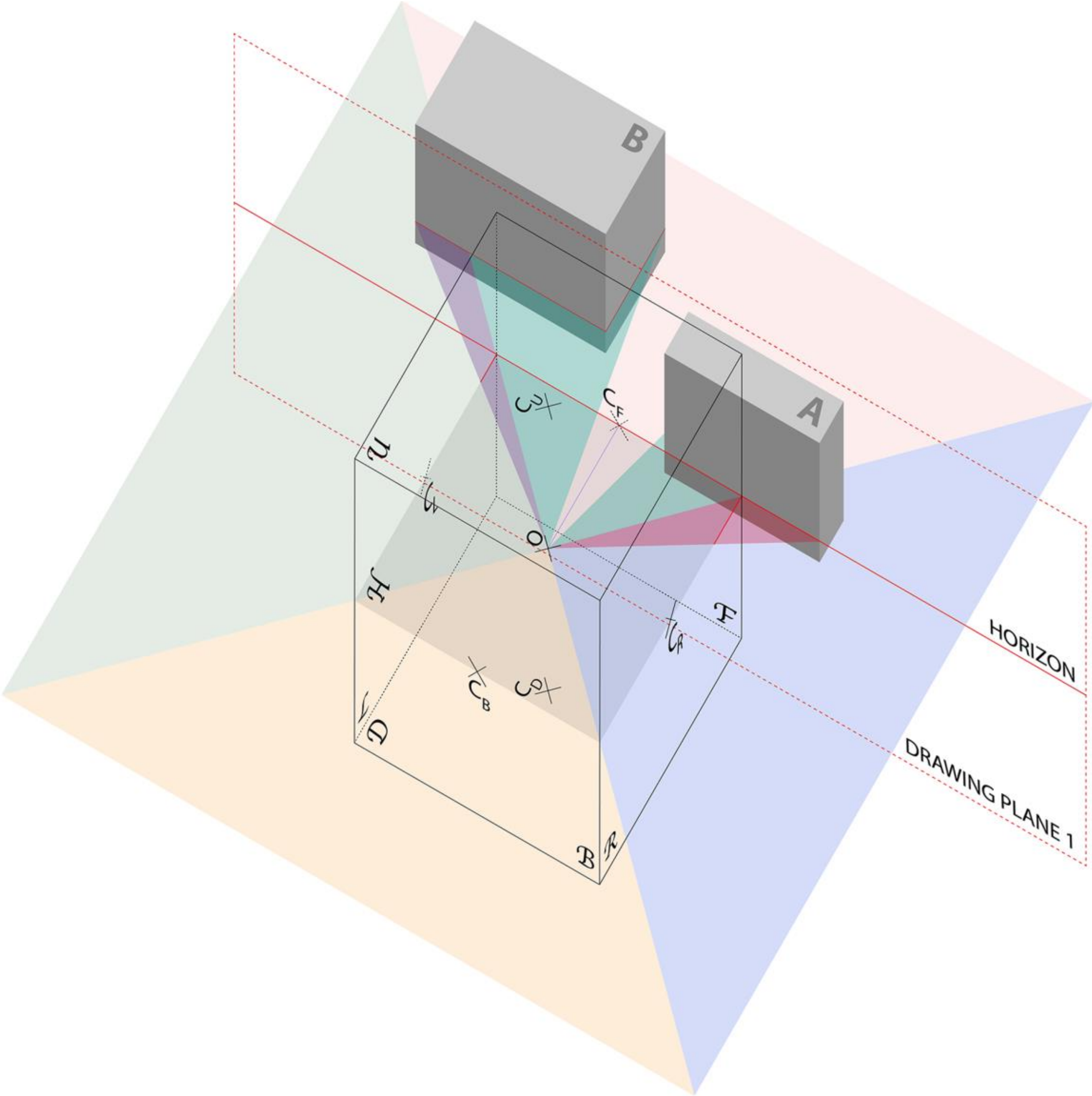


Figure 63 . General setup of the scene to be represented

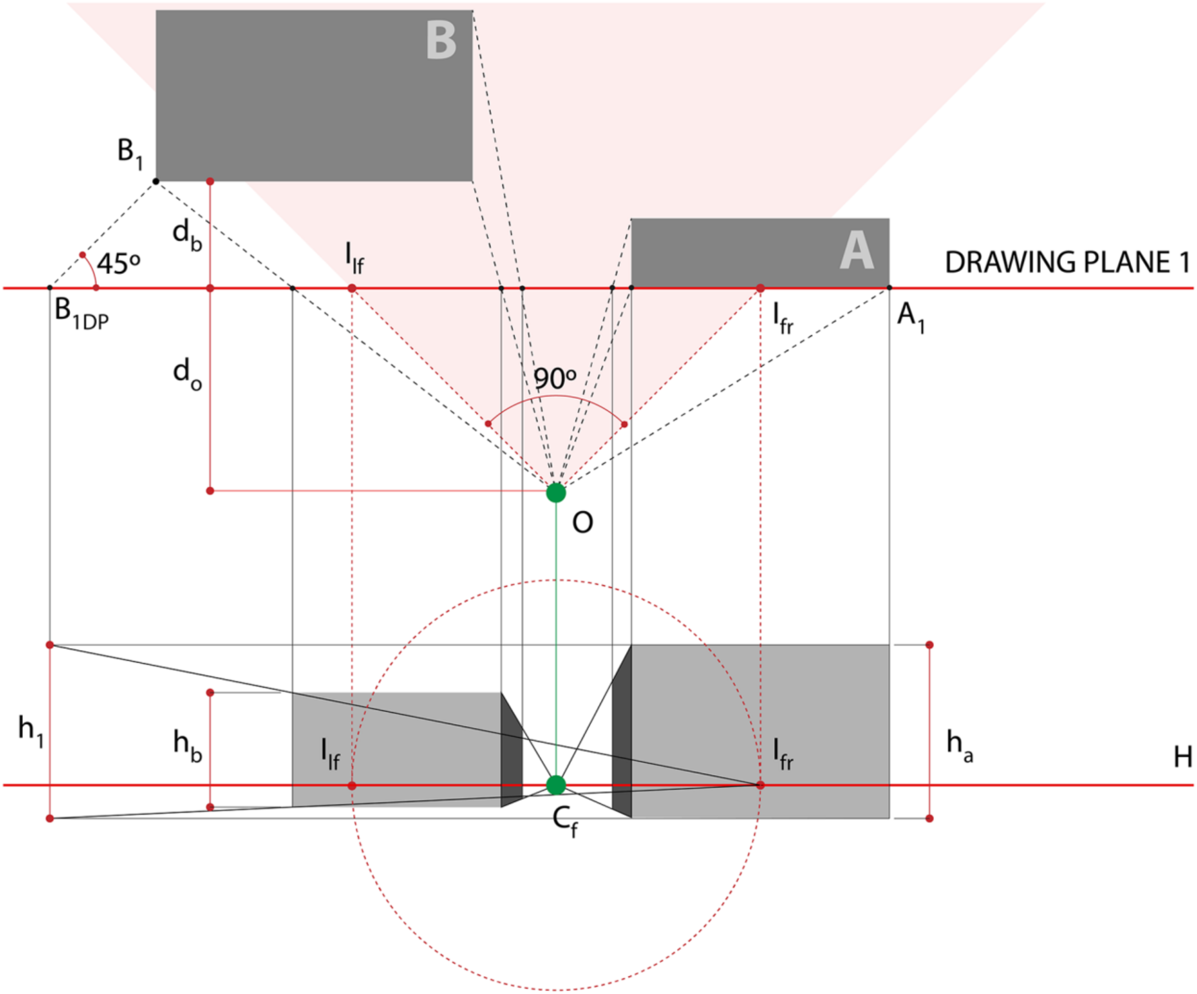


Figure 64 . Construction of the classical conical perspective of the scene

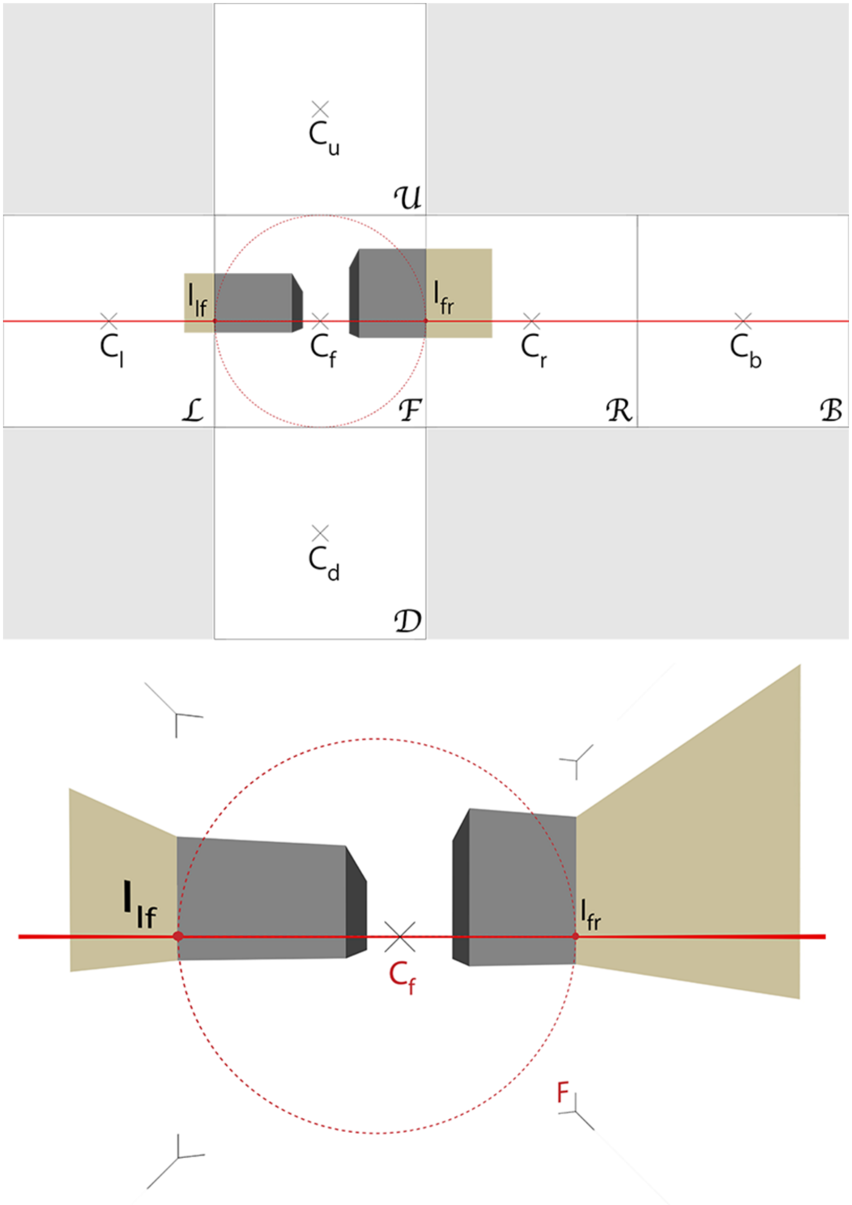


Figure 65 . The classical perspective placed in the cubical map (top) and its immersive visualization (bottom)

IV.4.2 Solving cubical perspective

Let us now introduce a generic scene composed for two buildings A and B (Figure 63) to be represented in cubical perspective. We start with a classical perspective using the central vanishing point C_f (Figure 64). Both buildings have the same height h_1. The facade of building *A* is parallel and coincides with *DP1*. Building *B* is also parallel to *DP1*, but it is located at a certain distance d_b. We locale the observer *O* at a distance *d* from *DP1* and perpendicular to it.

Thus, I construct a classical perspective: projecting every vertex of buildings *A* and *B* to *O*. Every intersection with *DP1* is corresponded with the bottom construction until the horizon line *H*. Since A_1 is in contact with *DP1*, it results that $h_1 = h_a$. Then h_a can be translated to *H* in real dimension and positioned directly accordingly to the observer's height (or *H*).

To find h_b instead, I use the previous method with the diagonals and project, for example, B_1 at 45° towards *DP1*. Following such a reasoning, one gets the complete scene with zones that exceed a 90° field of view (Figure 64). If we now insert this scene in the open cube map and then proceeded to the immersive navigation, the zones outside the 90° cone are incorrectly visualized (Figure 65).

Nevertheless, if the observer turns to its right, it is possible to reconstruct building *A* which is now the only one in its visual field (Figure 66). In this case we use projecting plane *DP2*, which is perpendicular to *DP1*, and the vanishing point *Cr*. We solve the part ahead *DP2* in the same way that at the beginning of this section, i.e., translating A_2 and A_3 with an angle of 45° to *DP2*. The real height of the building is in correspondence with point I_{fr}.

Using I_{fr} and projecting from C_r, we obtain the searched heights in correspondence with A_{2DP} and A_{3DP}. Placing this last content on the right face *R* of the cubical map, and centred on C_r, the immersive navigation results correct (Figure 67). Thus, the use of a projecting plane orthogonal to the first scenario gives as a result the correct anamorphosis.

IV.4.3 Drawing directly in the cubical map

I propose now a direct procedure to obtain the same complete scene, but independently of rotating the projecting planes. Indeed, we want to find the intersection points without the cumbersome need to rotate every 90°. To this end, I study the projections in the four planes of drawing *DP1* to *DP4* located around the observer in a holistic setup (Figure 68).

Previously, new projections appeared in the intersections with the projecting planes. For example, in the intersection of ray OA_1 with *DP2* there is now A_1'. This point is in front of *DP1*, so, to find its correspondence by translating it 45° towards the extension of *DP1*, which gives $A_1'_{DP}$. In perspective, $A_1'_{DP}$ effectively matches the intersection already built during the previous steps. Iterating the process, we effectively find the end of building *B* projecting B_1 in the same way (Figure 68). The new full construction verifies the correct composition of the anamorphosis in the immersive modality.

As last step, once a complete drawing has been done, digitalized and cut with the right proportions, it will be necessary to add it some metadata (see IV.7).

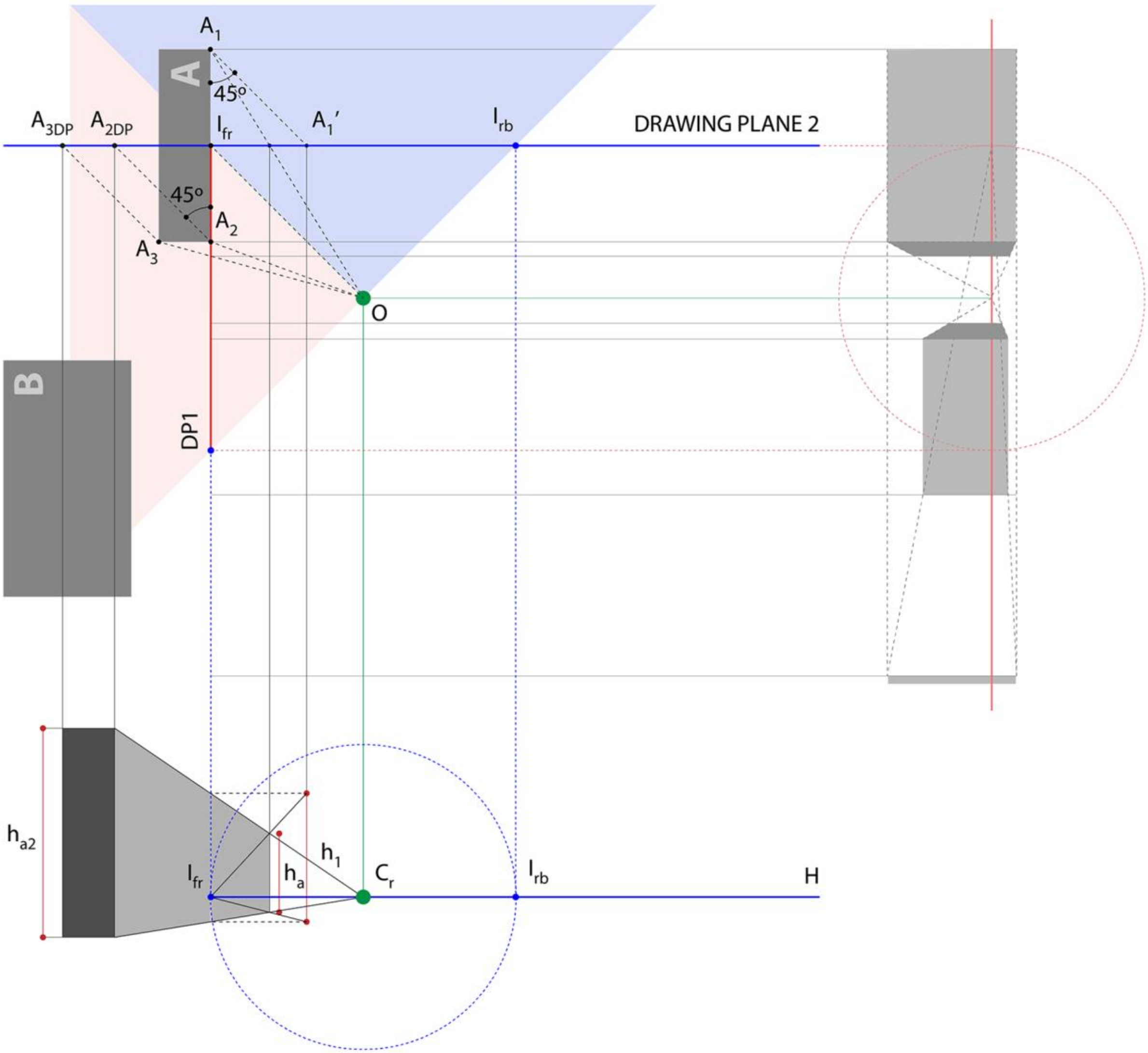


Figure 66 . Construction of the second perspective rotating the observer 90° to its right

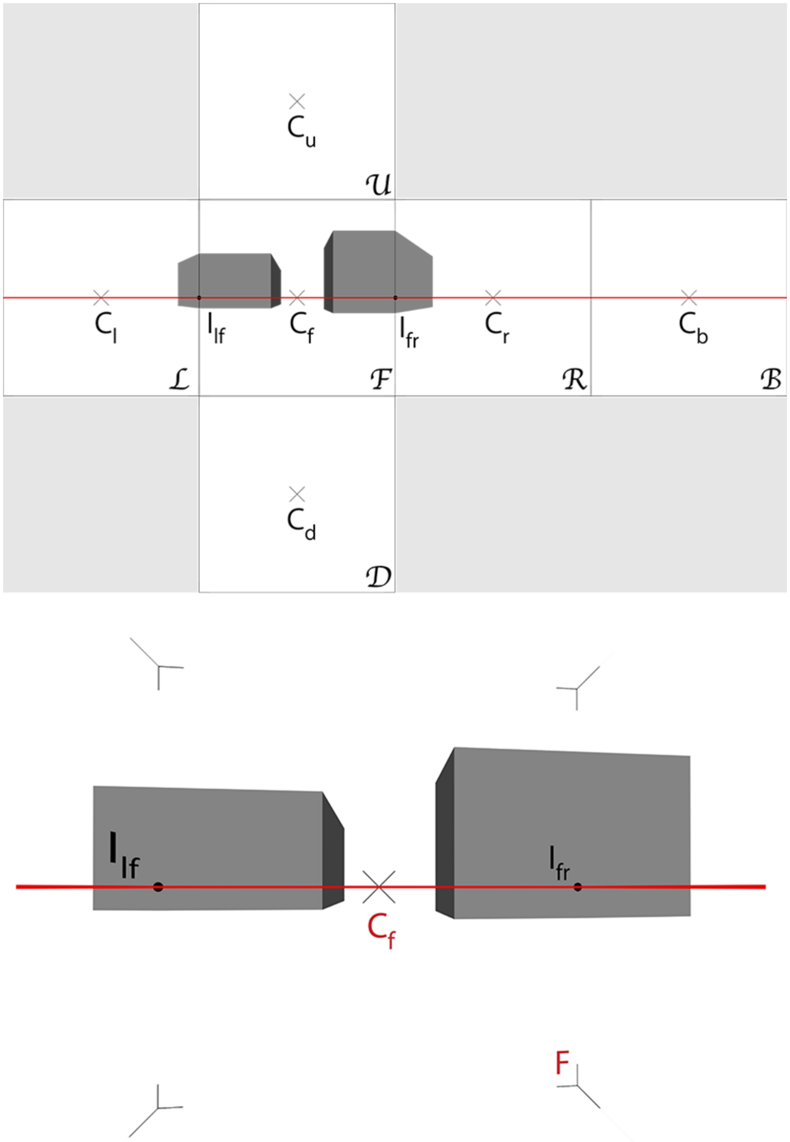


Figure 67 . Final construction placed into the cubical map and its immersive visualization

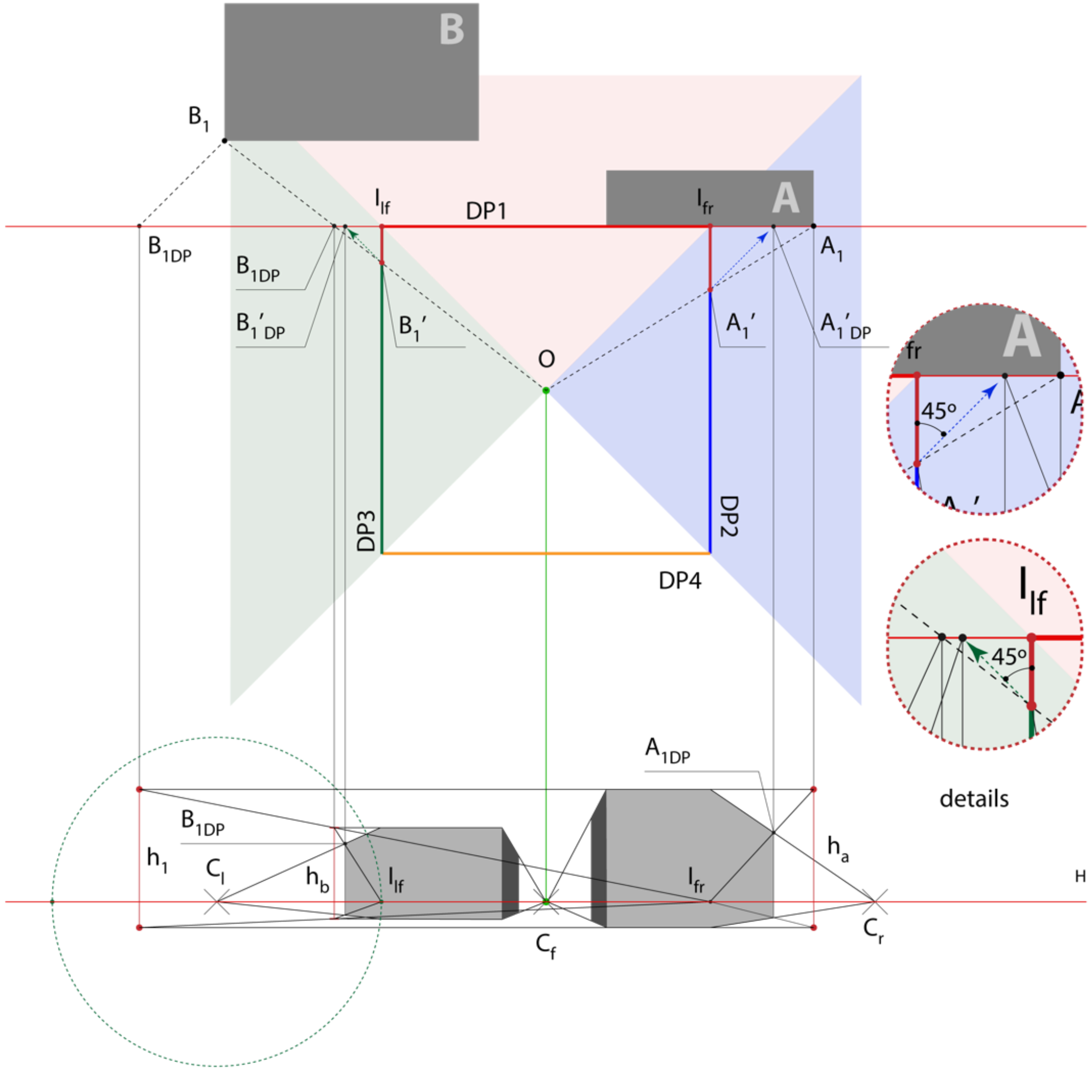


Figure 68 . Final composition of the scene solved with the use of conical projection

IV.4.4 Deductions

This first approach explores the possibility of solving cubical perspective as an adaptation of classical perspective's concepts and methods. However, it lacks many things, such as a complete classification of lines, and how to locate points from direct observation.

Nevertheless, the method opens the door for constructing an accurate cubical representation using floorplan and section. It is a first attempt of organising a different kind of representation that uses six different projecting planes at the same time. It also defines technically a hybrid immersive model made with cubical perspective, which up here was only verified for spherical perspectives or cubical drawings obtained from 3D models.

The method brings a possibility for understanding and manipulating the geometry of an architecture in immersive modality, that is, within the full field of vision around the observer. Thanks to this, architecture and objects can be studied in context, i.e., the object in the space and the single building in its urban context. At the same time, this final 3D environment gives a base for uploading interactive contents with the possibility of VR visualization.

Thanks to the use of anamorphosis and digital technology the setup of the drawing shows the possibility of covering the full visual sphere without needing any bulky support. Technology completes an open and "pocket" access through mobile devices and Internet.

I recall again the definitions of this method in a first algorithm that organises the described steps in an executive protocol (see Third Part VI.2.1).

The whole package of definitions of a hybrid model from cubical perspective drawings up here (anamorphosis, perspective, mathematics and computer sciences) is more than enough to open the path of an innovative system of representation.

IV.5 SECOND METHOD: CUBICAL PERSPECTIVE AS A SPECIAL CASE OF SPHERICAL PERSPECTIVE

IV.5.1 Towards a consistent and efficient method

The first approach that I follow to solve the fragmentation in cubical perspective, was in terms of *angles* (Figure 69). Afterwards, I developed the first method calculating the fragmentations using floorplan and section, but the problem of the angles was still unsolved.

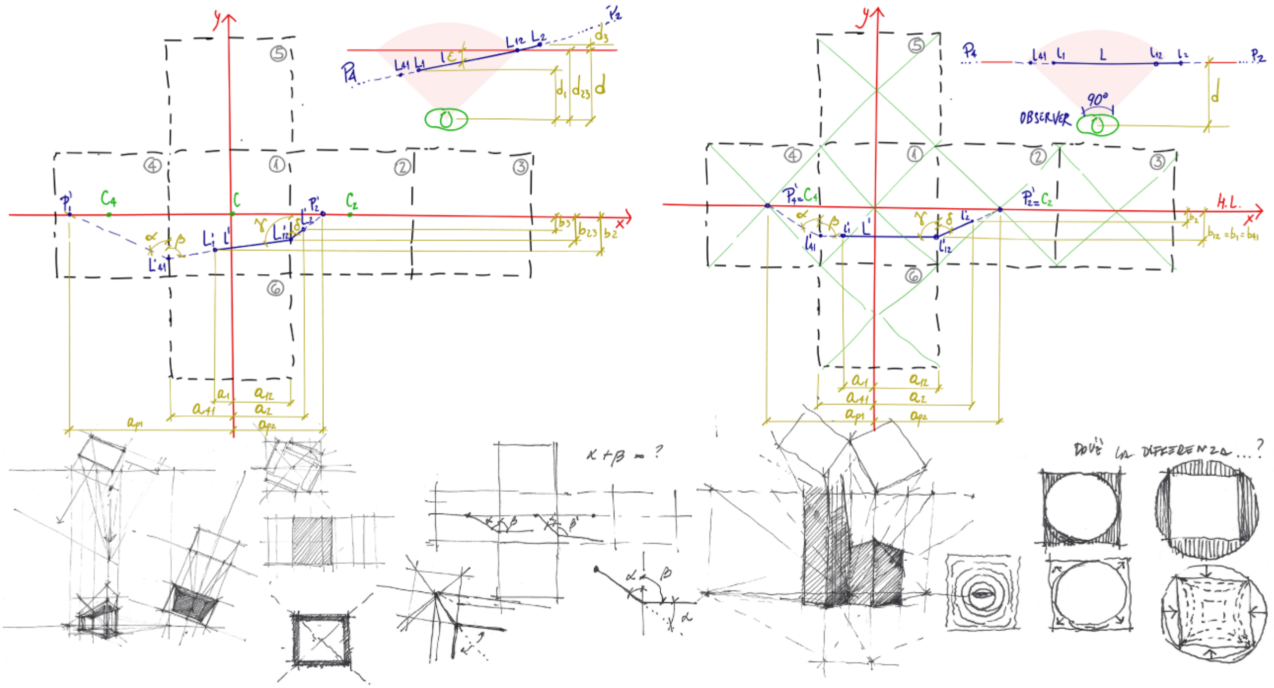


Figure 69 . First approach to cubical perspective fragmentation in terms of angles

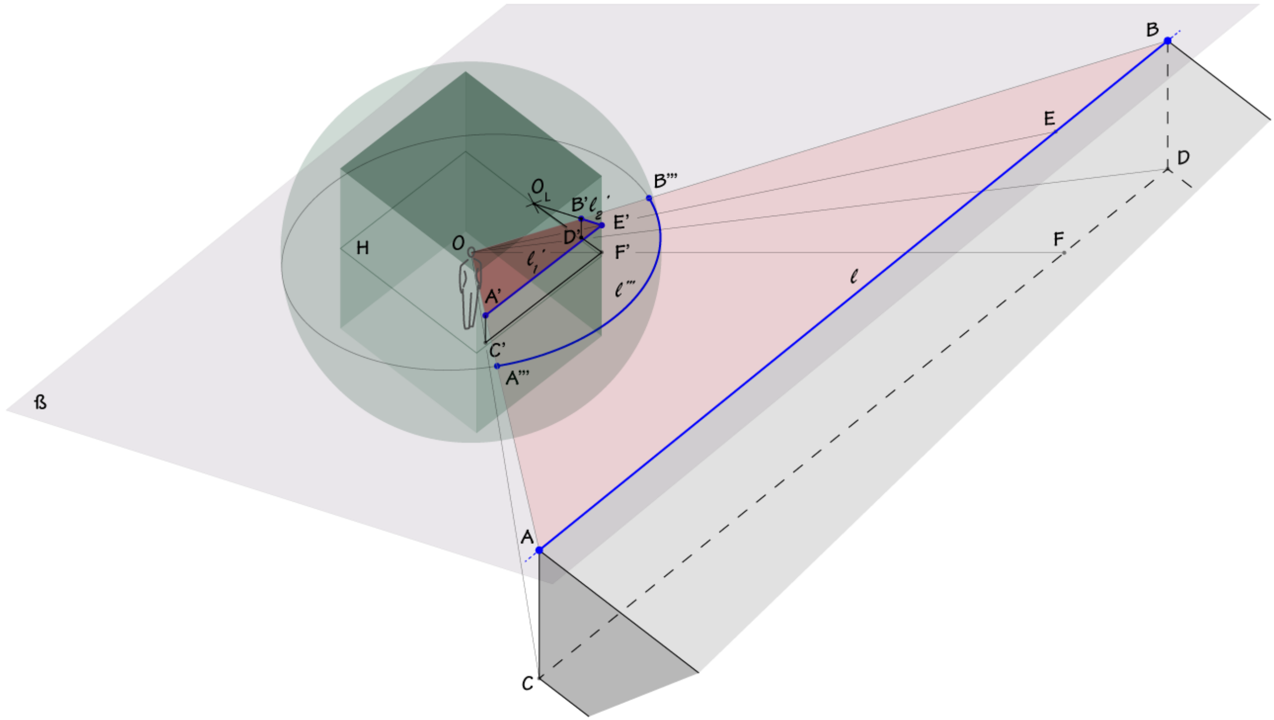


Figure 70 . The representation of a line is seen as a geodesic of the sphere

In this second method, the problem of the fragmentation follows a different interpretation: it regards a line as a subset of a geodesic and determine the full geodesic image from just two points. This avoids the consistency problem, is maximally economic, and solves the angle problem automatically.

IV.5.2 Immersive anamorphoses and spherical perspectives

In [9, p.149], a spherical perspective is defined as a central conical anamorphosis onto a sphere of centre O, followed by a flattening of the sphere that verifies certain continuity conditions. As explained in that work, the end result is a topological compactification of the spatial scene that preserves in the plane certain features of the spherical anamorphosis. We now recall some important properties of spherical anamorphosis:

A spatial line l determines a plane π through the centre O of the sphere. π defines, by intersection with the sphere, a *great circle*, or *geodesic* g. The anamorphic image of l is one half of g (a *meridian*[34]). That meridian's endpoints are the two vanishing points of l, hence any line has exactly two such vanishing points. These are found by translating l to O and intersecting it with the sphere; hence the two vanishing points are *antipodal* to each other, i.e., diametrically opposite on the sphere. Given a spatial object, its perspective is the plane drawing obtained from its anamorphic projection by flattening the sphere itself onto the plane.

Now consider that the cube is homeomorphic to the sphere (Figure 71) – the conic projection towards a centre O in common to a sphere and a cube defines a bijection between the two surfaces that is continuous both ways (a *homeomorphism*). So, the flattening of the cube is also a flattening of the sphere. Hence cubical perspective is a special case of a spherical perspective and it can be characterized as *cubical spherical perspective*.

Through this homeomorphism all concepts of spherical perspective, such as antipodal points or geodesics translate directly to the cubic case. In particular, two non-antipodal points on the cube's surface determine one single geodesic through them. This means that if we have two points P and Q like in Figure 72, on two faces of the cube, then there is a single correct way of connecting them that corresponds to a possible line segment between any two spatial points that project to P and Q. This will be part of a geodesic g. We also know that g must be made up of Euclidean line segments, as cubical perspective is a linear projection on each face.

[34] *Meridian* is understood in the terms defined at [9, p.149], that is, as "*one connected half of a great circle*" (i.e., not necessarily following the geographical convention of using North/South poles)

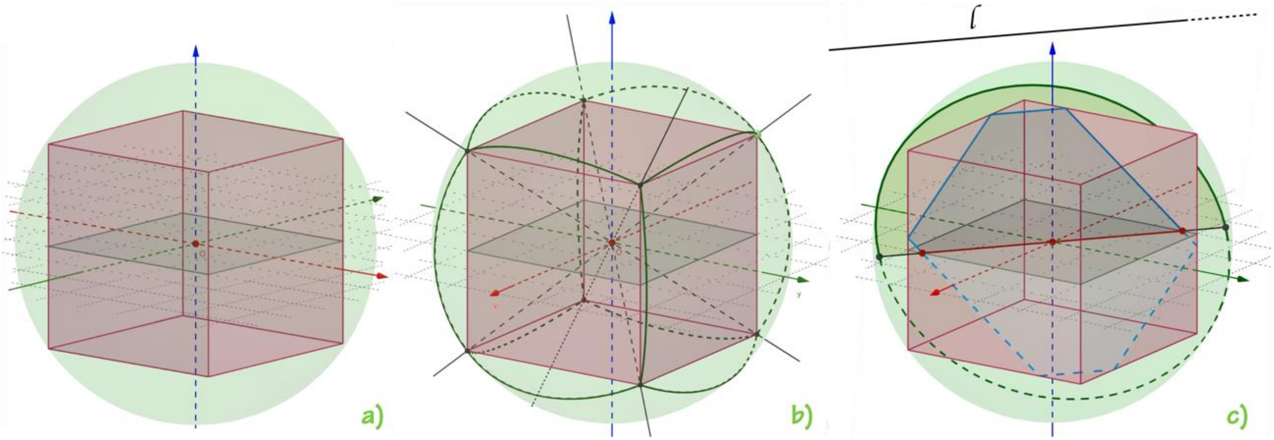


Figure 71 . a) A sphere and cube that are concentric are homeomorphic by conical projection. b) Image of the edges of the cube on the sphere as arcs of geodesic. c) Image of a spherical meridian (spatial line projection) on the cube

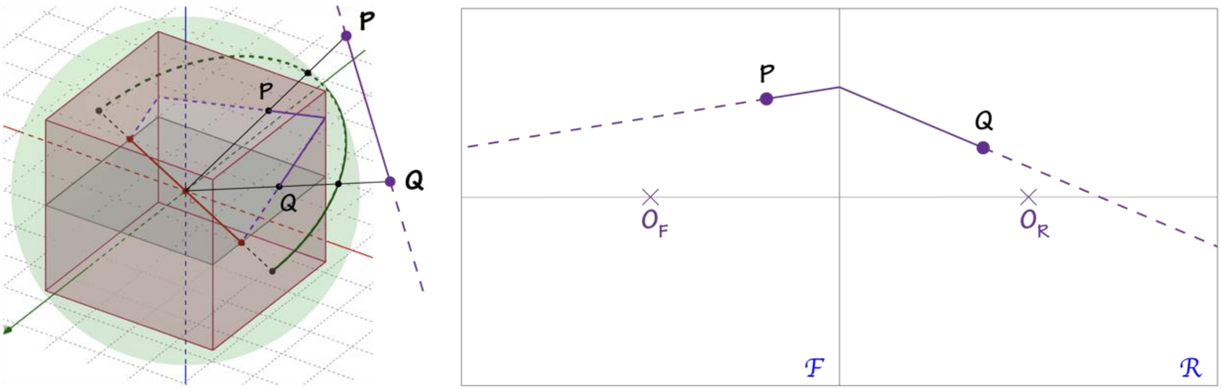


Figure 72 . A single geodesic image connects P and Q

In order to solve this perspective, we must show how to plot points from their angle measurements; plot antipodes; find vanishing points; classify and draw great circles. These are the common steps to the resolution of all spherical perspectives.

IV.5.3 Solving cubical perspective with descriptive geometry

We will see how to solve the cubical perspective through descriptive geometry constructions over the flattened cube. First some notation. We denote spatial points by bold font letters and both their conic projections onto the cube and their perspective projections onto the plane by the same letter in light font, unless context makes the distinction unclear. We call O_I to the centre of each face I of the flattened cube (for instance O_F for face F). This corresponds to the orthogonal projection of O onto each face. We say that faces F, R, B, L are horizontal faces and that U and D are vertical faces. This of course refers to relative bearings, not to absolute ones.

IV.5.3.1 Antipodes

Let P be a point on the cube. We call antipode of P to its diametrically opposite point on the cube and denote it by P^-.

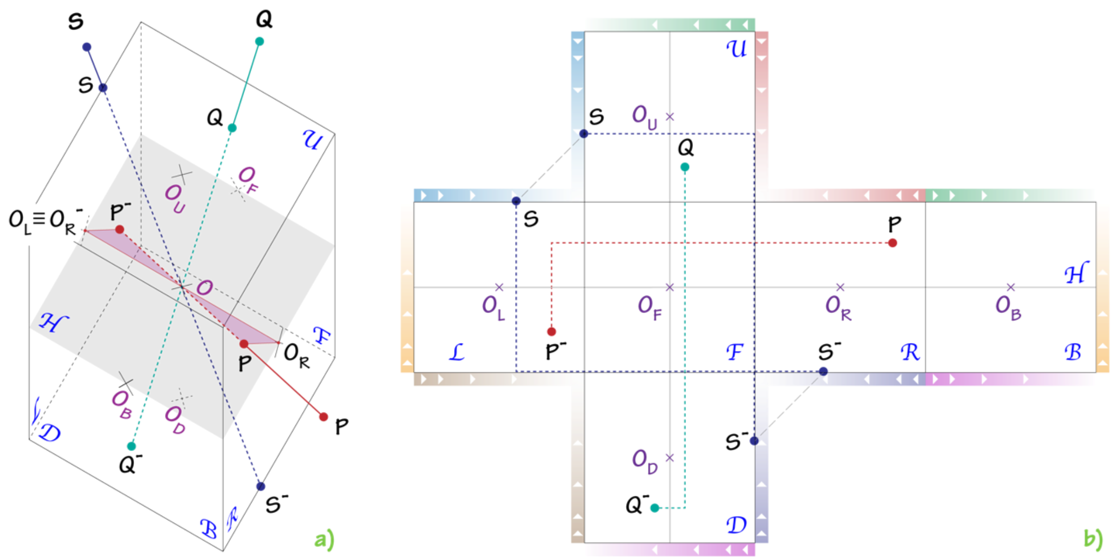


Figure 73 . Antipodes of three given points P, Q and S

Construction 1 (antipode of a given point P): If P lies on face I then P^- lies on the opposite face, and by the opposite angles' theorem, $\angle POO_I = \angle P^-OO_I^-$ (Figure 73a). P^- can be obtained by a sequence of two transformations on the perspective view (Figure 73b): first rotate P by 180° around the z axis, then reflect it across the plane of the horizon H.

There are two cases: If P is on a horizontal face, then the rotation becomes a translation of two cube side lengths to the right (resp. left), if P is on faces L or F (resp. R or B). If P is on face U (resp. D), then translate P down (resp. U) by two side lengths and reflect across the vertical axis through O_F.

IV.5.3.2 Construction of geodesics with two given points

Next, I expose how to obtain the images of spherical geodesics (great circles) on the cube surface and on the flat cubical perspective.

Two non-antipodal points P and Q on the cube's surface determine a plane $\pi = POQ$ through the centre of the cube and of its concentrical sphere, hence a spherical geodesic. The image of this plane on the cube is a set of connected line segments over the cube surface. We know this since on each face we have the intersection of two planes, hence a line segment. We know these must connect because this is a topological property, and the cube is homeomorphic to the sphere. We also know this image must be either 4-sided or 6-sided since for each of its points on one face, there is an antipodal point on the opposite face, hence the number of faces is even, hence it is 4 or 6, since just 2 segments wouldn't connect.

The next paragraphs develop the properties of the geodesic generated by two arbitrary points P and Q, according to the relative position of these points, and how to obtain its projection through descriptive geometry. There are several cases to consider, and it is useful to start by isolating the properties of geodesics according to their number of sides.

IV.5.3.3 4-Sided Geodesics

Suppose that a geodesic g contains a segment l on a face I such that l intersects two parallel edges of I at points P and Q respectively. Then P^- and Q^- are points of g on the respective antipodal edges of the face opposite to I. Segments P^-Q and PQ^- belong to g and are located on faces adjacent to I and opposite to each other. Joining their endpoints, we get a 4-sided closed loop, PQP^-Q^- (the full image of g). We call such loops 4-cycles (Figure 74).

When a 4-cycle only touches the horizontal faces, we say it is *panoramic*. We say that a geodesic g is *grid-like* if projects on a face I as a segment l parallel to one of the edges e of that face. Then l intersects the two edges of I orthogonal to e in two points P and Q, hence P^-Q^- is the projection of g on the face opposite to I, and $g = PQP^-Q^-$ is a 4-cycle. Also, by symmetry, QP^- and Q^-P pass through the centres of their respective faces, and, if l coincides with e, they are the diagonals of these faces (Figure 74).

Note that if a geodesic crosses an edge at two points, then its plane contains the line that joins them, hence contains the whole edge, hence is *grid-like*. Then a *non-grid-like* geodesic only crosses an edge at one point at most. Intuitively, *grid-like* geodesics are those generated by "*horizontal*" and "*vertical*" lines.

Suppose a geodesic g contains a segment l that cuts adjacent edges of a face I at points P and Q. Then let l_O be the line through O parallel to l. l_O intersects a face J adjacent to I at a point M that lies on the plane through O parallel to I. Either P or Q share a face with M. Suppose without loss of generality that it is Q. Then there is a point N on an edge adjacent to that of Q such that the image of g on J is QN. Then joining segments PQ, QN, NP^- , P^-Q^-, Q^-N^-, N^-P we get a the 6-cycle $g = PQNP^-Q^-N^-$ (Figure 75).

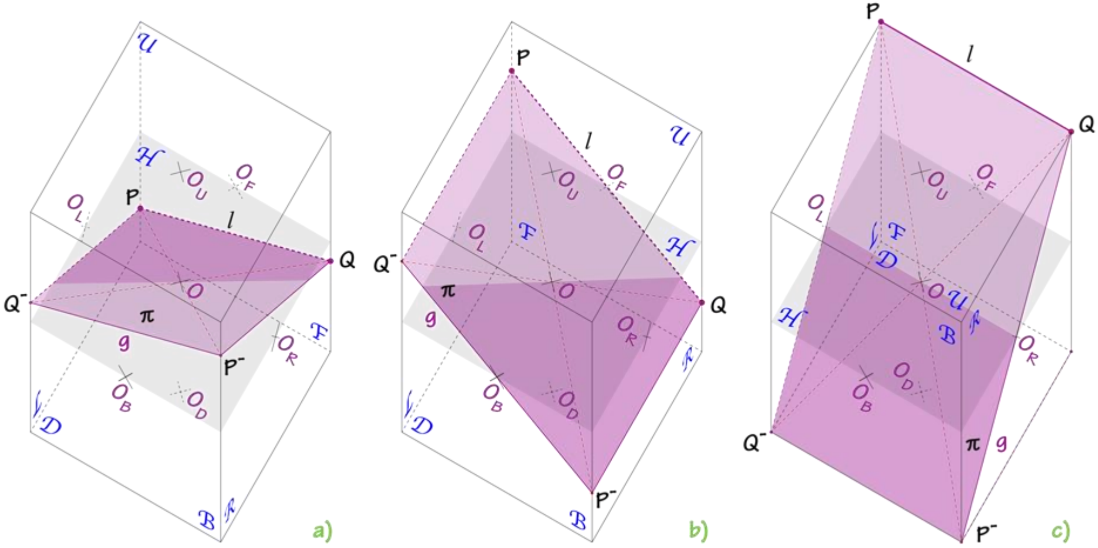


Figure 74 . Examples of 4-cycles geodesics. Cases (b) and (c) are grid-like

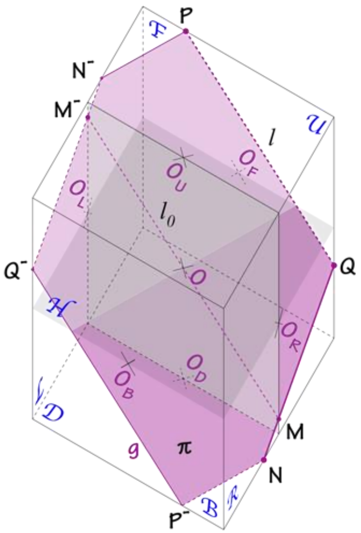


Figure 75 - 6-cycle geodesic

IV.5.3.4 6-sided geodesics

Given the perspective images of two points A and B which are not antipodal to each other, there is a single geodesic g through A and B. We will now use the classification above as a guide to draw the perspective image of g using descriptive geometry.

Case 1: Suppose A and B are on the same face I. Then line AB cuts the border of the face at two points P and Q. We must consider several sub-cases:

Case 1.1: A and B are such that P and Q are on opposite edges. We have described above the construction, from P and Q, that results in a 4-sided cycle. We now show its descriptive geometry implementation according to the faces and edges involved.

Case 1.1.1: P and Q are on vertical edges of one of the horizontal faces. Then P^- and Q^-, found by Construction 1, are also on two further distinct vertical edges of this same set of faces. Further, due to edge identification, one of the antipodes (assume it is Q^-) will appear repeated in the drawing, on a further distinct vertical edge. Hence there are points on five vertical edges, that can be joined to obtain a 4-cycle geodesic of the panoramic type. Figure 76 illustrates this case, assuming A and B on face F without loss of generality.

Case 1.1.2: P and Q are on horizontal edges. If P and Q are on the horizontal edges of face F or B (Figure 77) then P^- and Q^-, are on the horizontal edges of B or F (respectively). Then edge identification finds P^- and Q^- again on faces U and D. This gives a 4-cycle that crosses only horizontal edges of F, U, B, D. If P and Q are on one of the faces L or R then P^- and Q^- will be on the other. Without loss of generality, we can assume that P is on LU (resp. RU) and therefore Q is on LD (resp. RD). Then by edge identification, P^- is on RD (resp. LD) and Q^- is on RU (resp. UL). Together, these points determine 4 segments on L, U, R, D that define a 4-cycle. All the segments are disconnected on the plane (Figure 78b).

Case 1.2: A and B are such that P and Q are on adjacent edges. We have seen above that this is a 6-cycle constructed with the help of the auxiliary points M and N. We now construct these in the plane projection. To settle ideas, suppose that A and B are on face F and that P and Q are respectively on LF and FU, as in Figure 79. We obtain a further segment P^-Q^- on face B by taking antipodes and two further points, one on U and another on D by edge identification. We have seen above that we obtain two further points M and M^- in the geodesic by taking l_O, a parallel to AB through O, and intersecting it with the cube. Since AB is on F, that intersection must lie at the plane parallel to F through O, hence its plane projections must lie at the vertical lines through the centre of faces L and R, or at the horizontal lines through the centres of faces U and D. Then to obtain M, we take a parallel to AB through O_F. It must touch either the two vertical or the two horizontal edges of F. For concreteness suppose it intersects the verticals. Pass a horizontal line through the intersection on FR and intersect it with the vertical through O_R to obtain M. Draw a line P^-M and intersect it with UR to find point N. Taking an antipode, we find N^- on DL. We now have a segment in each face and a complete 6-cycle. The choices we made do not lead to loss of generality, as we can obtain all other cases by reflection through the vertical or horizontal line through the centre of O, or by cyclical translation of the face where A and B lie (Figures Figure 79 to Figure 81).

Case 2: Suppose A and B are points on faces adjacent to each other. In this situation we can have either a 4-cycle or a 6-cycle, depending on the relative positions of the given points. We need an auxiliary point to determine the geodesic through A and B. Let e be the common edge of faces F_A and F_B where A and B are located (Figure 82). Let $\pi = AOB$ be the plane of the geodesic determined by these points. Then segment AB is in π. Let δ_e be the plane through O and e. Then AB intersects δ_e at a point C. Since C is in AB, hence in π, then the ray $\overrightarrow{OC}$ is in π. Let l_e be the line that contains edge e. Ray $\overrightarrow{OC}$ intersects l_e at some point S, also in π. Then lines AS and BS will determine the images of the plane π in the faces F_A and F_B.

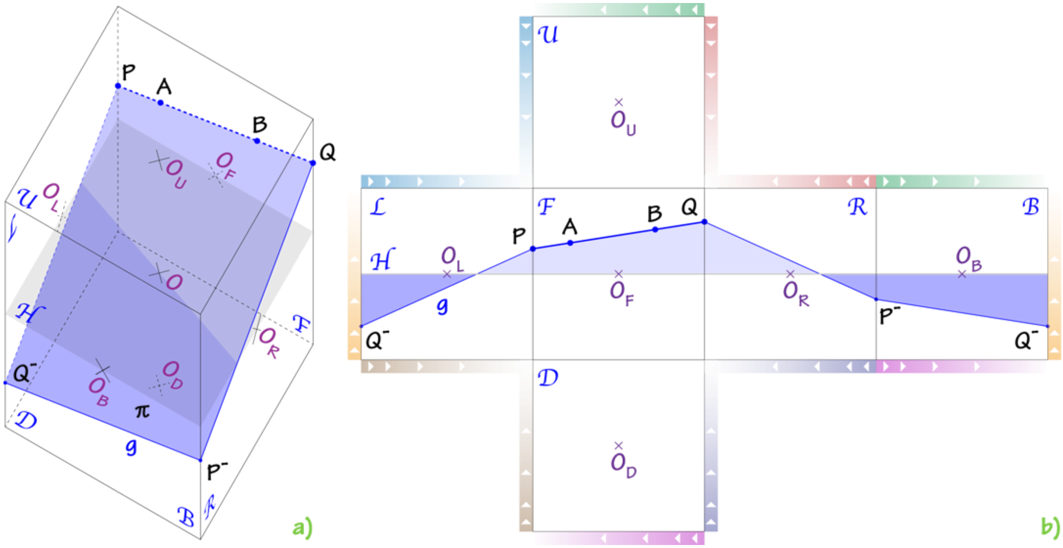


Figure 76 . P and Q are on vertical edges of one of the horizontal faces

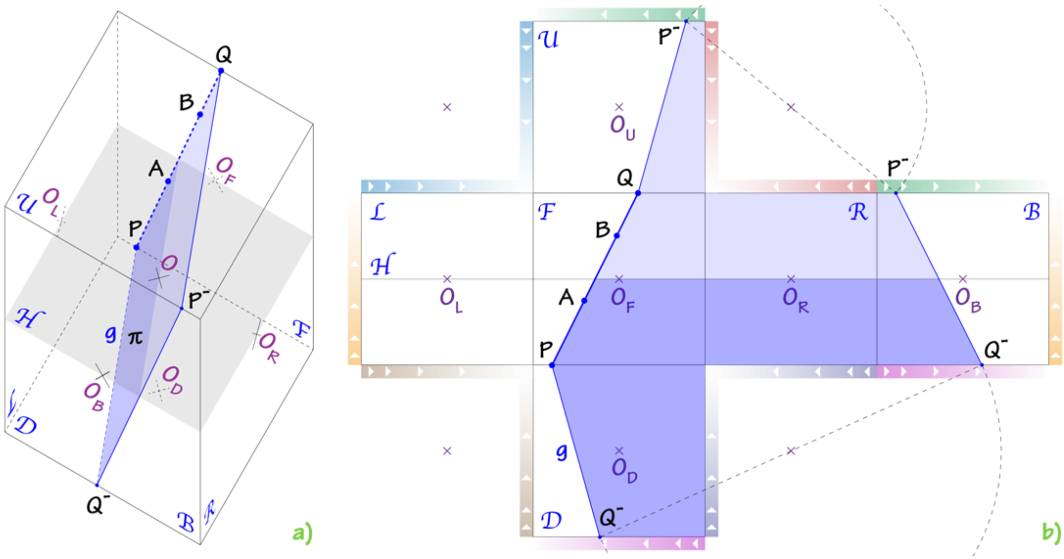


Figure 77 . P and Q are on horizontal edges of face F or B

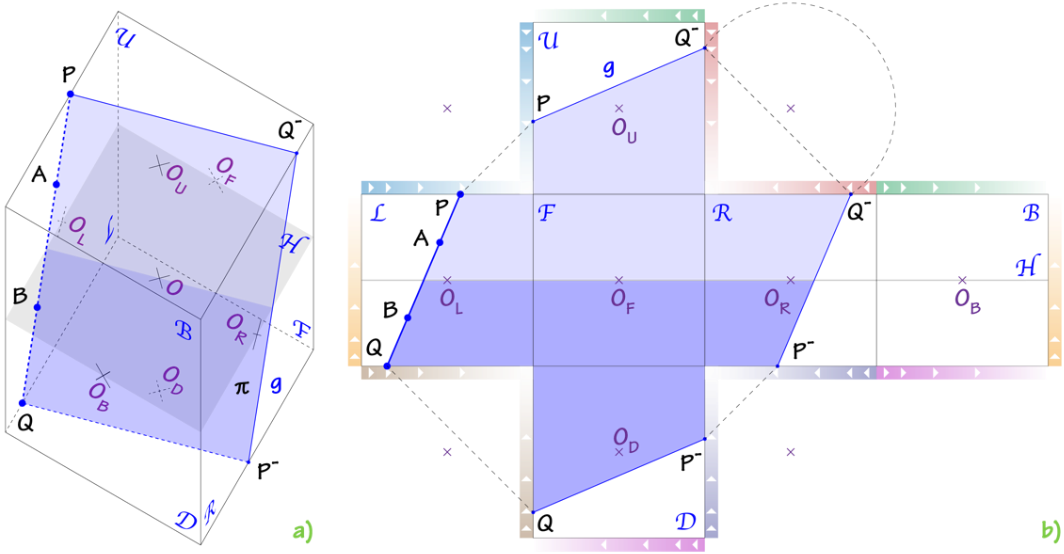


Figure 78 . P and Q are on horizontal edges of face L or R

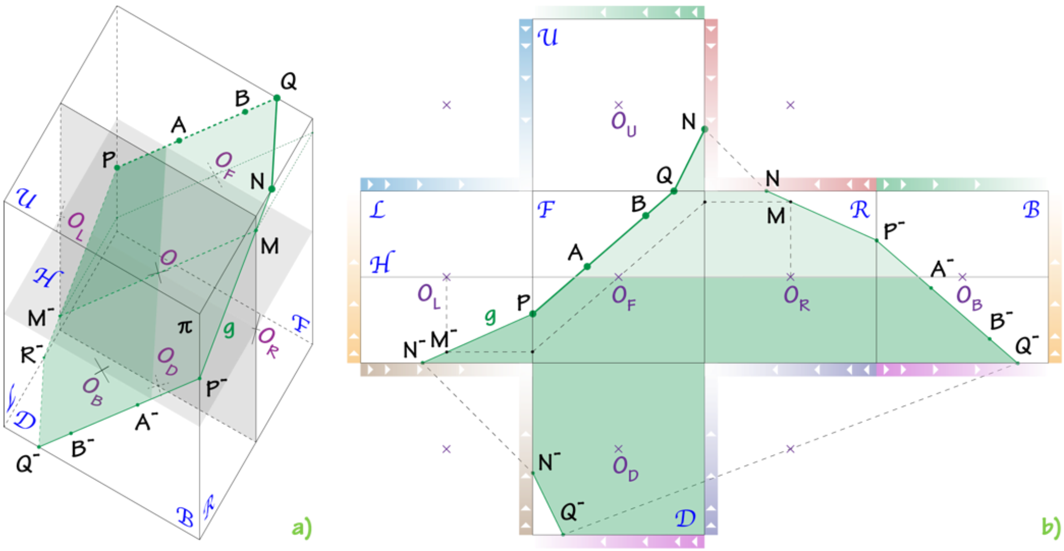


Figure 79 . A and B are such that P and Q are on adjacent edges, and M is on L or R

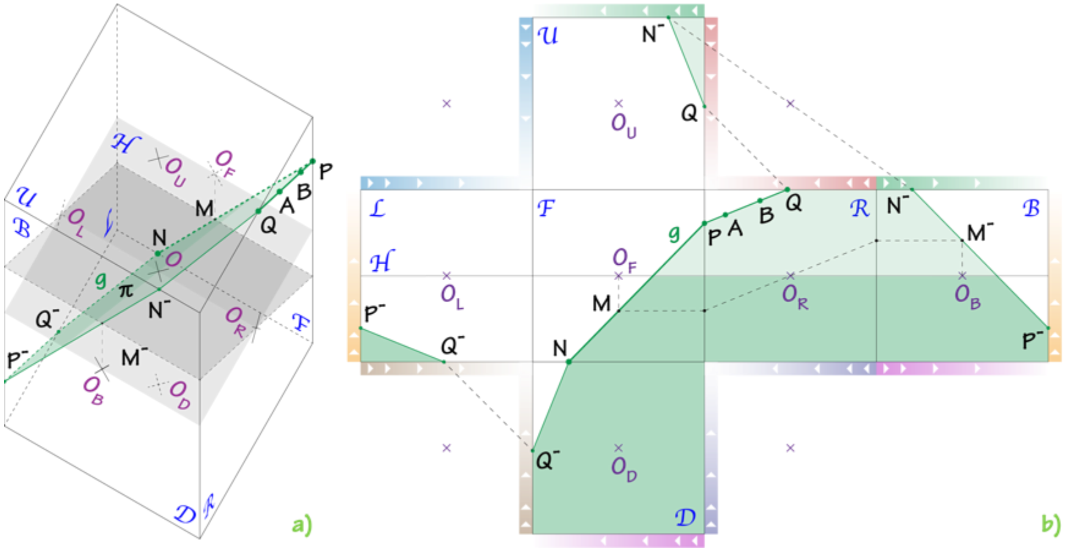


Figure 80 . A and B are such that P and Q are on adjacent edges, and M is on F or B

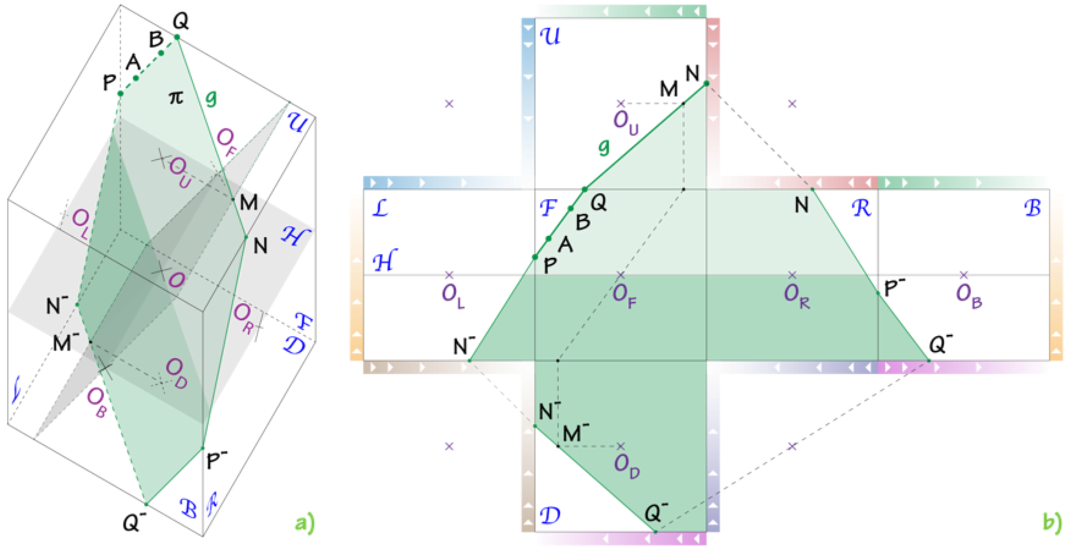


Figure 81 . A and B are such that P and Q are on adjacent edges, and M is on U or D

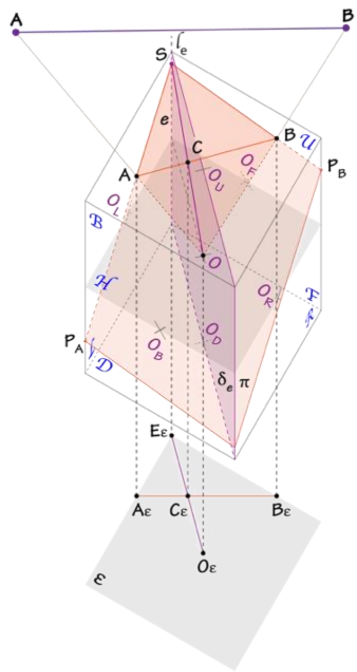


Figure 82 . A and B are points on faces adjacent to each other

To construct the auxiliary point S through a descriptive geometry diagram we take edge e as a folding line so as to draw F_A and F_B on the same plane. On the same drawing we consider a top view of the two faces, i.e., an orthogonal projection over a plane ε perpendicular to e. On ε, e projects as point E_ε and faces F_A and F_B form two adjacent sides of a square (Figure 83). We draw ε so that the image of F_A on it coincides with the bottom edge of F_A. The projection O_ε of O on ε is at the centre of the square defined by F_A and F_B, δ_e is a diagonal through O_ε and E_ε, with A_ε and B_ε on opposite sides of it. We find C_ε by intersecting with δ_e with $A_\varepsilon B_\varepsilon$. Let AB_e be the orthogonal projection of AB onto F_A. Then C is the intersection of the vertical through C_ε with AB_e, and S is the intersection of $O_A C$ with e. Joining A (resp. B) to S we find the projection of the geodesic of π on face F_A (resp. F_B).

This construction can be easily drawn on top of the flattened cube, thus dispensing with awkward auxiliary drawings. For instance, if the faces are F and R, the top view can be drawn on top of face D (Figure 85). S may or may not be on e (Figures Figure 84 to Figure 86). This, as well as the type of the segments obtained, determines the type of geodesic projection. Below we consider these several cases. Note that we insist that all constructions must be executed within the confines of the paper, that is, of the rectangle that contains the flattened cube. So, in Case 2.2, when S is outside e, we use an alternative construction to remain within the intended bounds (Figure 86b). This is a general philosophical principle in spherical perspective: just like the drawing itself, its construction should be *compact* [7], [200].

Case 2.1: S is in e. Then there is a point P_A on an edge of F_A such that $AS \cap F_A = \overline{P_A S}$ and a point P_B on an edge of F_B such that $BS \cap F_B = \overline{P_B S}$. We have two possibilities:

If P_A is on the edge of F_A parallel to e, then SP_A crosses parallel edges of the same face, hence we have reduced the problem to a previously solved case, and the projection is the 4-cycle $SP_BS^-P_A$. Note that this implies $P_B \equiv P_A^-$ (Figure 84). If P_A is on an edge adjacent to e, then we have reduced the problem to a previously solved case of a 6-cycle, with $P = P_A$ and $Q = S$. This implies that P_B must also be on an edge of F_B adjacent to e, and in fact $P_B = N$. So the 6-cycle is $P_ASP_BP_A^-S^-P_B^-$ (Figure 85).

Case 2.2: S is not in e. Then there are points P_A and Q_A in F_A such that $AS \cap F_A = P_AQ_A$ and points P_B and Q_B in F_B such that $BS \cap F_B = P_BQ_B$. P_A and Q_A must lie in two edges adjacent to each other, because if these edges were parallel to each other, they would both be perpendicular to e defining a 4-cycle that would not touch the face where B is, but this is absurd since B belongs to g. Hence P_A and Q_A are on edges adjacent to each other, and the geodesic is the 6-cycle with $P = P_A$ and $Q = Q_A$, $N = Q_B$, that is, it equals $P_AQ_AQ_BP_A^-Q_A^-Q_B^-$ (Figure 86).

Case 3: Points A and B are on opposite faces. Then A and B^- are on the same face, which reduces the problem to a previously solved case.

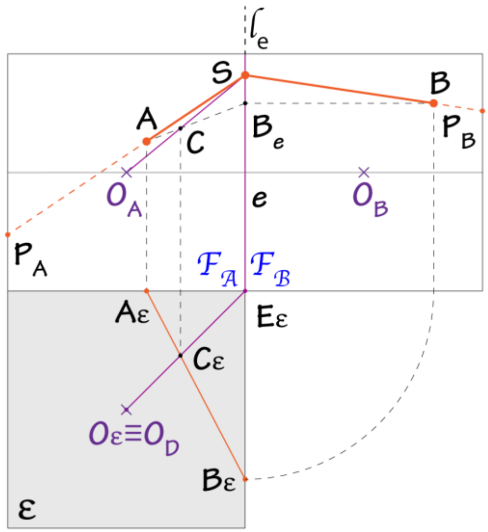


Figure 83 . Construction of the auxiliary point S

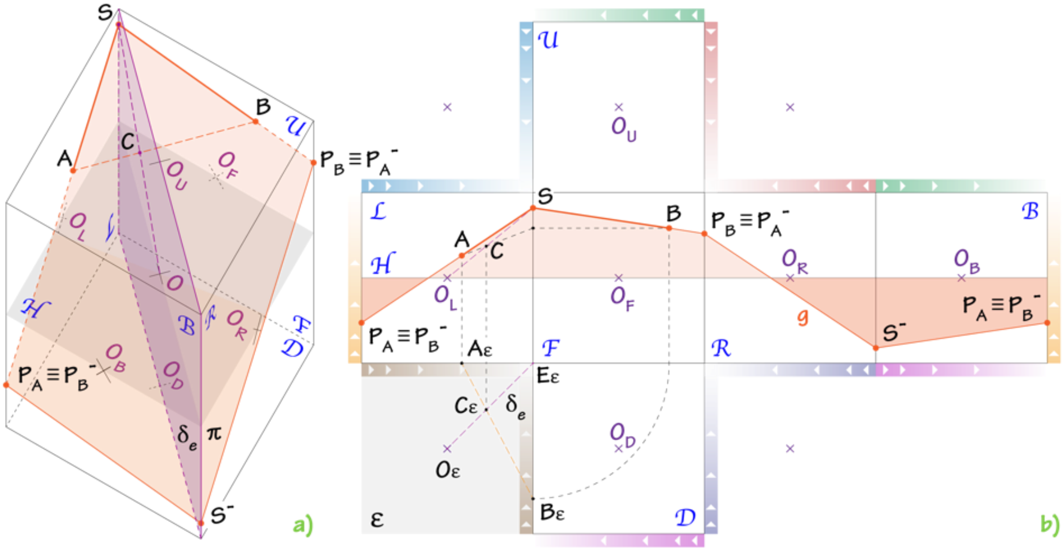


Figure 84 . P_A is on the edge of F_A parallel to e

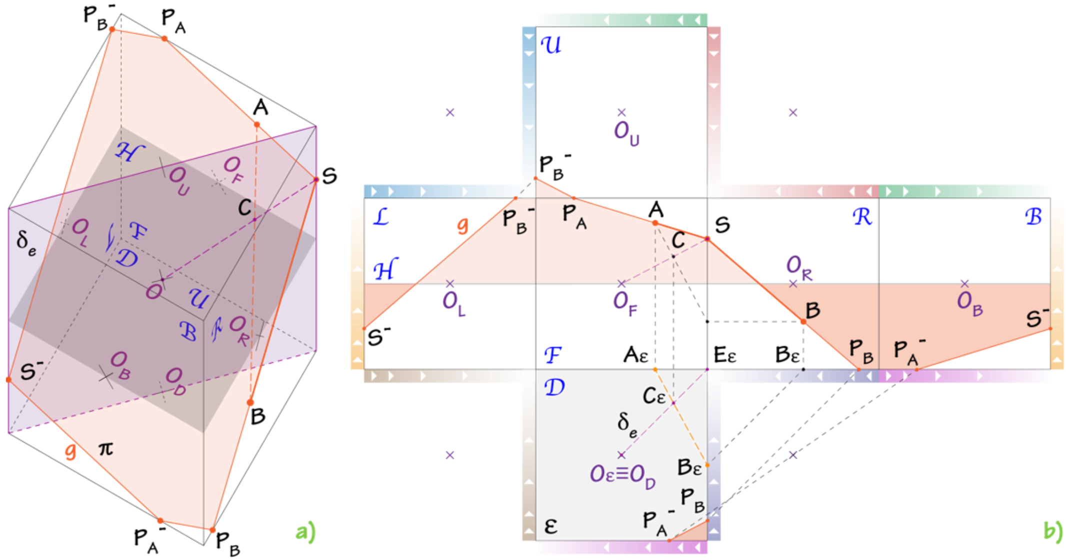


Figure 85 . P_A is on an edge of F_A adjacent to e

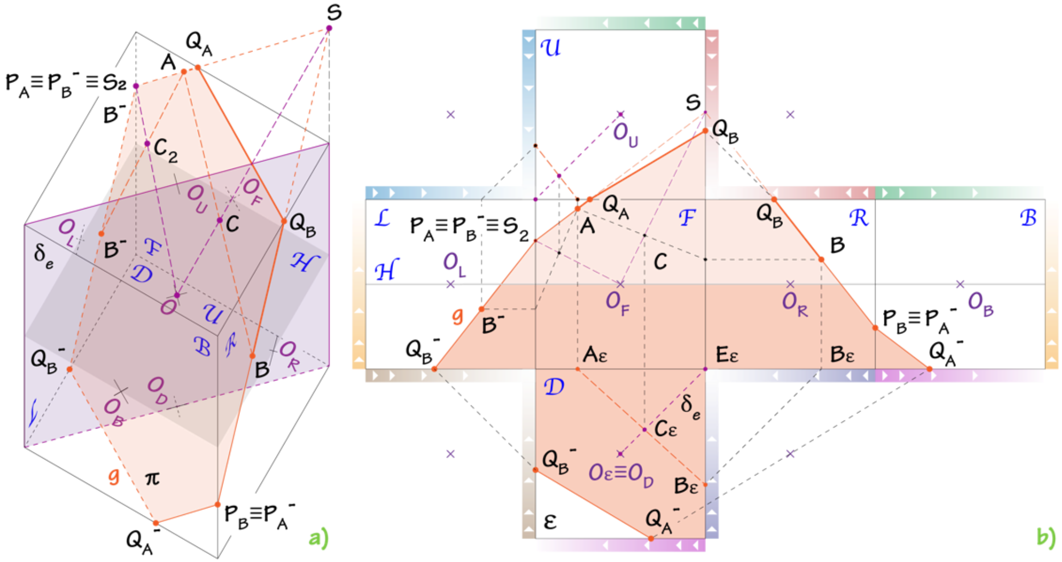


Figure 86 . S is not in e

IV.5.3.5 Measuring and plotting points

I have plotted antipodes and geodesics from given points, but I haven't said how to plot a particular point. It turns out that it is easier to plot points once we have classified the geodesics. The projection of a generic spatial point P is determined by its two characteristic angles λ (longitude or bearing/azimuth) and φ (latitude or elevation), which are the angles one measures when drawing from observation. λ and φ define two grid-like geodesics g_λ and g_φ, that intersect each other at P and P^- (Figure 87a). We find P by constructing these geodesics.

Suppose P is not on the vertical line through O (if it is, then it just projects as O_D or O_U). Let π_λ be the vertical plane through P and O. π_λ makes an angle λ with the vertical plane through O and O_F. Let g_λ be the geodesic of π_λ. π_λ intersects four faces of the cube. Let I be one of the faces not touched by π_λ. Then O_I and P define a non-vertical grid-like geodesic g_φ. The plane of that geodesic π_φ makes an angle φ with H.

Construction 2 (g_λ geodesic)**:** Let M_{FD} be the midpoint of edge FD. Let J be the point of the border of D such that $\sphericalangle M_{FD}O_DJ = \lambda$. The segment $b = O_D\,J$ defines g_λ and we construct it from the two points O_D and J as in section IV.5.3.4.

Construction 3 (intersection of g_φ with g_λ): Let J_λ be the intersection of g_λ with H. OP intersects the vertical through J_λ at a point P_I. Rotate the triangle $J_\lambda OP_I$ around the vertical through J_λ to bring it to face I. We obtain a triangle $J_\lambda O_H P_I$ such that $\sphericalangle J_\lambda O_H P_I = \varphi$ and $|O_H J_\lambda| = |b|$. If P_I is on face I, then $P_I \equiv P$ (Figure 87b). If is not, then triangle $J_\lambda O_H P_I$ intersects either the top (resp. bottom) border of face I at C_1 and at C_2, where C_1 is on the vertical through J_λ. Let c be the segment C_1C_2. On the top face (resp. bottom) we rotate c over the vertical through J_λ. Then the image of P will be the point on g_λ such that $|C_1P| \equiv |C_1C_2| \equiv |c|$ (Figure 88b).

Note that when I is the face B, it is easier to plot P^- and then use Construction 1 to get the antipode.

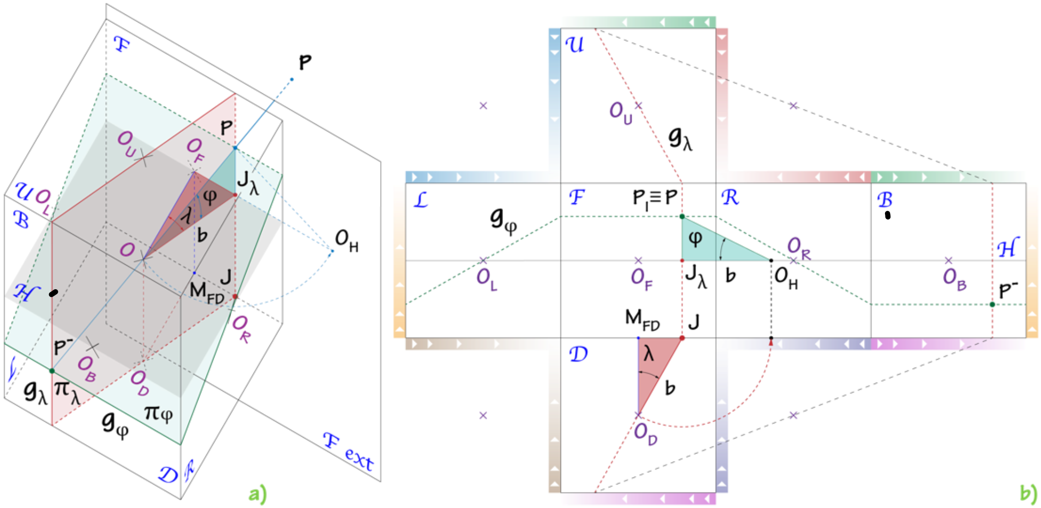


Figure 87 . Image of P using segment b

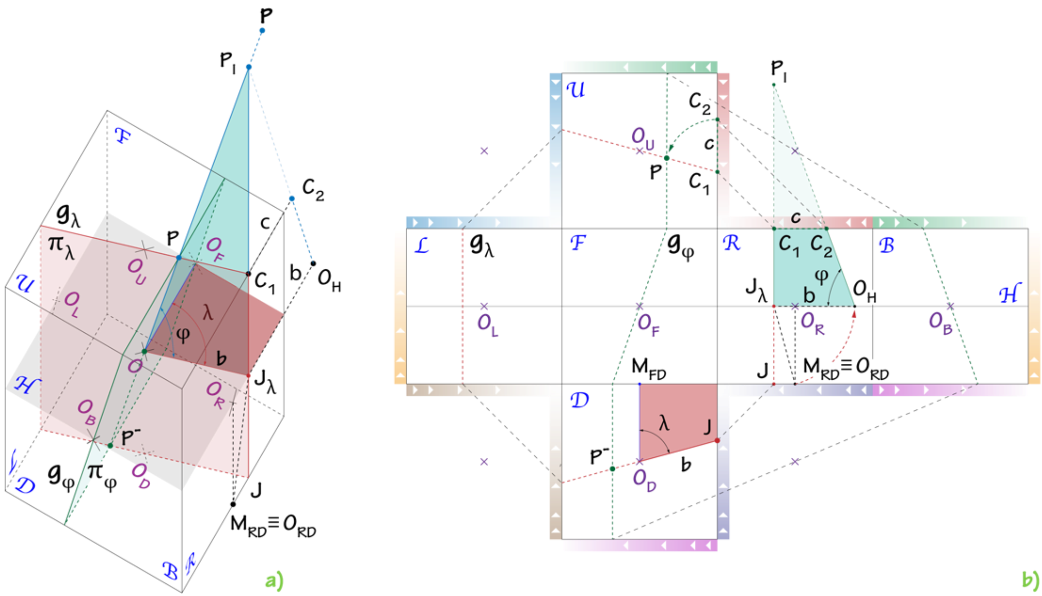


Figure 88 . Image of P using segments b and c

IV.5.4 Uniform Grids

The constructions of geodesics obtained above allow us to solve any problem in cubical perspective. I illustrate this with a couple of examples which are generalizations of classical perspective constructions.

I recall and generalize the standard construction of a tiled floor (uniform grid) in classical perspective: assuming the floor is horizontal and below O, and one of the vanishing points of the grid is centred on a face, then it is possible to assume without loss of generality (since the anamorphosis is independent of the cube's size) that face D touches the floor. Hence the grid projects on D in true size, as an orthogonal grid of horizontals and verticals (Figure 89) that intersect each horizontal face in uniformly spaced points. These lines can be extended as halves of grid-like geodesics, vanishing to O_F, O_L, O_R, O_B. From the bottom left vertex of face F we send a diagonal to vanish at the middle point of edge FR. We get the exact construction of Piero de La Francesca's uniform grid [1, pp. 102, 366]: the 45-degree line intersects each row of lines going to O_F at exactly one point per row, and through these intersections we pass the rows of perpendiculars, to finish the grid. These lines are all grid-like, so they extend to points O_R and O_L as seen in Case I.1. The grid can be completed either by symmetry or by using another 45-degre line on face B to repeat the construction.

Note that in classical perspective the location of the vanishing point of the 45-degree line will depend on the distance of the station point to the projecting plane. But in the cubical perspective that is no longer true: although the distance of O to the projection plane varies with the size of the cube, the position of the 45-degree vanishing point is invariant. It is always located exactly at the midpoint of edge FR (Figures Figure 89 and Figure 90). The geometric constraint between the various faces of the cube keeps it there, invariant for change of scale. Angles, not linear measurements, determine the cubical drawing. In a way, the cube is just apparent: the underlying structure is that of a sphere. This invariance of the position of the 45-degree vanishing point is at the basis of the first method.

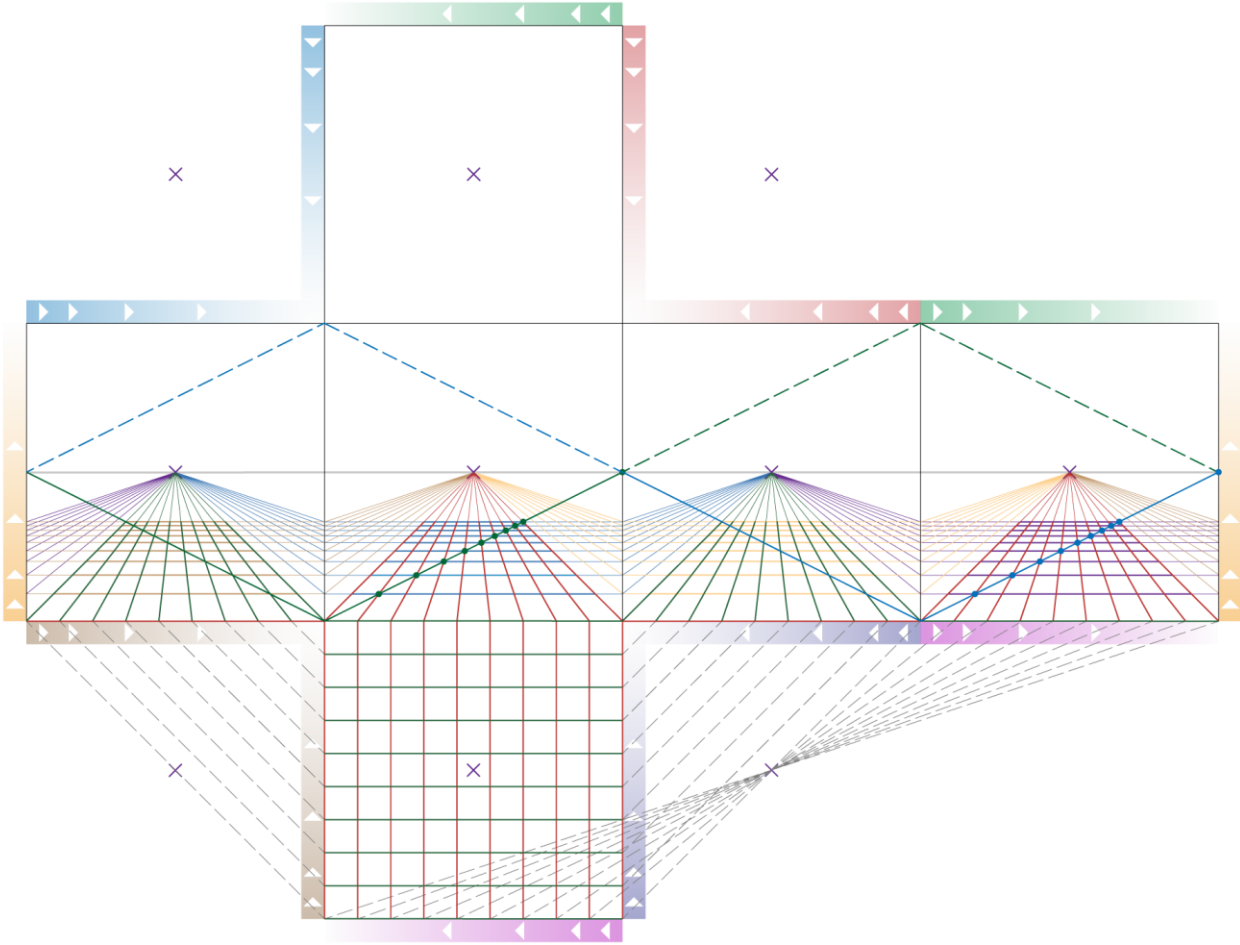


Figure 89 . Grid construction

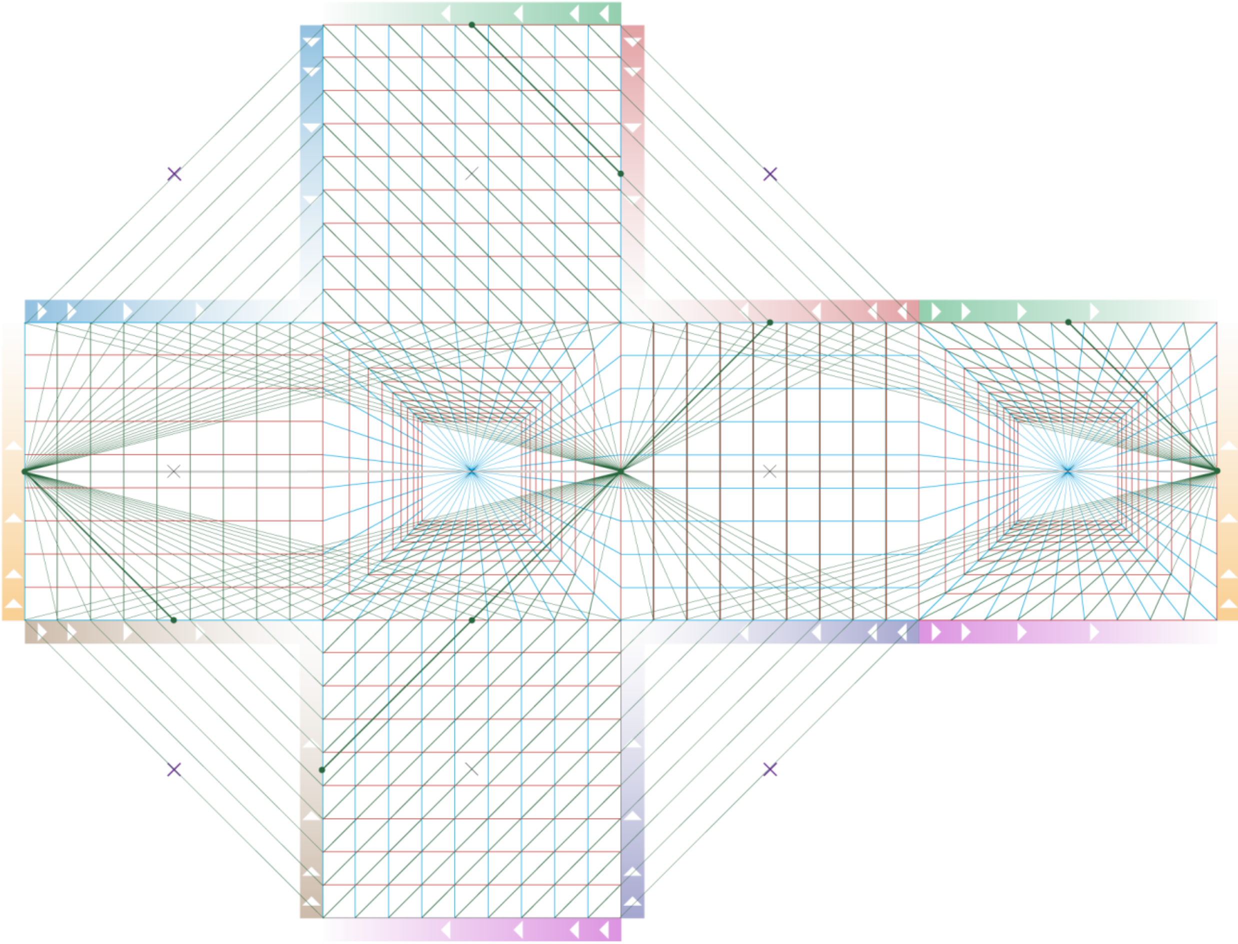


Figure 90 . Extending the uniform grid

IV.5.5 The telephone pole problem

Consider now an example involving the plot of equally distanced elements (Figure 91) from two measured ones. Imagine a scene with equidistant thin columns (e.g., telephone poles) along a vertical plane π that makes an angle with the plane of face F. Suppose we measured two points from direct observation and plotted them as in section IV.5.3.5: point A in the upper extreme of the first column and point B in the bottom of the second one. Assume also that we measured the angle of π with F and we found π to project on face R at 10° to the left of O_R. We will show that these three measurements are enough to construct the whole scene.

Following Case 2 of section IV.5.3.2, we construct the two geodesics g_1 and g_2 that pass through points A, V and B, V. Passing verticals through A and B we find Z on g_2 and C on g_1 respectively. Then segments AZ and CB are the first two columns.

To find the other columns we will define an iterative process based on the vanishing points of the diagonal line $d = AB$. This is a generalization of a well-known construction in classical perspective.

Let g_d be the geodesic of d. Since A and B are in the same face, we construct g_d by Case 1. Extending AB we get P and Q in adjacent edges of F. Therefore, g_d is a 6-cycle, and we construct it using points M and N as in Case 1.2. The vanishing set of π is the geodesic $g_{\pi o}$ obtained by translating π to O. $g_{\pi o}$ is generated by the vertical on face R that passes at 10 degrees to the left of O_R (second case of 1.1.2) and is a 4-cycle with segments all disconnected. Because d is on π, its vanishing points V_d and V_d^- must be in the vanishing set of π, hence we find them by intersecting g_d with $g_{\pi o}$. We join point C with V_d or V_d^- to construct g_{d2} following Case 2 (in Figure 91 the construction is done above face R). Let X be the intersection of g_{d2} with g_2. Pass a vertical through X to obtain point Y on g_1. Segment XY defines the third column. We can iterate the process to get as many columns as we like. Since the diagonals go to the same vanishing points, the columns will be equally spaced.

It is important to highlight that in order to construct the same scene using only classical perspective in the plane of the face F, the (unique) vanishing point of the diagonals d, d_2 and of lines AC, ZD would be outside of the drawing (by quite a lot in the first case). This worsens without limit as the angle of π with F goes to zero. Instead, using geodesics, we draw in a compact way by using whichever of the two vanishing points that happens to be more convenient for the draughtsman. In fact, unlike in classical perspective, we can guarantee that both the vanishing points of a scene and the diagrams required for their construction are within the bounds of the drawing.

In Figure 61 we have an elaborate example of both the uniform tiling of the previous section and of the present construction with regard to the columns. The column multiplication is in that case simplified, since π will be parallel to face F and the vanishing points of the diagonals of the columns will lie on face R and L rather than U and D.

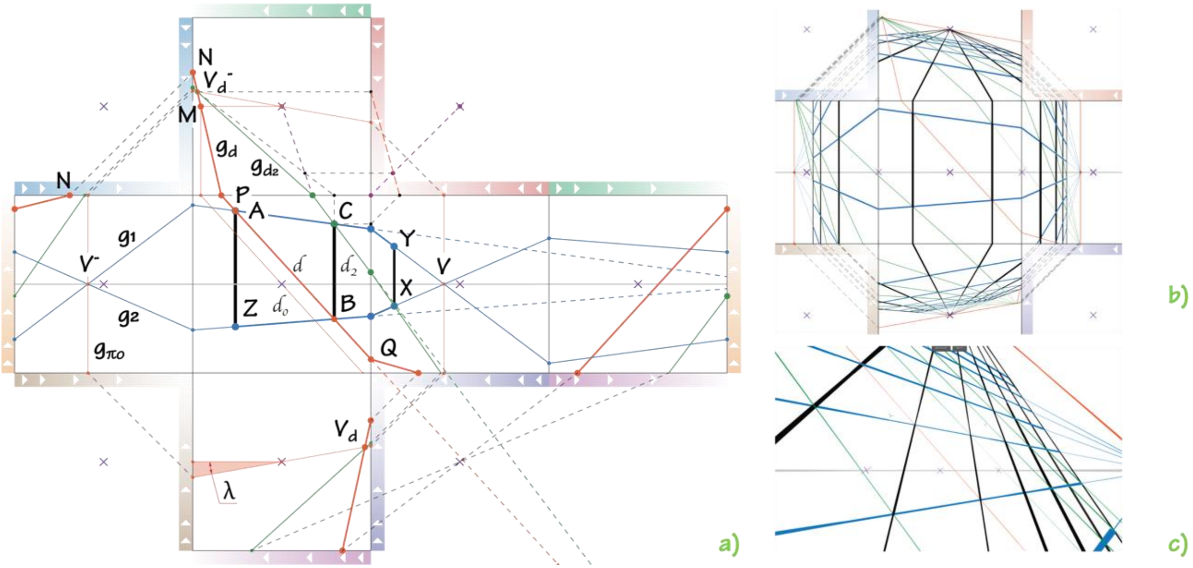


Figure 91 . a) Construction of an example involving the plot of equally distanced elements. b) Final drawing repeating elements in faces L, R, U and D. c) VR visualization

IV.5.6 Deductions

Each spherical perspective, just like each cartographic map and exactly for the same reasons, has its positive and negative aspects. Cubical spherical perspective is no exception. Its positive aspects, when compared to the other main contenders - equirectangular and azimuthal equidistant perspectives - are clear: it works as a classical perspective in each face, and therefore requires much less effort from the user's intuition. Also, if classical perspective can be characterized as the single spherical perspective that is still an anamorphosis [7], [201], hence retains the property of *mimesis*, then cubical perspective holds a close second place, being a set of six local anamorphoses.

Finally, from the point of view of construction, unlike the other two contending perspectives, all geodesic segments can be constructed from the angular measurements of two given points by descriptive geometry diagrams. In both the azimuthal equidistant and equirectangular cases [2], [34] this can only be done with the measurement of specially chosen points which may sometimes be inconvenient to measure. Further, this construction is exact, without requiring approximations or interpolations, due to its linearity.

As for negative points, the main one is the enumeration of cases in this solution, that is comparatively complex when set up against the other two perspectives and the sometimes troublesome process of dealing with the discontinuities from one face to another. The abrupt changes of plane reflect themselves in a comparative inelegance of construction, unseen in the curvilinear cases.

All in all, cubical perspective, when treated properly as a spherical perspective, must hold an important place in the growing bestiary of immersive perspectives from which the architect, artist and engineer can choose according to their needs.

IV.6 GOOD PRACTICES. SHORTCUTS

When the second method was finished, one of the main disadvantages of cubical perspective was the large casuistic to solve geodesics. Moved by that negative aspect, I tried to find shortcuts that will make the practice lighter. The following shortcuts are the result of that intense graphic experimentation. I tested them in many practical examples and especially during the elaboration of the case studies shown in Third Part CHAPTER V (Applications).

There are some more shortcuts that have not been proved, mainly due to factors of time and deviation from the central goals of the research, although all of them were indirectly proved graphically with vector-based programs and with Geogebra constructions as well. Hence, for these unpublished narrowed constructions, I know how they work, but I cannot say why.

IV.6.1 Introduction

Considering characteristics, advantages and the setup of both methods, I introduce narrowed constructions for a more fluid drawing in cubical perspective. The presented shortcuts apply to find an auxiliary point to construct a geodesic, find special auxiliary lines to repeat elements, use centres to find the correspondence between discontinuous faces and plot geodesics. To illustrate these constructions the content is presented using an applicative case. The iconic architecture recalled synthetize some well-known problems of classical perspective, such as the regular repetition of elements. Furthermore, I show how to represent any line focusing on those non-parallel to any face of the cube, since they are the most difficult to represent.

With these shortcuts, I improve the constructions by simplifying them to a shorter and more understandable expression. I also recall other properties of the flattened cubical map such as key points to translate projections within discontinuous faces.

IV.6.2 Shortcut for finding the auxiliary point of a geodesic

In IV.5.3.3 and IV.5.3.4 is presented how to construct geodesics for representing a line l in the cubical map. I recall the solution for the most difficult case, which is when A', B' are in adjacent faces of the cube. The proposed method gets projection S using a descriptive geometry construction from A, B. S is a point of the geodesic within A, B such as $S \in l_e$, where l_e is the line through e, that is, S is an auxiliary point that also belongs to the searched geodesic but can be located in the edge (Figure 83).

Spatially, such a construction considers the line through A, B and the plane δ_e passing through the edge e and the centre O. Then, point S is at the intersection of AB with δ_e. In the cubical map, S is obtained using the top and side parallel projections of the spatial construction.

After many experiences with cubical perspective, it has come to the light a shorter way to find this auxiliary point. The spatial construction remains equal, but new parallel projections are inserted to prove the shortcut. I prove this construction solving the most difficult case, which is when the projections of two points of a line are in two adjacent faces.

IV.6.2.1 Solving the shortcut

Let $F_{A'}$ and $F_{B'}$ two adjacent faces of the cube and e the edge between them. Let A', B' be the two conical projections towards O of two points A, B such that $A' \in F_{A'}$ and $B' \in F_{B'}$. To proceed, we will need to translate one of the points until e. Let suppose without loss of generality that point is A'. We trace $p_{A'}$ perpendicular to e and passing through A'. We now have A'_e in the intersection of $p_{A'}$ with e. We trace the two lines starting at B' and passing through A'_e and A'. Line $A'B'$ intersects e at some point X_e. Let p_{Xe} be the perpendicular to e passing through X_e. p_{Xe} intersects A'_eB' in some point X_i and it turns out that $X_i = C'$. Thus, the construction can be completed from there on to obtain D'.

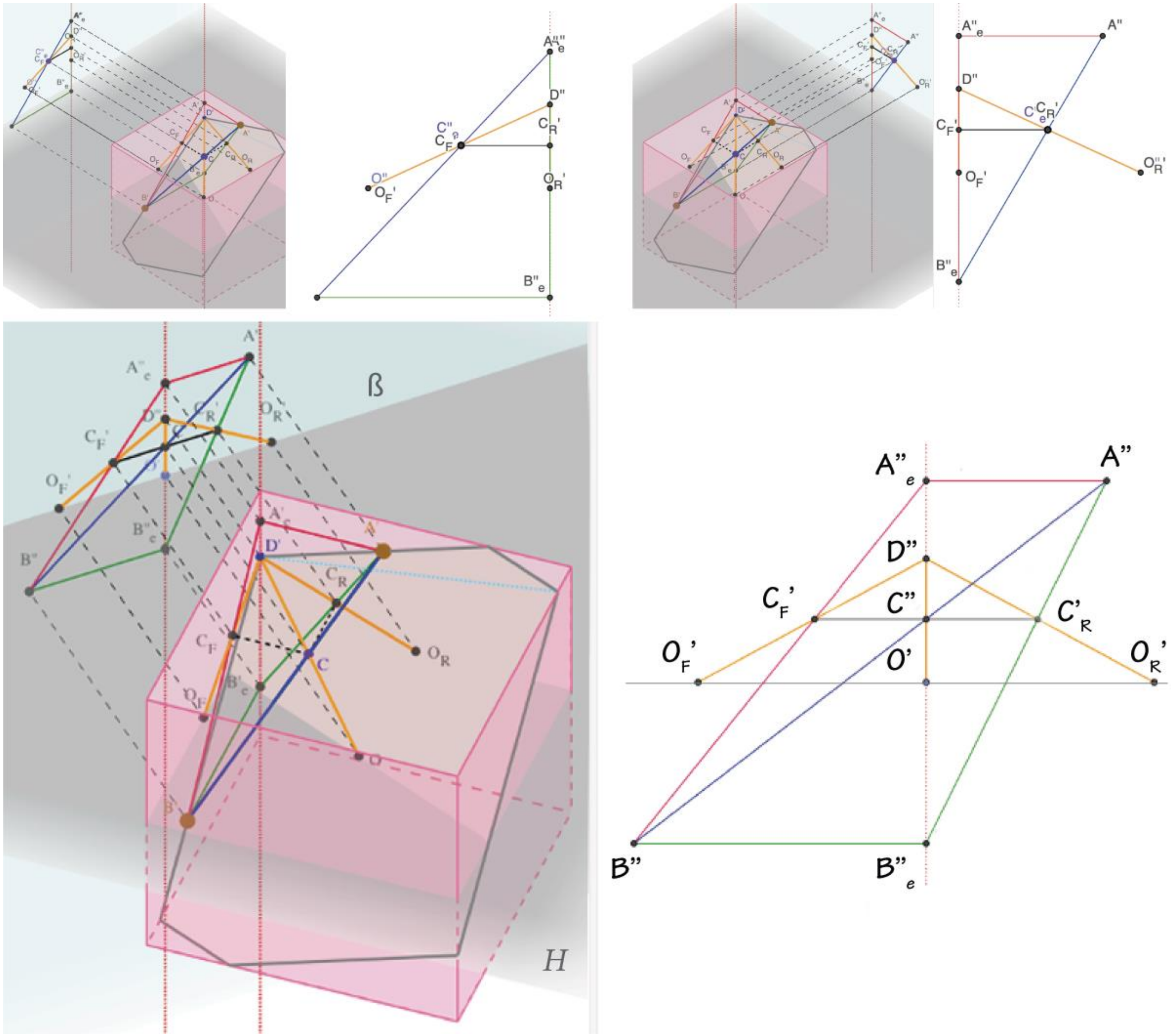


Figure 92 . Construction projected on ß is the same than in F (top). Construction on ß with $\alpha = 45^{\circ}$ *(bottom)*

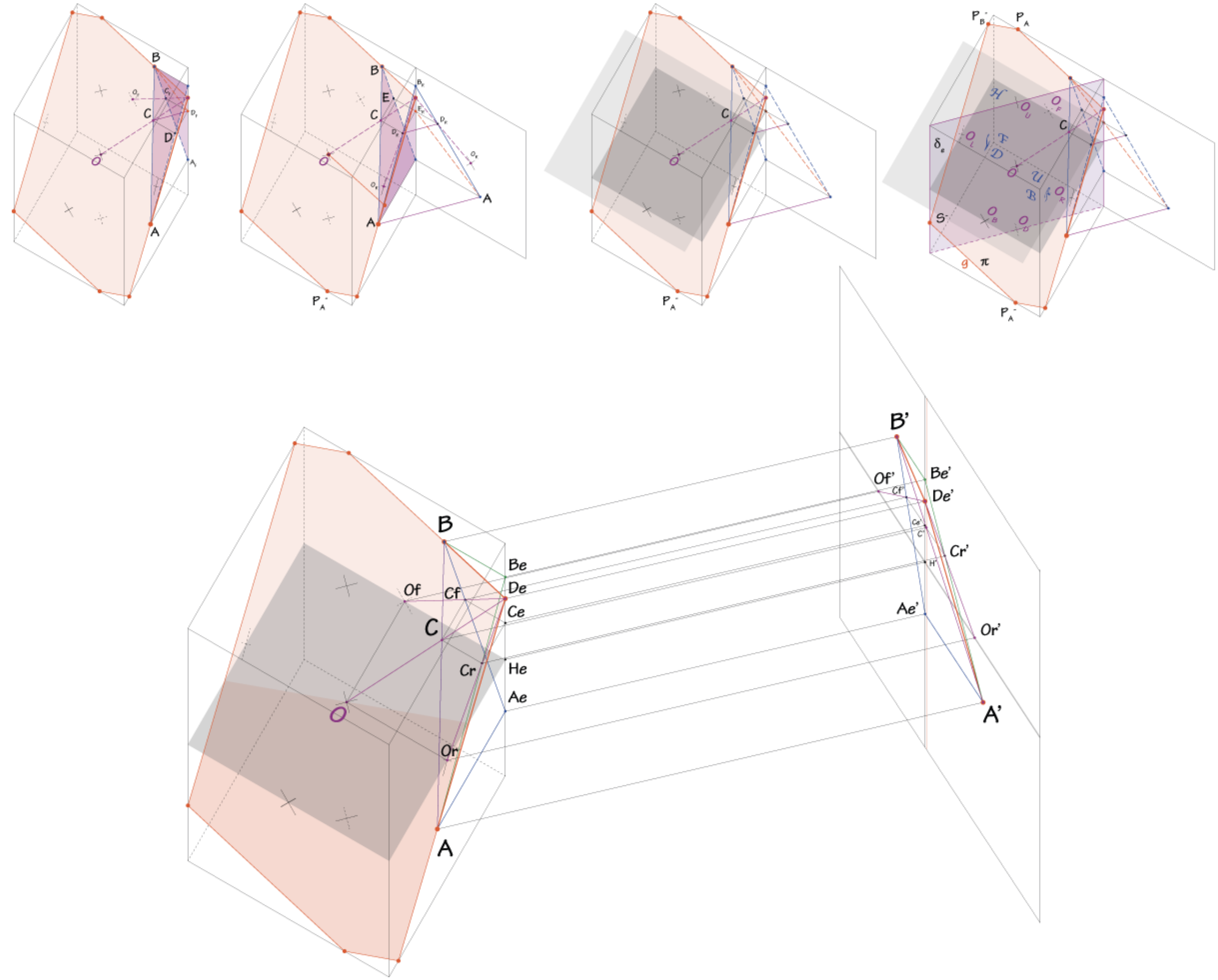


Figure 93 . Reasoning of the shortcut

To prove this is true, we must show either that $A'B'$ is actually a valid projection on the cube map (first condition) and that X_e is a projection of C (second condition).

Let's now consider again the spatial construction with a specific example. Let $F_{A'} = F$, $F_{B'} = R$ and $e = FR$. From here on, we consider projections parallel to the horizontal plane H.

If we project C towards faces F and R we get points C_F and C_R respectively. As they are parallel projections, distances HC, HC_F, HC_R are the same. Therefore, C_R and C_F have the same height h in the cube map.

Let plane ß be a projection plane independent to the cube. Let α be the angle between F and ß. If we project the whole construction with $\alpha = 0^{\circ}$, that is, ß $\parallel F$, we get on ß exactly the same projection that we have on face F. If we do it with $\alpha = 90^{\circ}$ (ß $\parallel R$), then we have on ß what we have on face R (Figure 92 top).

But, if we add a third projection with $\alpha = 45^{\circ}$, we get on ß the construction from Figure 92 bottom. Let $e_{ß}$ be a vertical line, projection of e on ß. C projects as C' on e and as C'' on $e_{ß}$ while O projects as O' on e and as O'' on $e_{ß}$. Segment $A'B'$ projects on ß as a straight segment with no breaks $A''B''$ (first condition). $A''B''$ intersects $e_{ß}$ at point C''. As C'' and O'' are on $e_{ß}$, segment OD is projecting as a vertical segment on $e_{ß}$ and therefore D'' must also lie on $e_{ß}$.

Thanks to the parallelism of the projection, distances HC, HC', HC'' are equal. As $HC = h$, then $HC'' = HC' = HC_F = HC_R = h$. Thus, C_F, C_R and C'' are three different projections of the same point C with equal height regarding H (second condition).

Note that O'' must lie in the horizontal plane and in $e_{ß}$ since we are working with parallel projections and ß is a vertical plane. This condition will not verify for any other position of plane ß. If ß is not vertical, segment $O''C''$ will not start at the horizontal plane and therefore C'', C_F, C_R will not align. If $\alpha \neq 45^{\circ}$, C'' will not lay on e since ray $O''D''$ will not be vertical (Figure 93).

IV.6.3 Centres of translation between different faces

Within the practice of cubical drawing, I used some points that were not included into the first definition of the perspective since they do not belong to the cube itself. Still, these points proved to be very effective for translating projections among discontinued edges.

For example, the projection *P'* of a point *P* lying at edge *UB*, finds its correspondence in its twin edge by passing a straight ray through RC_1 (or rotating with a compass around it). In turn, a point R' lying at edge *LU* finds its pair by rotating with a compass making centre in RC_1 or with a 45° line (Figure 94).

The utility of a clear setup for drawing, including these extra points, makes the task easier especially when the construction starts to be dense of lines (Figure 95 and Figure 96). With this shortcut, it is possible to maximise the utility of a compact setup, i.e., with all the vanishing points and constructions are within the margins of the paper, in contrast with classical constructions where is normal to have improper points.

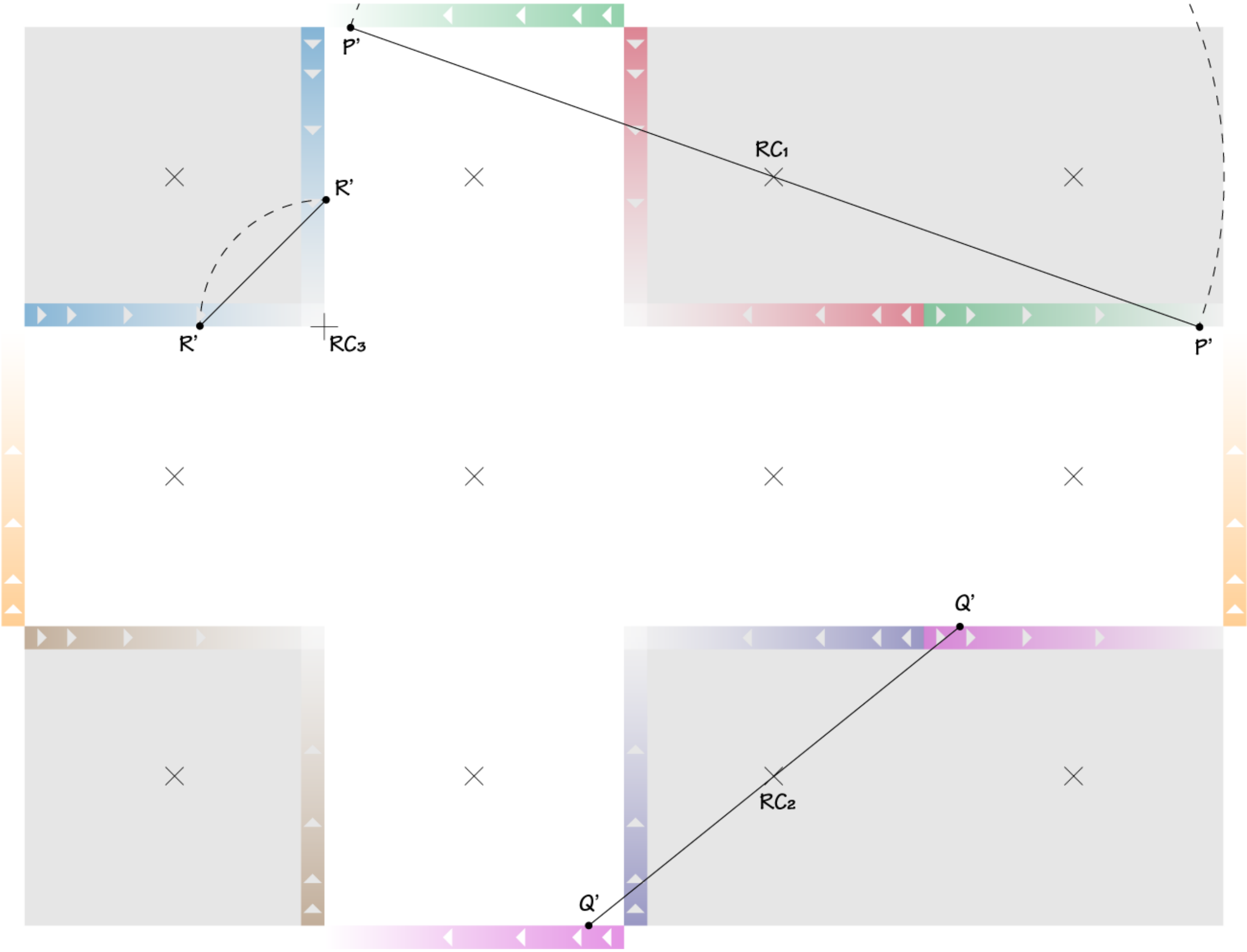


Figure 94 . Linear correspondences between edges

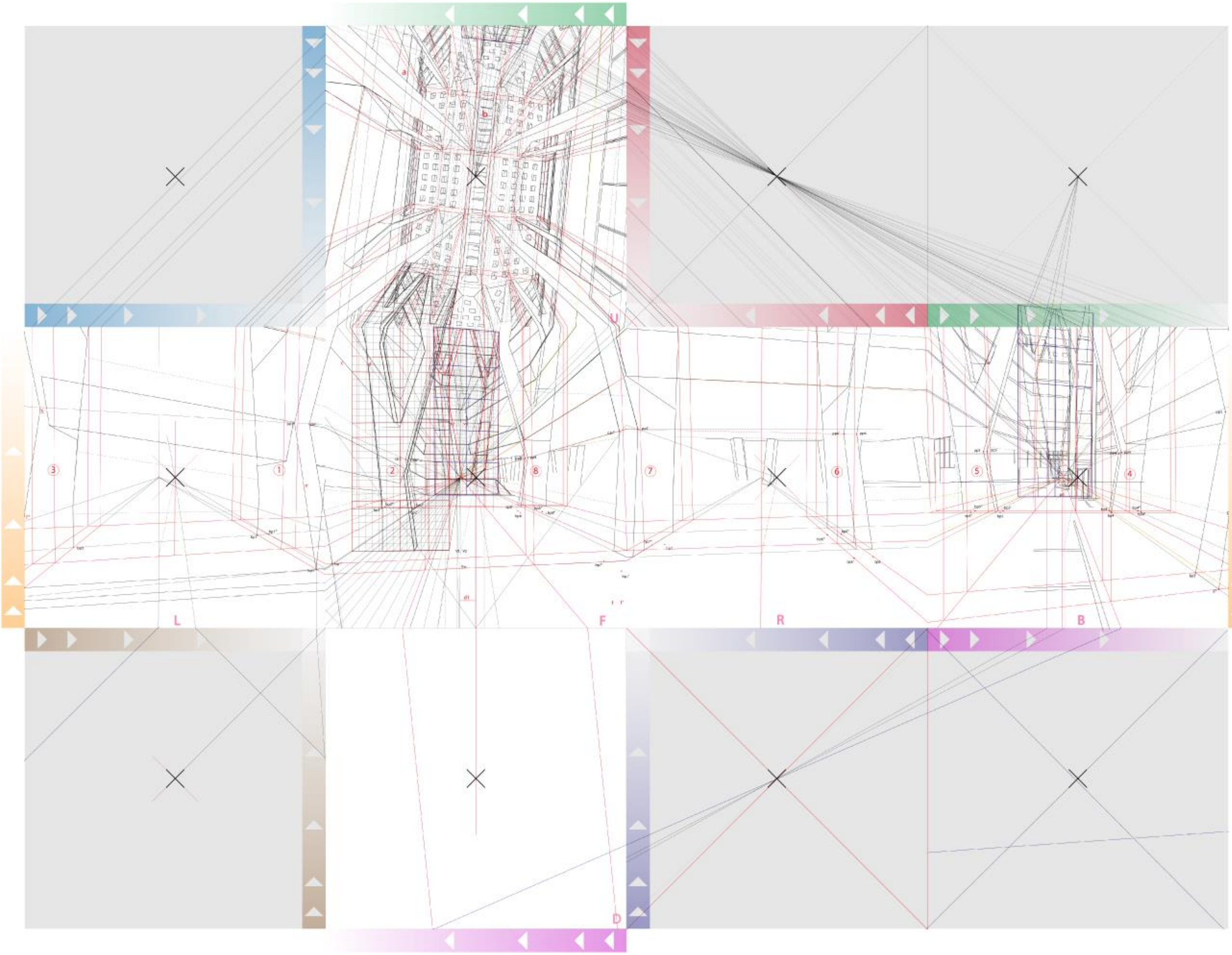


Figure 95 . Shortcuts applied for drawing the Solimene's Factory (2020) by Assia D'Alessio, Teresa Di Palma, Caterina Crispino, Marta Campanile, Maria Petrillo, Lorenzo Villani (see full case study at Third Part V.3)

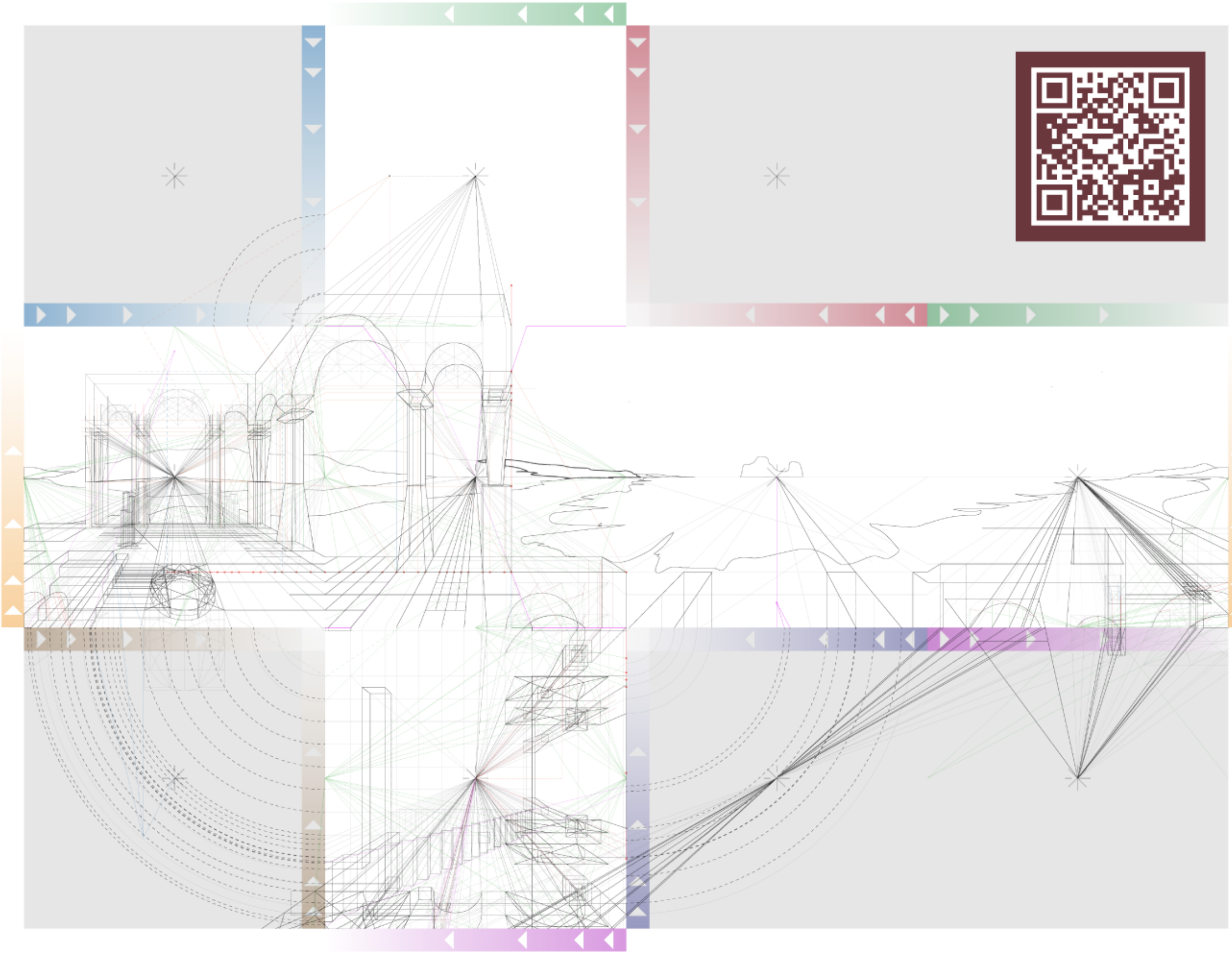


Figure 96 . Construction of an ideal architecture using shortcuts (see full case study at Third Part V.2)

IV.6.4 Deductions

When the two methods were finished, cubical perspective had both advantages and disadvantages. Among the former it had great simplicity thanks to the use of straight lines, viable apparent distortions, precise geometric constructions and integration among traditional and digital techniques. Among the latter, there was its large casuistic and multiplicity of cases, the not always easy discontinuity of the cubical map, a limited diffusion and too many steps until achieving the VR visualisation.

On the other hand, by the moment that this chapter ended, shortcuts represented a great strategy for improving limitations and softening the practice.

In particular, the first shortcut reduced the large casuistic of constructions to a single one that works for both 4-sides and 6-sides geodesics.

Thanks to the centres of rotation, the discontinuities of the cubical map started to be more flexible to manage, both during digital drawing and handmade analogical one by using a ruler.

An important contribution for improving the limited diffusion of cubical perspective is the definition of essential exercises for transmitting the whole theory to a neophyte (see Third Part V.2.1). These exercises were the motor for both finding and proving the shortcuts among new operators.

Many other shortcuts are not published here since they have not been proved more than graphically or satisfactorily written, such as an important relation between cubical perspective and optics. They are on their way to be proved and they will continue the path for bringing cubical perspective drawing to a simpler and more elegant and efficient way.

IV.7 Setting the digital model

Having ready the cubical drawing, the last step for achieving the immersive model is to complete the digital part. The next paragraphs synthesise a *how-to* procedure using a drawing made in the board during a lecture (Figure 97 top). The geometry in the example is not really complex, but it is good enough for showing the basic principles such as vanishing points, correspondences among edges and discontinuities.

IV.7.1 Shot

The final end of this path is to pass from the cubical format to the equirectangular one, since the latter is the standard input for immersive tours. So, to start is necessary the individual cubical map with its correct proportions. One may digitalise the drawing by taking a picture or scanning the paper if is using analogical means or exporting the image file if is working with digital means. The final artwork *must* have a 4:3 proportion. The precision of proportions is a key for having good results within further steps.

Another option is to take individual pictures or export individual faces, an option especially useful for working with traditional means for having a higher final resolution (Figure 97 top). In this case, I recommend using programs like *Adobe Bridge* [202] to correct the deformations induced by the lens of the camera, especially if one is working shooting with a smartphone. The proportion of each individual file it must be squared.

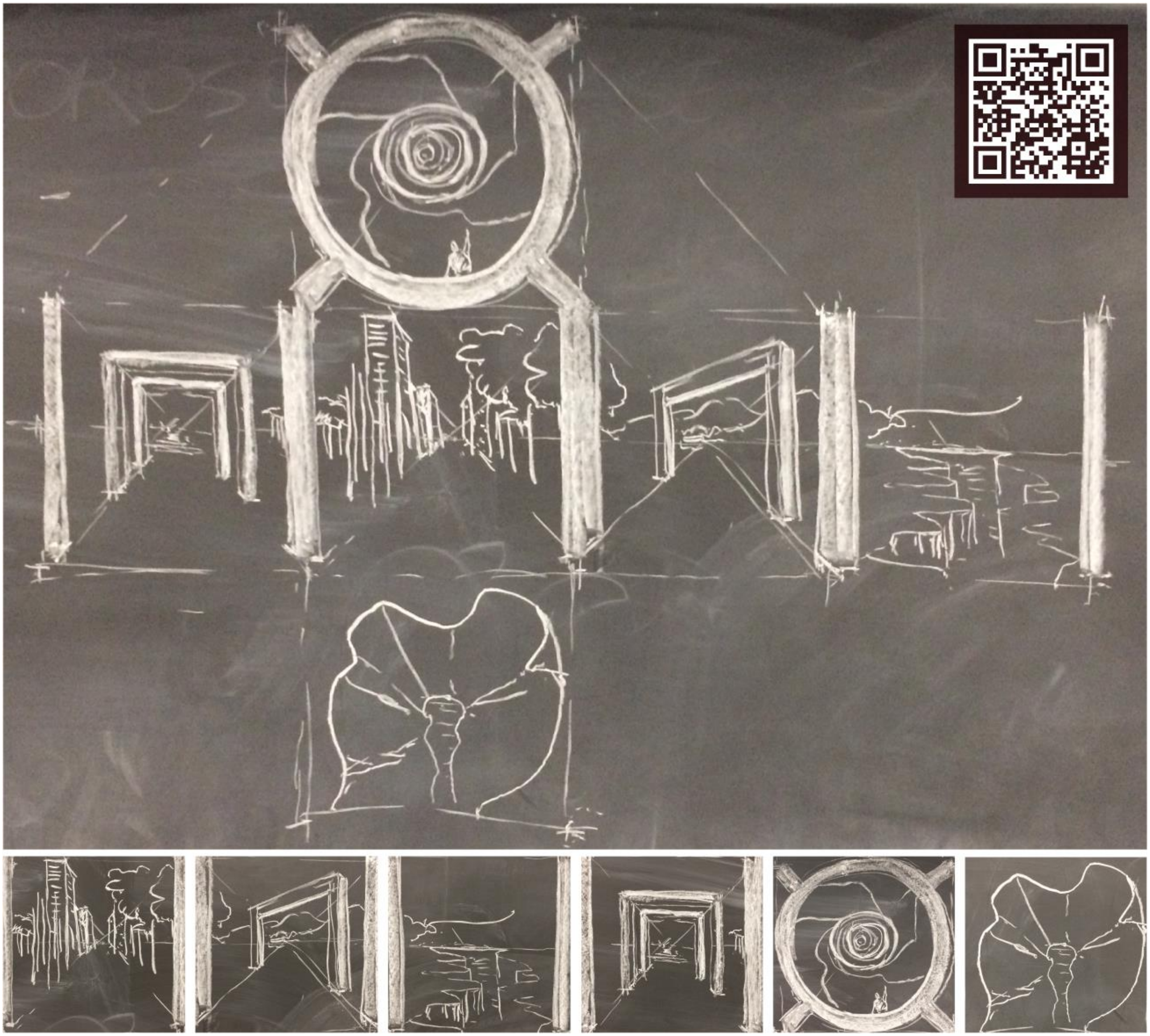


Figure 97 . Drawing in cubical perspective made in the board (top). Independent faces (bottom)

IV.7.2 Getting the equirectangular panorama

If we took one single picture, it may directly use some converter, such as website 360Toolkit [203], the application *Bixorama* [198] or any other similar program for switching among immersive projections.

If the previous tools are not available, another option is cutting the individual faces and mounting the equirectangular with a stitching software. For such an end, any raster-based program can be useful to cut the previous shot and obtain the six individual squares. We name them as front, left, right, back, up and down (Figure 97 bottom).

These six individual faces can be thus loaded in stitching programs like *PTGui* [204]. In this particular example I used the free source program stitcher *Hugin* [97]. I imported the six files and inserted the missing data as if they were pictures: the more basic setup is FOV = 90° and Focal Length = 15,3 (Figure 98 top left).

As a stitching software, *Hugin* is prepared to find automatically homologous points among pictures. In this case, it is clear that such points do not exist due to an inexistent overlapping. Therefore, we must force the stitching by giving a specific position to each image within the full panorama. To do so, setting up "equirectangular projection", we can set the following values respectively for Pitch, Yaw and Roll: front (0, 0, 0); right (90, 0, 0); back (180, 0, 0); left (270, 0, 0); up (0, 90, 0); down (0, 270, 0) (Figure 98 top right).

At this point it is possible to preview and to have a visual inspection controlling the panorama before the final rendering (Figure 98 bottom left). The final rendering may be done at any resolution allowed by the native resolution of the single shots. Yet usually, 8000 x 4000 pixels is enough for a fast but good visualization. Before rendering, the blending mode should be put as "hard seam", so to keep the stitching neutral without colour processing which may give unexpected results.

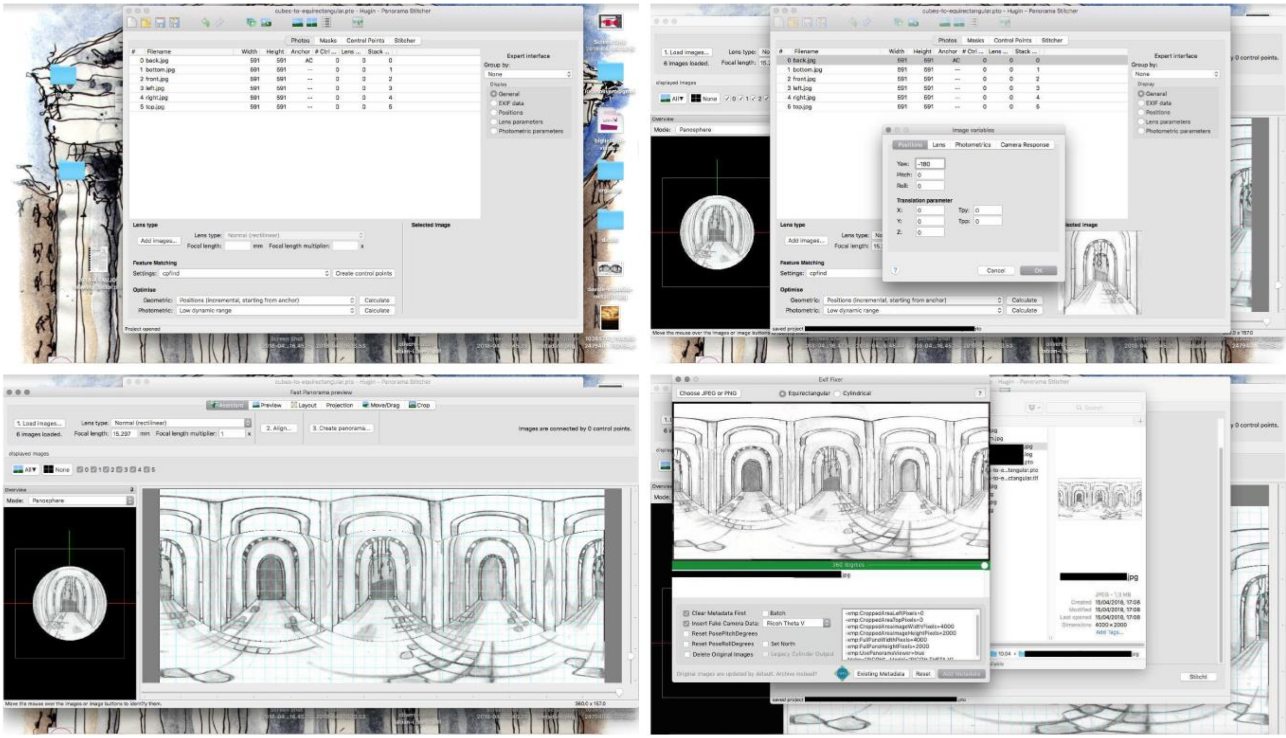


Figure 98 . Setting the equirectangular panorama using Hugin and Exif Fixer

IV.7.3 Metadata

The injection step is not entirely necessary since due to the current spreading of panoramas it is possible to navigate, load and share the panorama without metadata, using, for example, the website *Roundme* [103] or the free spherical visualiser *FSPViewer* [102].

Nevertheless, metadata may be required for some programs or websites, such as Facebook, so to recognise and treat the panorama as such and allow the VR navigation. Indeed, an equirectangular panorama obtained from a cubical drawing needs the addition of metadata to be treated as a regular panorama. This is due to the fact that regular panoramas are usually the output of a stitching process from pictures and not drawings. Consequently, the computer needs to be "tricked" with non-native information so to make it process the drawing as a panoramic picture.

There are many different ways to inject metadata, I use *ExiFixer* [205], which has both an automatised website and a desktop application (Figure 98 bottom right). *ExiFixer* is a free source software running in both MacOS and Windows operative systems. The software has predefined templates oriented to share the panoramas on social media.

Another option is using *Adobe Bridge*, which allows to create, apply and conserve a customized template file. In that case, it is possible to create a template extracted the metadata from a native panorama. The template can also be completed with personalised data, like a copyright notice or contact details.

A third option is opening a photographic panorama with *Adobe Photoshop* and replacing the content with our equirectangular image. The old metadata is conserved after saving the file.

IV.7.4 **Sharing**

Nowadays, VR panoramas navigation is very common and spread in the last few years from a few specialised websites to be included by default in the system of *WordPress* [206]. This visualisation allows a first degree of interaction with the panorama, in which the end user drags the picture with the mouse or finger to rotate the visual cone (Figure 99).

On the other hand, a more complete option is the composition of a full virtual tour. In that case, it is possibly to access further tools, such as the VR modality. This modality creates a fake stereoscopy by dividing the screen in two parts and projecting two slightly different images (Figure 100).

Hence, it is possible to watch the scene in total immersion by means of mobile phone + VR glasses or a proper VR headset (Figure 101). In that case, the image projected in the screen replies to the rotation of our head by means of the gyroscope. Within the immersive view, the lenses increase the two images that we overlap naturally completing the third dimension during the watching.

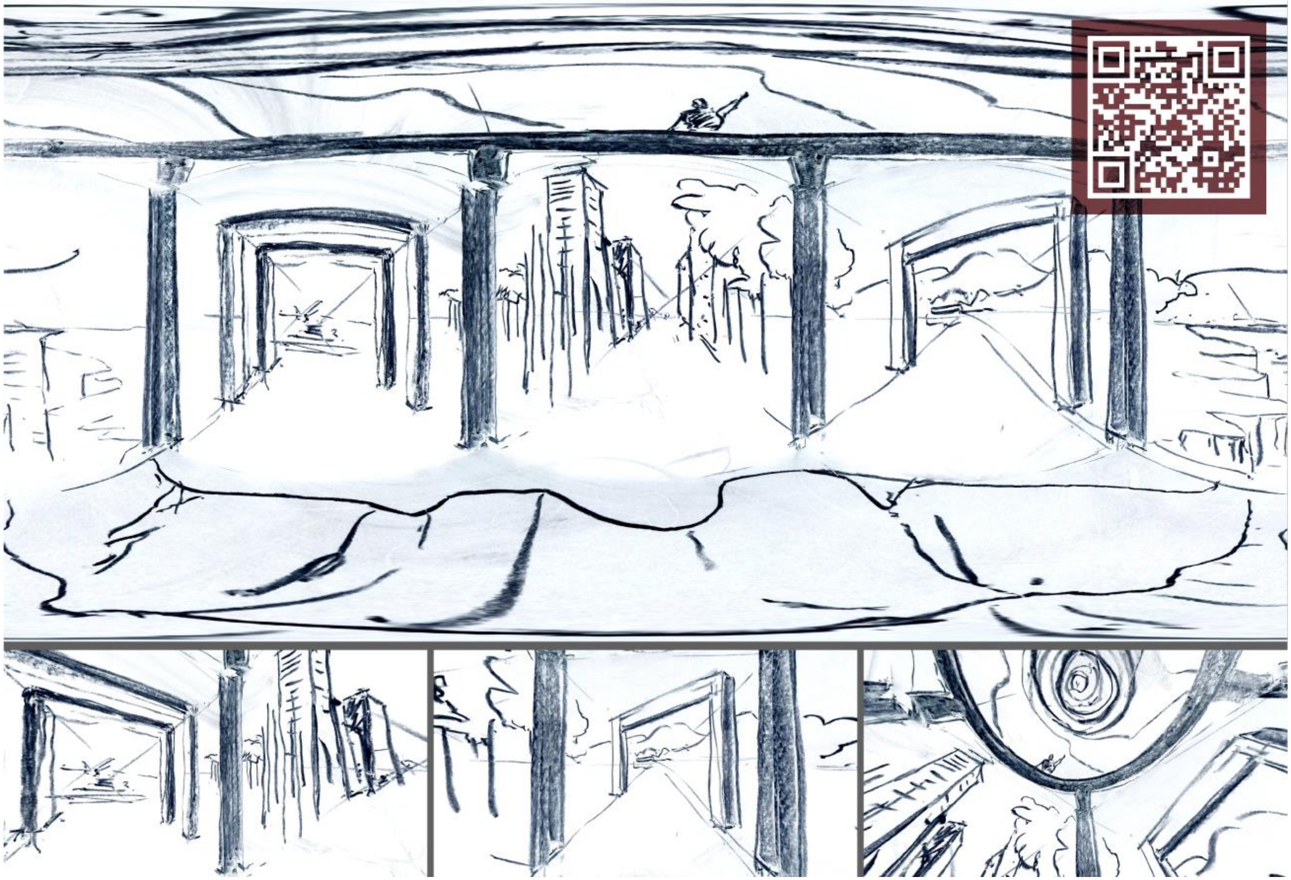


Figure 99 . Equirectangular panorama (up). VR visualisation (bottom)

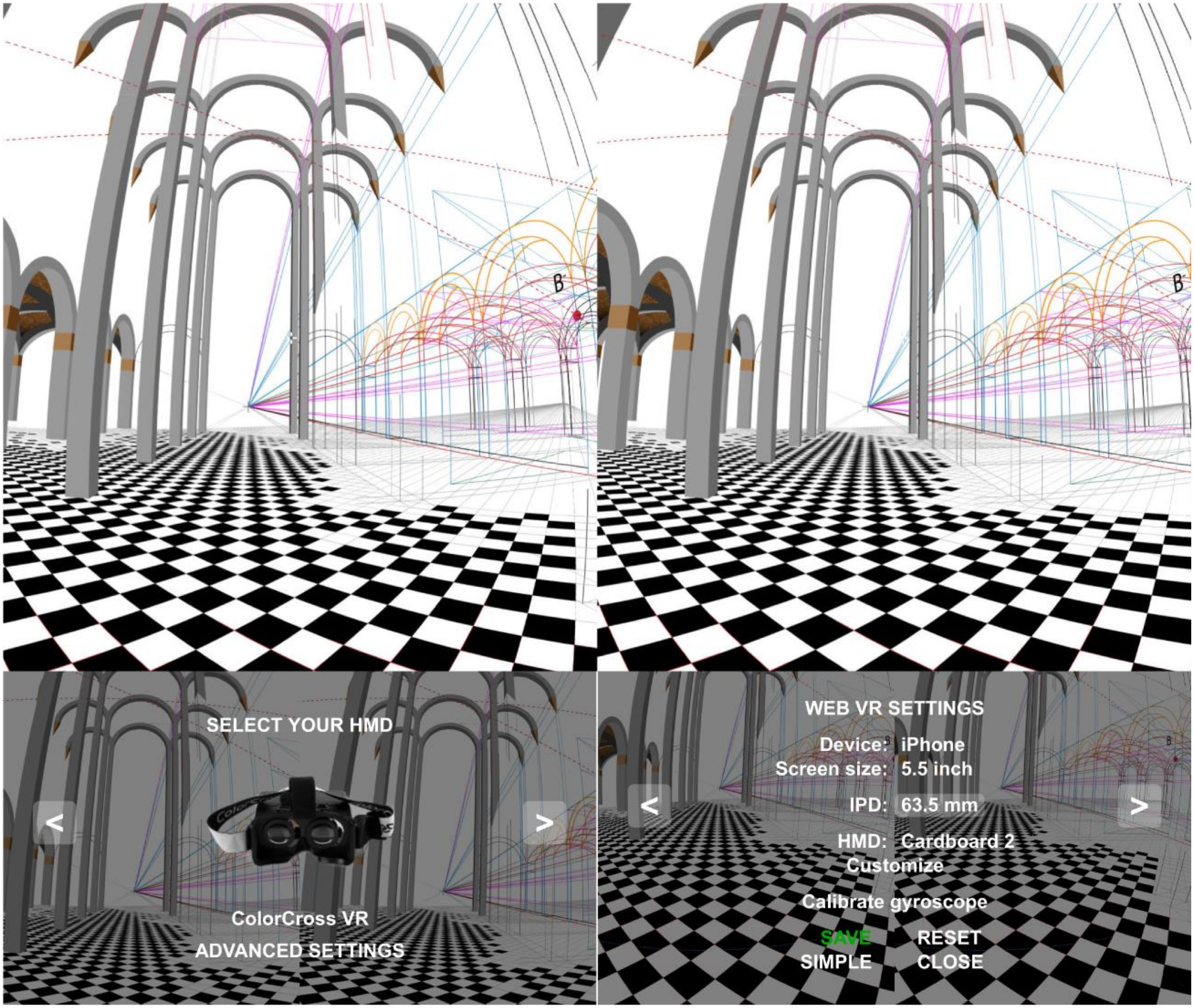


Figure 100 . Forced stereoscopy using software deviation. It is possible to setup different kind of carboards

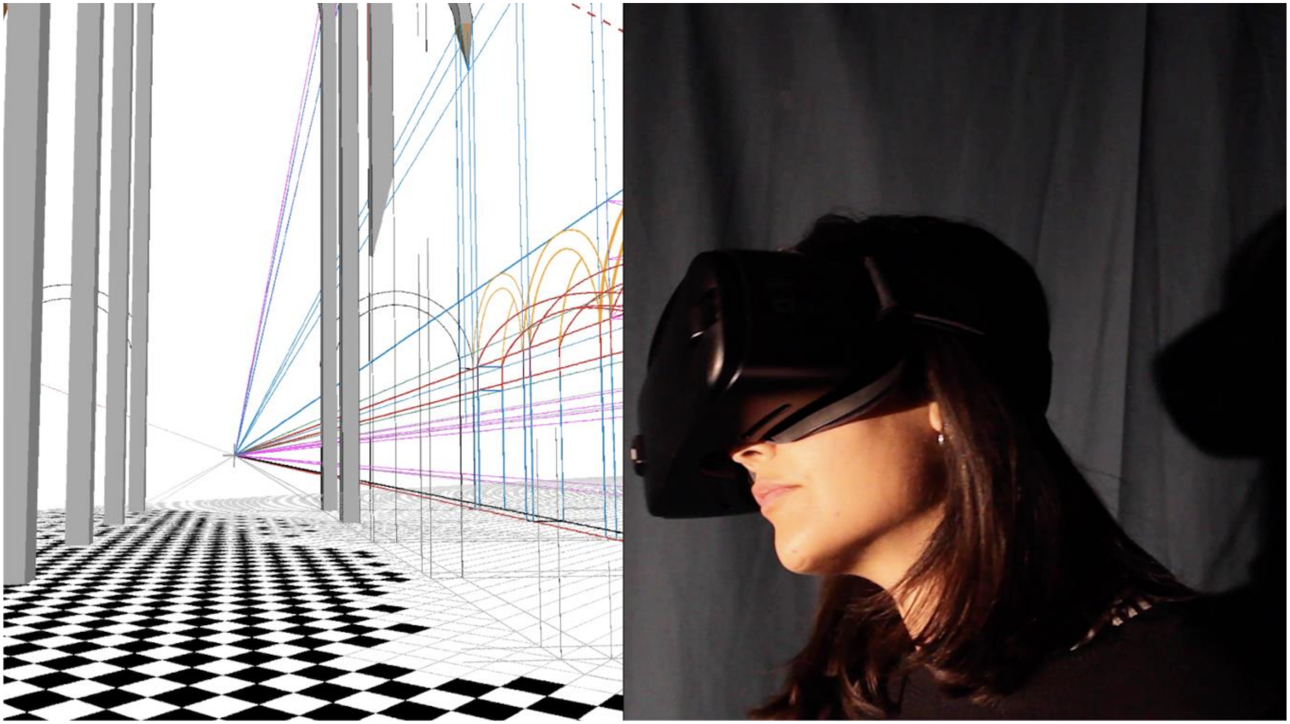
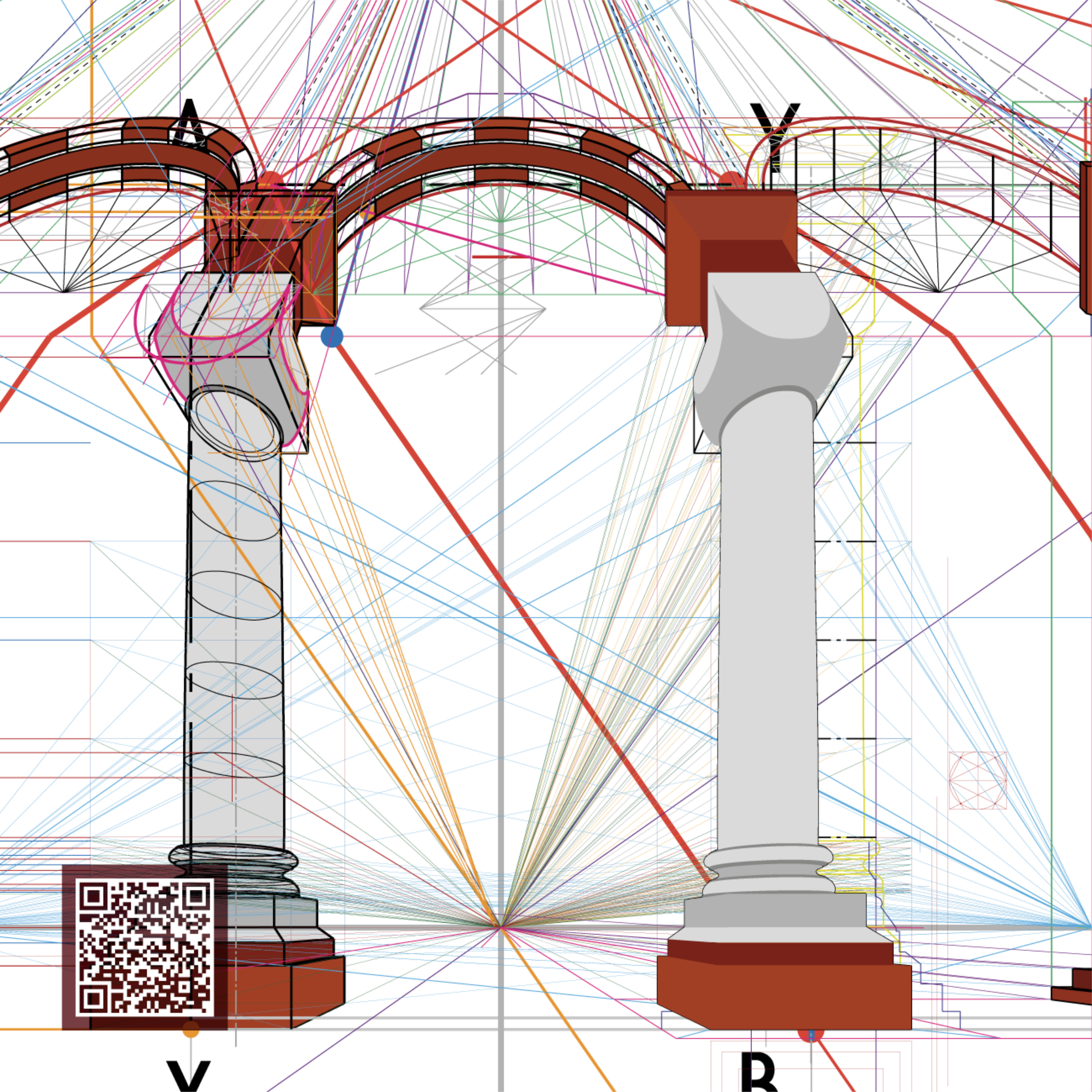


Figure 101 . Immersive VR visualisation using headsets[35]

35 Thanks to Ana Elisa Perez Saborido for her colaboration

THIRD PART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CHAPTER V APPLICATIONS

V.1 INTRODUCTION

Until now, I have developed the current panorama of conic projections, the state of the art for the cubical case and the descriptive geometry rules for plotting a cubical perspective. The state of art and the mathematical developments represented more than necessary stages to describe the cubical perspective. However, the research needs to show its utility in the field of representation. In words of Mario Docci and Riccardo Migliari, the mathematic knowledge by itself deprives descriptive geometry of the contact with the reality, the same reality from which representation comes and from where the discipline gets its nourishment. Hence, without applications, the research risks to get stuck in the pure theory and far from that experimentation laboratory that is *drawing* [45, p. 10].

Therefore, in the following chapters graphical examples will take the lead, summarising the results of applying cubical perspective drawings for surveying existing architectures, creating ideal spaces, reconstructing a remote or inexistent place and for product design.

The first two case studies are divided paths closing a same circular and complementary application where the one feeds the other in a constant interaction. They are oriented to operators that need to learn cubical perspective first. Consequently, the general exercise among these two paths goes from transmitting the cubical perspective's rules with ideal architecture (culture of project), up to their application in a real architecture (culture of survey).

The third case opens the use of cubical perspective as a way of surveying and reconstructing a space. Such a space can exist or not, but in both cases the application opens the possibility of exploring remotely a place that is not physically possible to visit.

V.2 PROJECT: LEARNING CUBICAL PERSPECTIVE

The purpose of this first path is to learn the theory of cubical perspective. To that aim, I developed exercises in which a neophyte with basic knowledge of classical perspective can acquire the first notions. I used ideal elementary architecture examples, created intentionally to introduce progressively the complexity of the representation system. The operator starts with these essential exercises and finishes projecting an ideal architecture that verifies proves the assimilated concepts.

I use the paper artefact (Second Part IV.2.2) to familiarise the operator with the cube, its flattening and the correspondences among edges: I draw basic lines in the physical artefact and fold it back to illustrate the concept of anamorphosis and the fragmentation of a line.

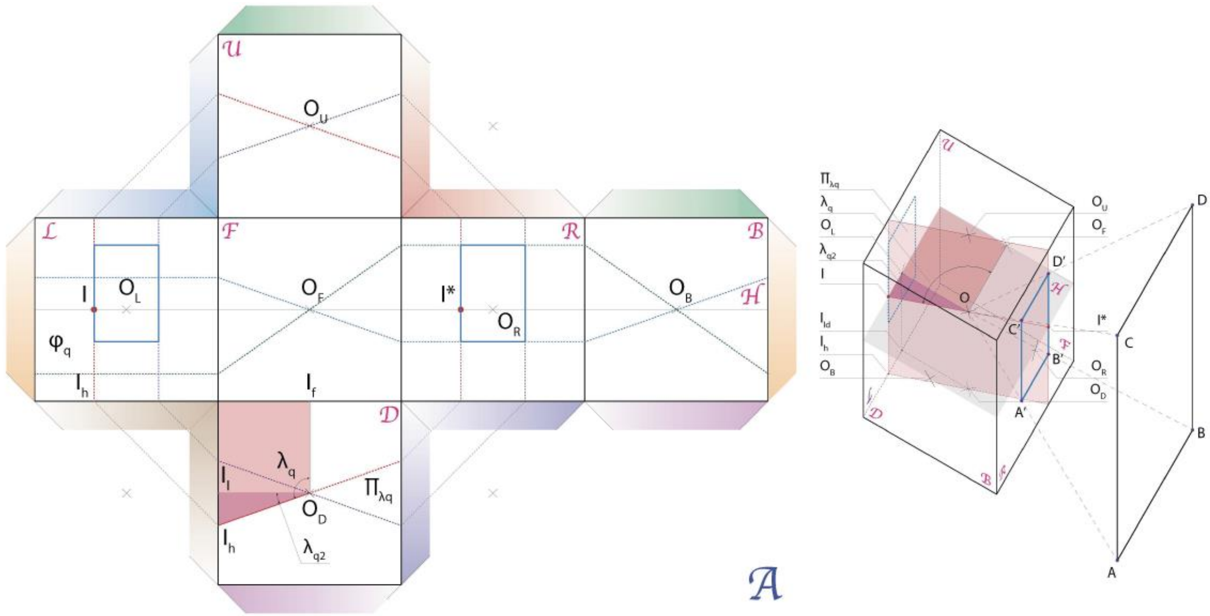

Figure 102 . Exercise for practicing basic geodesics and representations within one face of the cube

As well, I use two simple examples to practice geodesics, planes passing through the origin and plotting of angles. These two cases exemplify when a representation is within one single face (Figure 102) and when is fragmented in more than one face (Figure 103). The same examples may be printed and fold physically in a cube.

Being oriented to designers, it is important to clarify that the essential skill of cubical perspective can be achieved using knowledge that the operator (should) already possess, such as classical perspective notions and a daily interaction with representation systems.

In a further step, the learner follows the next seven exercises E I to E VII (Figure 104), introducing with each of them a further complexity. By the end, all the concepts are synthetised in a creative example where the operator will condense the previous exercises in one (E VIII).

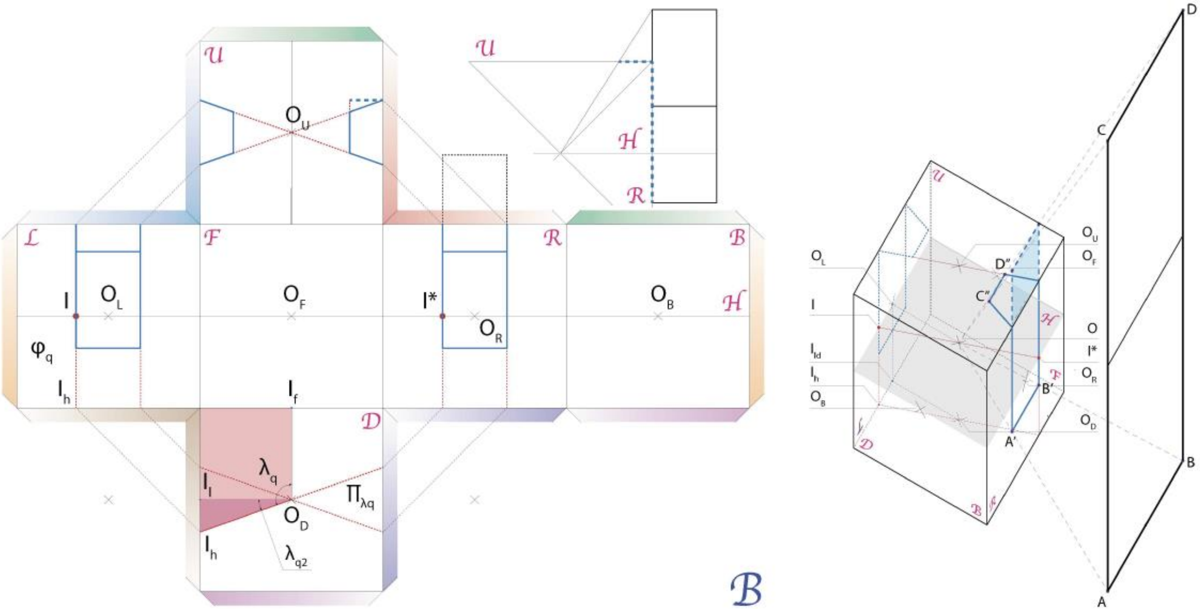

Figure 103 . Exercise for practicing basic geodesics and representations within more than one face of the cube

V.2.1 First path: from ideal architecture to VR

In exercises E I to E VII, the operator constructs the cubical perspective of a basic room or architectural elements such as columns. By the end of each construction, the operator verifies the results in VR modality. In case of using digital drawing programs, it is possible to create a dynamic workflow to verify the results almost in real time by means of linking the raster or vector-based illustration with Adobe Photoshop and using the plugin *Oniride Art 360* (see Second Part III.4.2.1).

V.2.1.1 E I

The operator represents a basic room, considering the architecture and the position of the observer such as in the figure of reference (Figure 104 E I)

This exercise uses a simple architecture to practice essential concepts, such as central vanishing points and the development of a geodesic in the cubical map. It introduces the operator to the measuring of angles from floorplan and section and their transportation to the cubical map.

V.2.1.2 E II

The operator represents a basic room, considering the architecture and the position of the observer such as in the figure of reference (Figure 104 E II).

It uses the same room than the previous exercise, but it changes the position of the observer in such a way that now there are some hidden profiles. The reconstruction of hidden profiles may be easily done using geodesics' intersections. E I and E II find their projections in panoramic faces, that is, Front, Left, Back and Right.

V.2.1.3 E III

The operator represents a room, considering the architecture and the position of the observer such as in the figure of reference (Figure 104 E III).

A new room introduces more elements to consider such as doors, windows and stairs. The observer has now a position such that the image of the architecture has correspondences also in the vertical faces, that is, using the six faces. The new complexities rely into finding the correspondences among unplugged edges. This exercise also considers putting in practice the previous concepts. Up here, all represented edges are verticals and horizontals.

V.2.1.4 E IV

The operator represents prismatic columns equally distanced, considering the observer symmetrically positioned in front of them (Figure 104 E IV).

In this case there is no room to be represented. The exercise focuses into how to repeat elements regularly distanced. The practice introduces the plotting of diagonal geodesics and reasons in terms of two key points (from columns 1 and 2) and using geodesics for finding the position of 3 and 4 (see Second Part IV.5.5). Up here drawings are composed with 4-sides geodesics.

V.2.1.5 E V

The operator represents prismatic columns equally distanced, considering the observer asymmetrically positioned in front of them (Figure 104 E V).

Like the previous case, this exercise focuses into how to repeat elements regularly distanced. Yet, it introduces the plotting of non-centred diagonals, and therefore, the drawing 6-sides geodesics.

V.2.1.6 E VI

The operator represents columns with variable section equally distanced, considering the observer asymmetrically positioned in front of them (Figure 104 E VI).

This exercise introduces the complexity of drawing non-orthogonal objects. To solve the problem, one constructs grids with vertical and horizontal lines.

V.2.1.7 E VII

The operator represents prismatic columns equally distanced and regular tiles, considering the observer asymmetrically positioned in front of them (Figure 104 E VII).

This exercise focuses into reading and repeating elements structured by one same module. Indeed, in the image of the exercise can be seen that one same unit the criterium for defining the columns' section and height, the distance between them, the extension of the pavement and the tiles. The concept of cubical perspective introduced is the repetition in depth of modules and submodules by means of vanishing points at 45 degrees and the use of diagonals.

V.2.1.8 E VIII

This first path concludes with this summary exercise aimed to prove the previously acquired knowledge. The operator builds the cubical perspective of an ideal space. In this case, is on the user's creativity to define the architecture, proportions, modules, elements, and the position of the observer.

However, there are some requirements: the scene must include all the concepts learnt up here, such as the regular repetition of elements disposed both symmetrically and non-symmetrically (e.g., columns, tiles, subdivisions of a façade), the existence of modules and submodules ruling the design, and representations in the six faces. The position of the observer must be such to guarantee images fragmented in more than one face.

Operatively, the user defines first a geometrical criterion, it composes a floorplan and sections based on that. Then, it places the observer in a non-central position and, if is possible, near some tall object such that the projection coming from the upper extreme of this object must intersect the top face.

For constructing the perspective, the operator translates the minimal quantity of references from the orthogonal views, and it builds the perspective reasoning in terms of vanishing points, geodetics, intersections, etcetera.

Such a construction guarantees the coherence of the content within the VR navigation. That is, if one decides to measure and translate all points from the orthogonal views, it will build the perspective with more points than the essentially necessary ones. And, as exposed at Second Part IV.2.1, the problem with more points is a much probably wrong immersive visualisation, situation accentuated by drawing precision.

In Figure 105 and Figure 106 are some results of this first path, made by neophytes that followed these exercises.

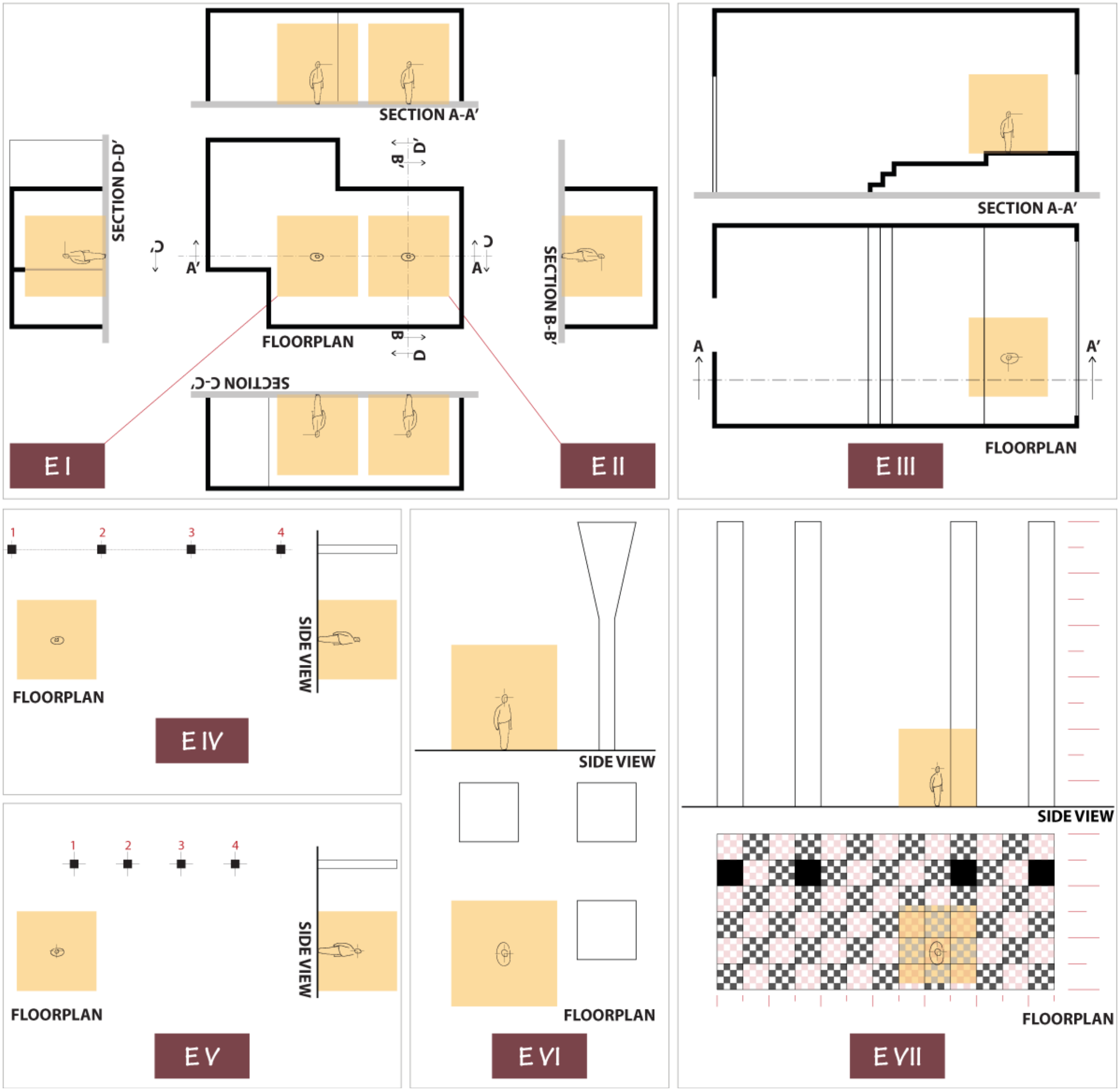

Figure 104 . Exercises E I to E VII. E I) Single room using panoramic faces. E II) Representing hidden walls. E III) Using superior and inferior faces. E IV) Symmetric repetition of elements. E V) Asymmetric repetition of elements. E VI) Elements with variable-section. EVII) Lecture and repetition of modules and tiles using 45° vanishing points

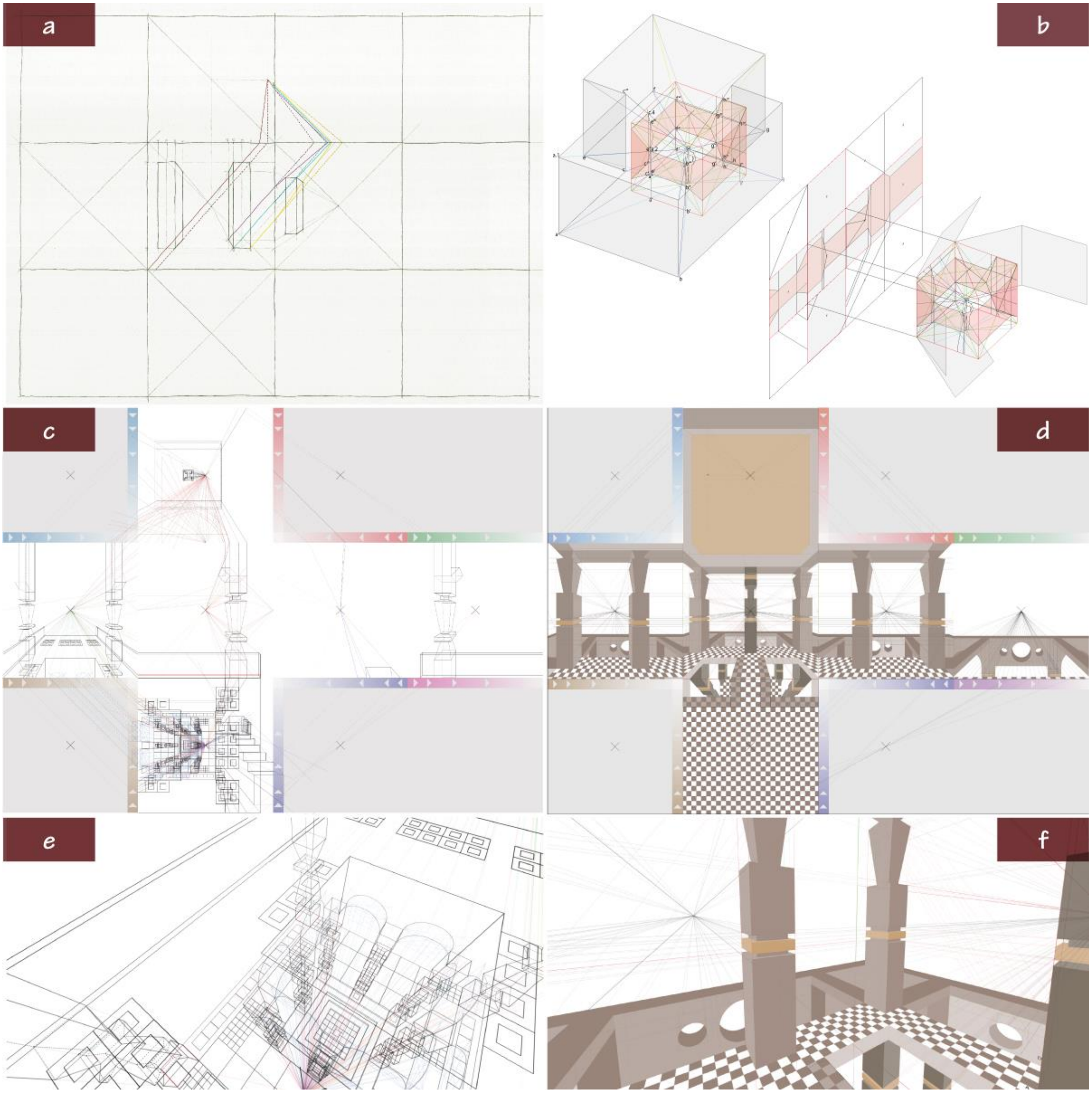

Figure 105 . a) E VI made with analogical drawing. b) Spatial interpretation of E I. c, d) Cubical perspective of E VIII. c, d) VR navigation of E VIII. Authors: a, b) Teresa Di Palma, c, e) Ibtissam Jayed, d, f) Assia D'Alessio. T.A.R. Unicampania a.a. 2019-20 (prof. Adriana Rossi)

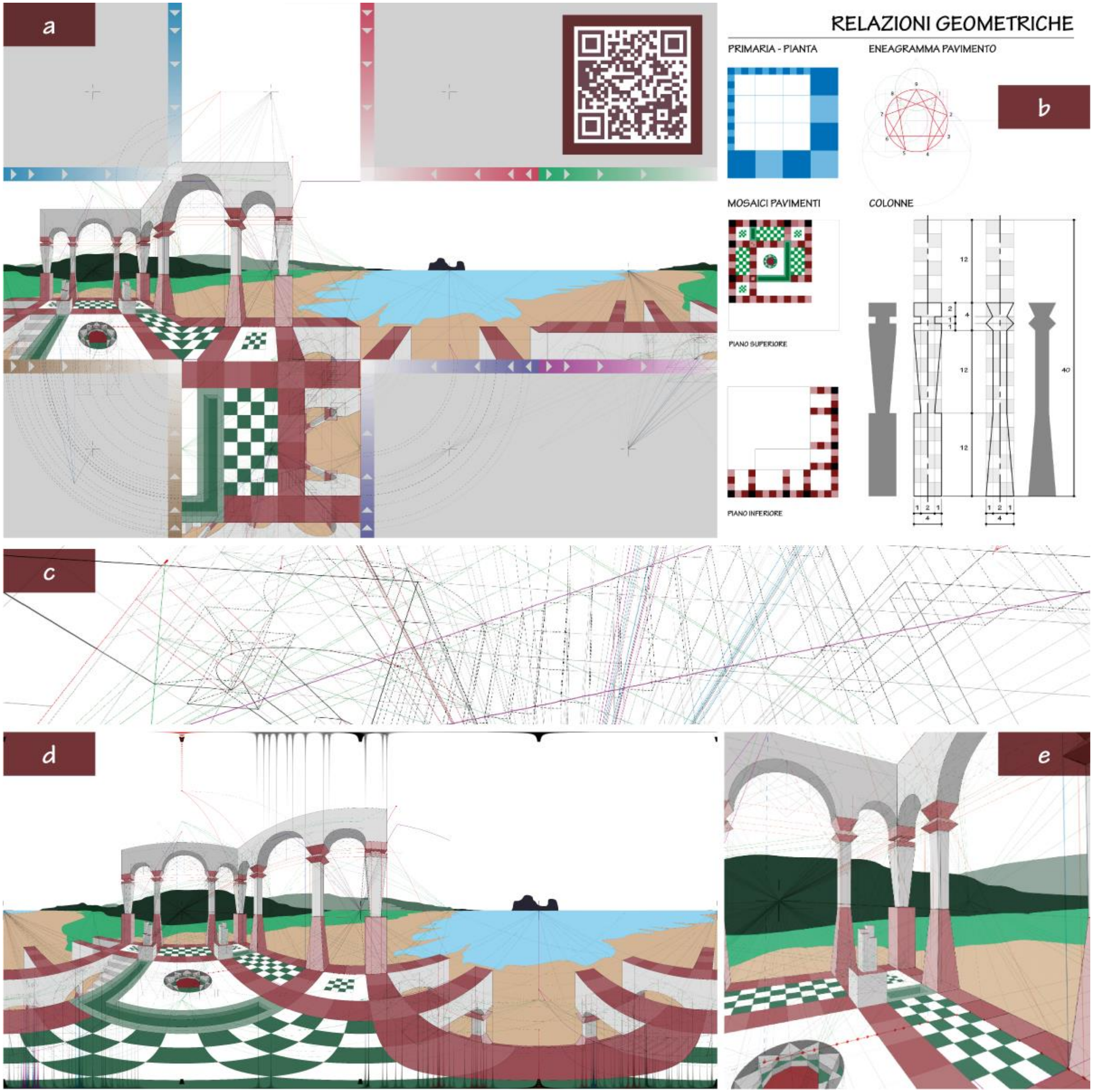

Figure 106. Exercise E VIII. a) Cubical perspective. b) Geometrical proportions of the project. c, e) VR navigation. d) Equirectangular panorama

V.3 SURVEY: SOLIEMENE'S CERAMICS FACTORY

In this second path the operator uses an HIM for surveying an existing architecture already having familiarised the basic knowledge of cubical perspective. The case study centres the Solimene's ceramics factory, located at Vietri sul Mare, Italy.

Here, the user highlights its own criterium while reading the space's geometric principles. Such a lecture structures the way that spatial elements relate to each other, being the key of both the personal lecture and a possible reference for projecting new ideas. Indeed, the HIM promotes the meditated exercise for the acquisition of the knowledge that shapes a given architecture or, in other words, a new and different way of reading the same reality in a **holistic approach**. Such notions are important for the professional development of designers, but they can also be used in further interventions as a support for the ideation process and collaborate within the dissemination of architectural survey as well.

The operator conducts two exercises: E IX, in which a) it extracts data from a panoramic picture, analyses the building and compares critically the results.

V.3.1 The building

Architect Paolo Soleri built the Solimene ceramics factory at Vietri sul Mare, Italy, in 1954. The construction represents an iconic example of the modern architecture organic trend, rising on the terracing of the cliffs to the northeast of Salerno's gulf. This architecture example had several surveys, giving as result interesting geometrical studies synthetised in models obtained from both direct and instrumental high-tech surveys [207]–[215]. This new survey collaborates to that state of art of the building by adding a new vision of the same reality, synthetised in personal lectures of modular laws. These lectures, though subjective, are methodologically possible to compare: the operator defines a criterium and writtes in a report.

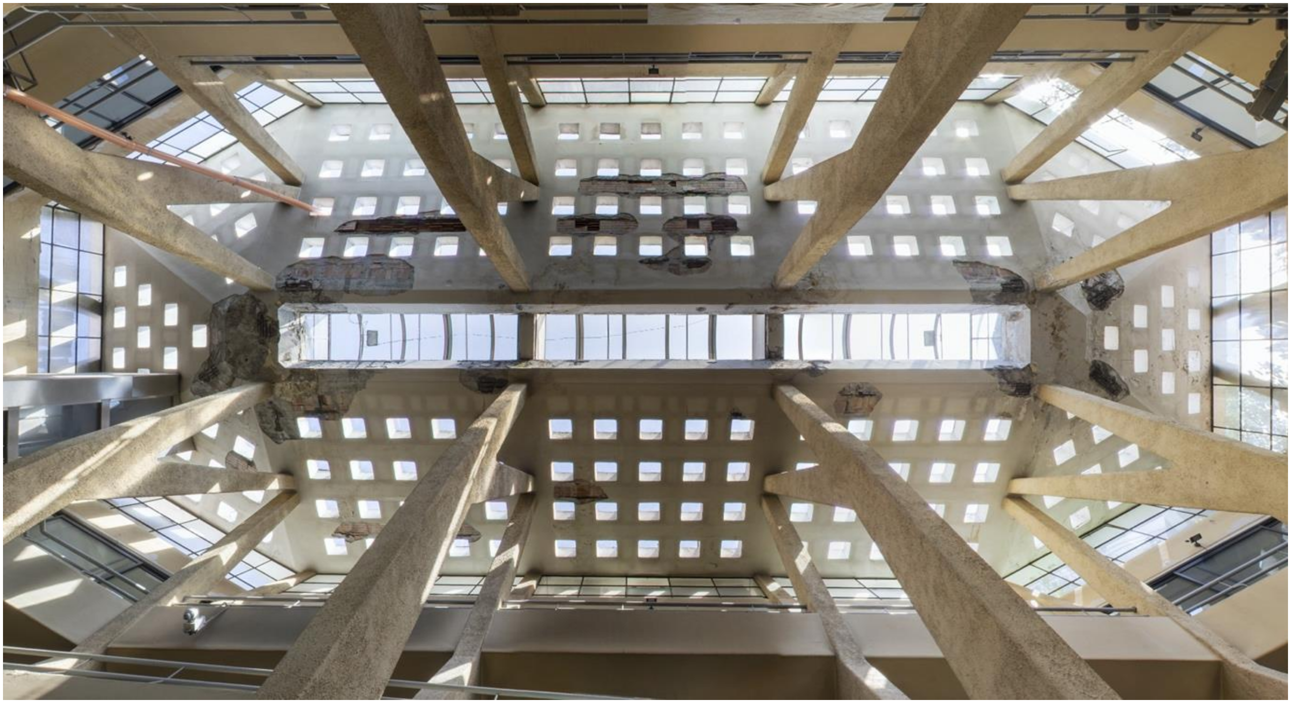

Figure 107 . Solimene's Factory (2019), Vietri sul Mare, Italy. Interior of the main space

The interior of the building is characterized by columns regularly repeated in pairs (Figure 107, Figure 108). Some elements such as these columns and the ramp around the interior space, add the last case of geodesic construction, synthetising good enough all the constructions from E I. At this point, the operator is able to combine and use the full casuistic of cubical drawing to represent a simple environment or a complex architecture.

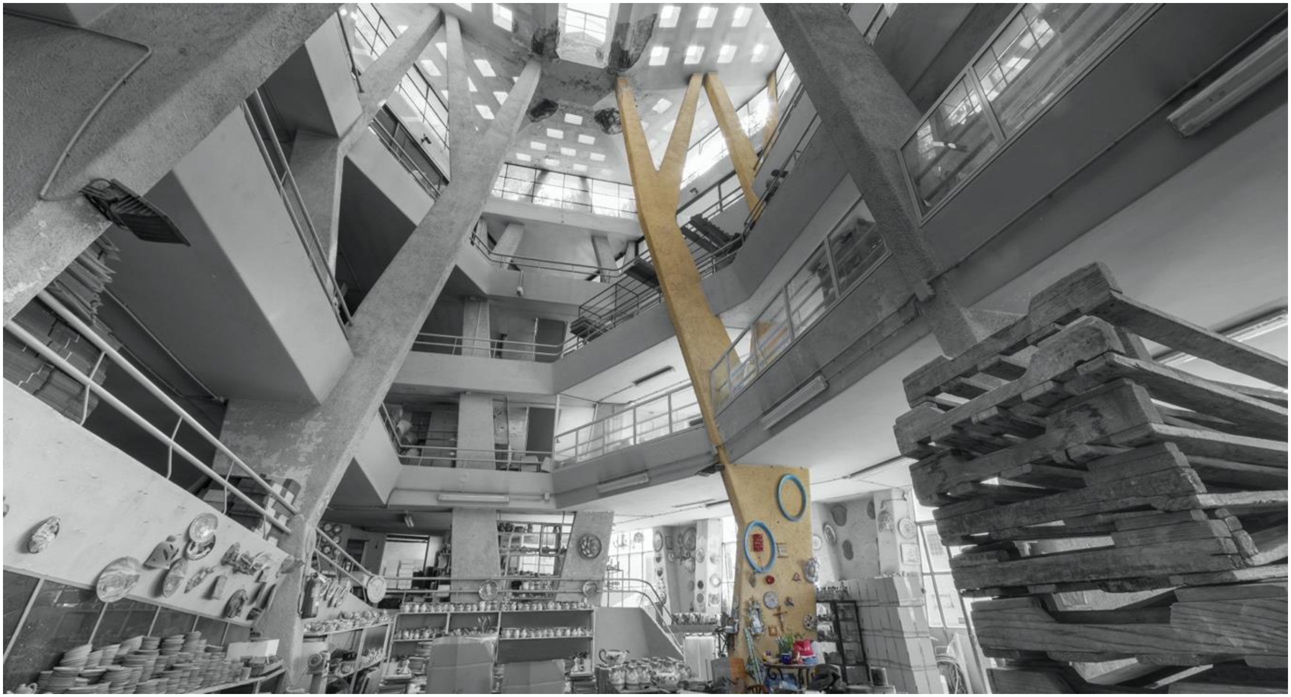

Figure 108 . Solimene's Factory (2019). Columns' detail

A singularity of this building is the direct correlation between its architectural shape and the products produced in it "*there is in this singular building a concept of circulation…that has always existed in using the potter's wheel to create and transform clay into a vase*" [216]. Its organic geometry will be useful to point out some particular problems on cubical representation, such as the geodesic construction from two points in adjacent faces, i.e., the most complicated case of lines non-parallel to any face of the cube.

V.3.2 Second path: from reality to VR

V.3.2.1 E IX

The operator receives a panoramic photography shot on-the-spot, from which it elaborates a base data tracing the picture and understanding the surveyed geometry. Then, it analyses the data searching modular laws of composition of the architecture's elements. Finally, it brings such a law to orthogonal views so to compare it with other available graphic documents, such as floorplan and sections.

Operatively for the first step, the user converts the given photographic panorama from the equirectangular to the cubical format. Then, it traces main elements of the space and applies the knowledge in cubical perspective so to verify (and eventually correct) the result. During this stage the user makes a personal and autographic selection of information (Figure 109).

Regarding the learning of cubical drawing, E IX introduces the most complex case of geodetic construction, which is when two points of a line lie in adjacent faces (see Case 2 of Second Part IV.5.3.4).

Note, that in the latter theoretical model, vanishing points and geodesics are a way of verifying both the first tracing and the material reality. Indeed, the graphic elaboration of this case study gave as a result an incoherence among columns. The operators' hypothesis[36] is either a partial demolition of a part of the column, or an incoherence in the used original floorplan.

Another part of the exercise leads to the extraction of the modular law that rules the analysed reality and its comparison with other available data. In the case study, the comparison was made with a floorplan made with traditional survey and a section got from a point cloud. Finally, the operator elaborates a written report with the followed criteria, so to document the

[36] Not verified yet

followed path. Such kind of reports are loaded in the final virtual tour of E X. To get such a modular law in the practice, the operator extracts a skeleton of horizontal and vertical planes and synthesises them in an orthogonal grid. Such a grid is re-presented in floorplan and section, highlighting the key in which the geometry was read (Figure 110). Once understood the rule, it is possible to apply the cubical perspective for studying new project possibilities, in harmony with the existing architecture.

A particularity of this exercise is the introduction of unknown difficulties, something inherent to the material reality of an architecture. For example, the no-correspondence among vanishing points of presupposed parallel elements (Figure 111 b, c). The case study is solved creating a new model by drawing a theoretical geometry using average values. To such an end, were used reference points got during the tracing and defined an ideal geometry by tracing new lines and using both 4-sides and 6-sides geodesics.

V.3.2.2 E X

The last exercise is the composition of a virtual tour, predisposing the content in a digital project of communication interactive through “hotspot”. Thanks to the hotspots, the operator connects environments and manage several other kinds of models, such as 3D models, plane pictures, texts, sounds, etcetera.

V.3.2.3 Summary

These exercises focus on the human capacity of releasing an element from its material aspect and keeping its geometrical shape. Thanks to this geometrical research through drawing, it is possible to deduce formative principles of the project idea. Furthermore, the obtained results can be punctiliously discussed since there is a criterium written and defined to follow.

Hence, cubical perspectives can be confirmed as a key to read, interpret or shape the space. The digital side of a HIM is used as a “*grimaldello*” (a picklock) to force the limits between the material and the immaterial. Digital and traditional techniques are not alternative but complementary means. The cognitive analysis of drawing results in a meditative exercise in which the hybrid model allows to touch the limit among what can and what cannot be done [217]. The hybrid model can be converted into a platform to organise informative systems, or also be a container for digital media interaction. Sensorial receptivity is enhanced in the virtual tour, thanks to the inside first-person navigation of the immersive perspectives. Thanks to the self-perception, the operator/creator can also become visitor, interacting with a collaboration space that goes beyond the geographic barriers and the cultural and linguistic diversities.

V.3.3 Deductions (from V.2 and V.3)

The operator passed from an external and centrally static position to the immersive fruition by learning and applying cubical perspective. The former refers to the draughtsman's position during the cubical perspective elaboration. The latter concerns the observer first visiting and then critically analysing, zooming in the details of the environment. Hence, the mechanical eye of the camera replaces the empirical point of view.

We pass from a traditional scheme to a contemporary way to see the reality, or in other words, from the perspective frame to the use of prosthesis (elements external to our body). Such a way of perception put us in the ambiguity perception where physic and virtual world get mixed. With this door open, it is now lawful (indeed *dutiful*), to witness the changes that such a mix will bring from now on into our perception's essence.

Quatremere de Quincy (1788-1825) exposed how one trains itself to acquire eye rightness and faculty in execution by copying an original (real-life drawing) [218]. On the other hand, the notion of the project is associated with copy and therefore, with the interest in imitation. But imitation requires talent and intelligence and is structured by rules and a manneristic way. The former is the modular law found by measuring an observing with a new eye the same architecture. The latter are the reasons behind those measurements, the personalised criterium for selecting some or another information and the planner's criterium from which the building is the consequence. Hence, the exercise of tracing an architecture from a photographic survey and its consequent elaboration, helps and guide the imitation, the gathering of visual data, and it stimulates the brain reading compositive criteria.

Both case studies focus the limit on which the hybrid converge stratified ways of consuming ideas. This is one interesting aspect (a part of the creative possibilities of mimetic nature). In this position, traditional and innovative techniques interlace to reveal the renewed aspects inside the "stereoscopic" vision of Virilio (see First Part II.4). Looking carefully after a first "immediate amazement", the exercises bring the operator back to reflect on the conscious or unconscious sensitive mediation with the external reality. Considering the current use of the smartphone, television and the Internet, it is not difficult to agree that the experience of being simultaneously in many places is typical of our times [156].

In this optic, known spaces get unusual when one sees what normally do not. This fresh vision creates operations in which theoretical and empirical visions collapse, two different appreciations[37] [159]. In other words, there are two different visions from the empirical and the theoretical ego. The representation joins and mixes them in different ways according to the circumstances. Such a dual perception is already evident in Durer's well-known painting[38]: the draughtsman's eye looks, the object is watched, the painting captures mimetically the impression, but there is also an external eye that gives the immersive vision of the represented scene. This last external eye is represented nowadays by the "video camera" from which one navigates digitally. Thanks to - and in virtue of - the current digital procedures, one can produce, disseminate and compare these hybrid experiences with new potentialities.

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[37] If not opposite appreciations, even if they normally are considered as equivalents
[38] See Albrecht Dürer, method of perspective construction from "*Underweysung der Messung*" (1525)

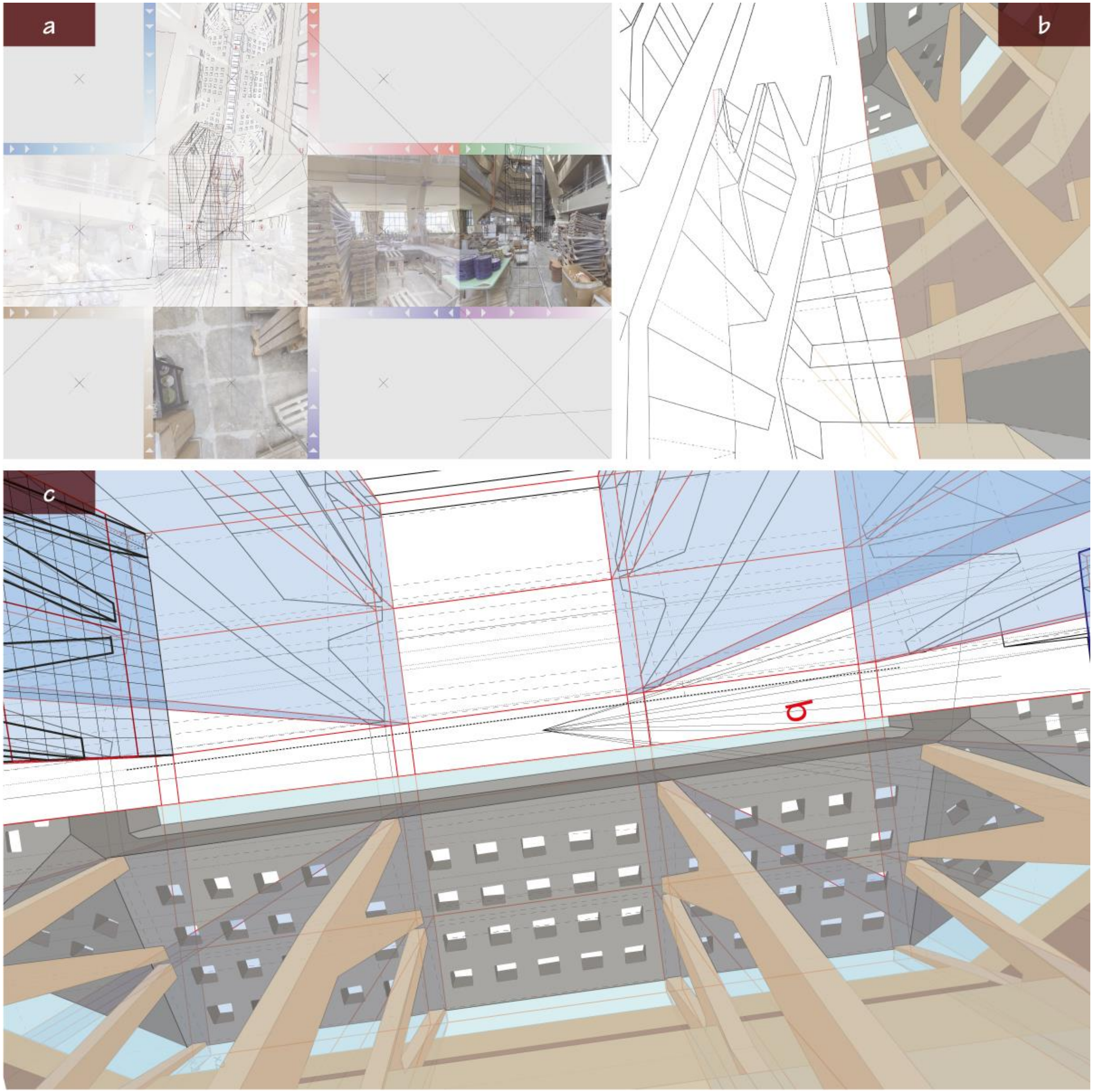


Figure 109 . Exercise X: a) Photographic panorama converted and traced with vector-based tools. b) VR comparison between structure and textures. c) Immersive view of a re-modulation proposal (2020), by Assia D'Alessio, Teresa Di Palma, Caterina Crispino, Marta Campanile, Maria Petrillo and Lorenzo Villani, T.A.R. Unicampania a.a. 2019-20 (prof. Adriana Rossi)

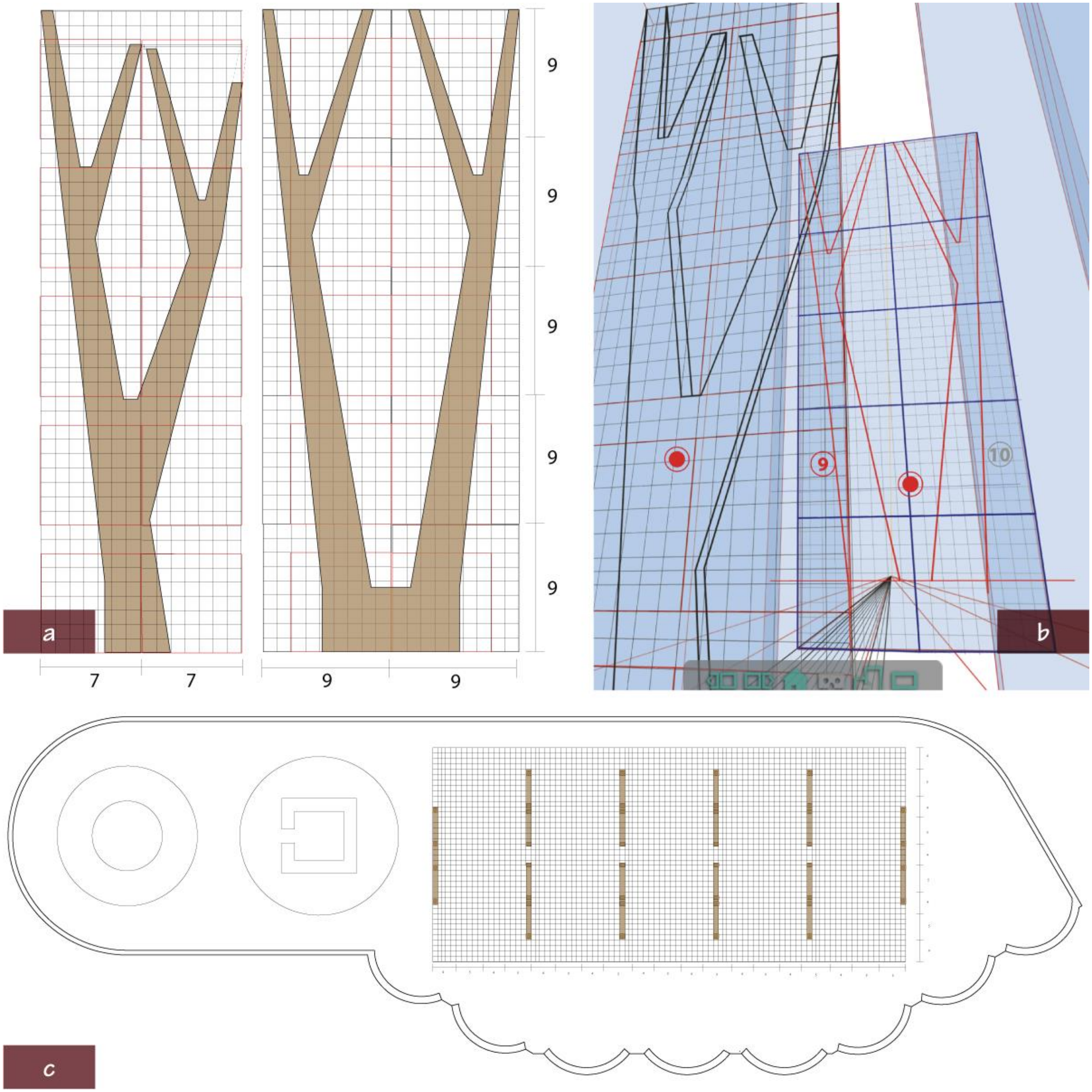


Figure 110 . Exercise XI: Column's modules in section (a) immersive view (b) and floorplan (c) (2020) by Assia D'Alessio, Teresa Di Palma, Caterina Crispino, Marta Campanile, Maria Petrillo and Lorenzo Villani, T.A.R. Unicampania a.a. 2019-20 (prof. Adriana Rossi)

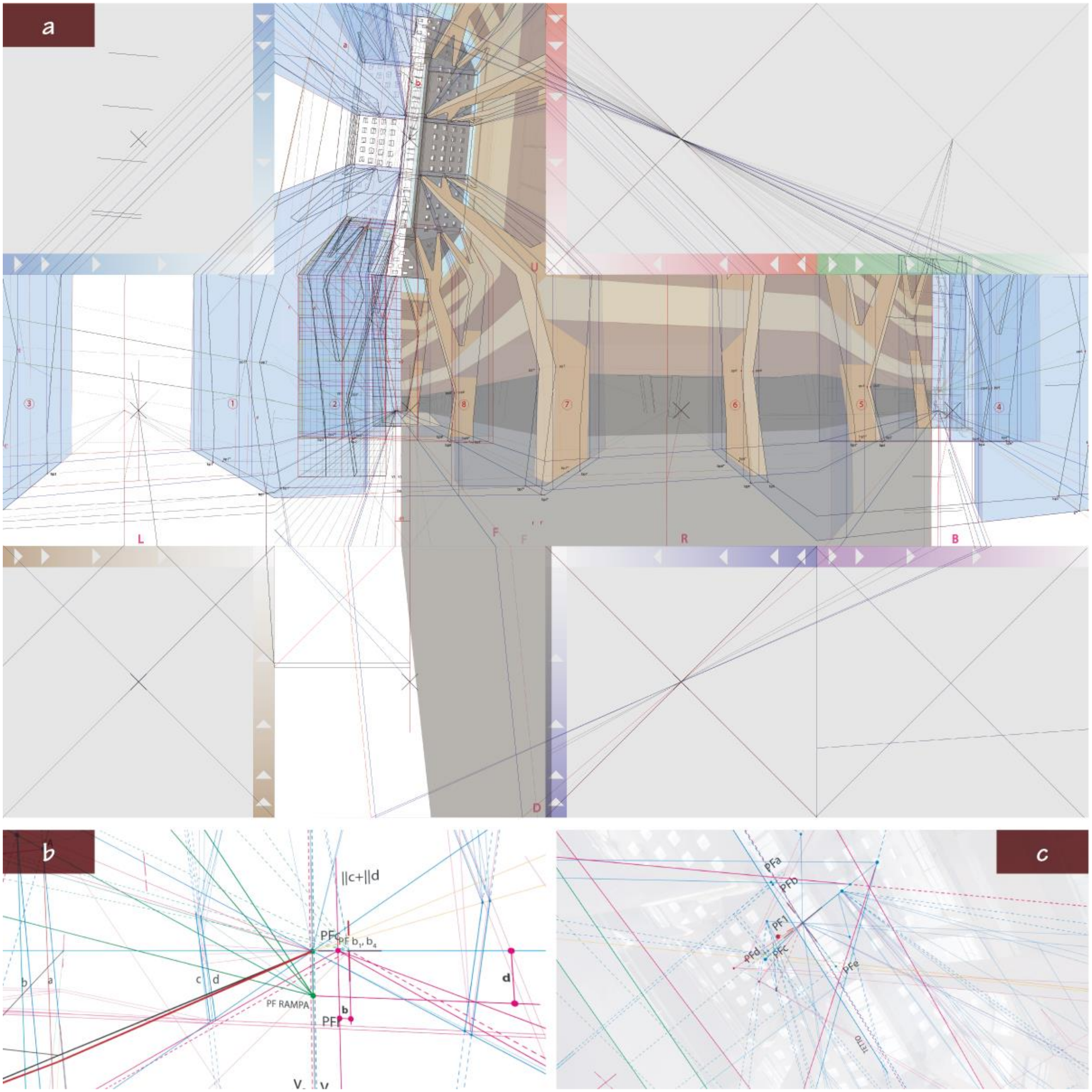


Figure 111 . a) Full construction: geometric analysis, vanishing points, proportions, geodesics and solid textures. b, c) Immersive view of average geodesics and selected criteria (2020), by Assia D'Alessio, Teresa Di Palma, Caterina Crispino, Marta Campanile, Maria Petrillo, Lorenzo Villani and Lucas Fabian Olivero, T.A.R. Unicampania a.a. 2019-20 (prof. Adriana Rossi)

V.4 SURVEY AND RECONSTRUCTION: ST. MICHAEL'S CHURCH IN HILDESHEIM

In this case study I use immersive representation as a contribution to document St. Michael's Church in Hildesheim, Germany. I synthetise the geometrical structure through cubical perspective, reconstruct the space with the available documentation, and visit the place remotely through VR. In a further step, such geometry may be used as a base for new project hypothesis and lectures of the building.

V.4.1 The building

St. Michael was consecrated in 1022 and is part of the UNESCO World Cultural Heritage. After a history of destructions and reconstructions, fires and WWII bombings, it still conserves many key elements from middle ages and in particular from the Romanesque period. It represents an invaluable heritage with inscriptions and artefacts from its foundation [219].

The church's geometry is characterised by a squared unit of measure (Figure 112 and Figure 113). The main nave measures three squares, alternating squared pillars with pairs of round columns. The width of the crossings measures one square. The capitals of the round columns are the intersection of a cube with a sphere [220].

V.4.2 Construction of the scene

The construction is made using the available graphic documentation. I start taking bearing λ and elevation φ angular measurements of characteristic points of the module. Thus, I plot them in the cubical map by means of section Second Part IV.5.3.5. I chose points A' and B' such that point A' is in the upper extreme of a first column and point B' in the bottom of a contiguous one.

Then, I iterate the module applying the method for repeating regular elements using geodesics and vanishing points (see Second Part IV.5.5) since I want to plot equally distanced elements such as the columns of Figure 113.

After having plotted references for the main elements, I added further details such as the organ, arcs or other complex geometries using orthogonal grids from Second Part IV.5.4.

In particular, columns can be reconstructed from the graphic documentation extracting transversal sections. Each of these sections can be thus vanished to the central point and used so to define a circular shape in depth through characteristic points of the circle (Figure 114).

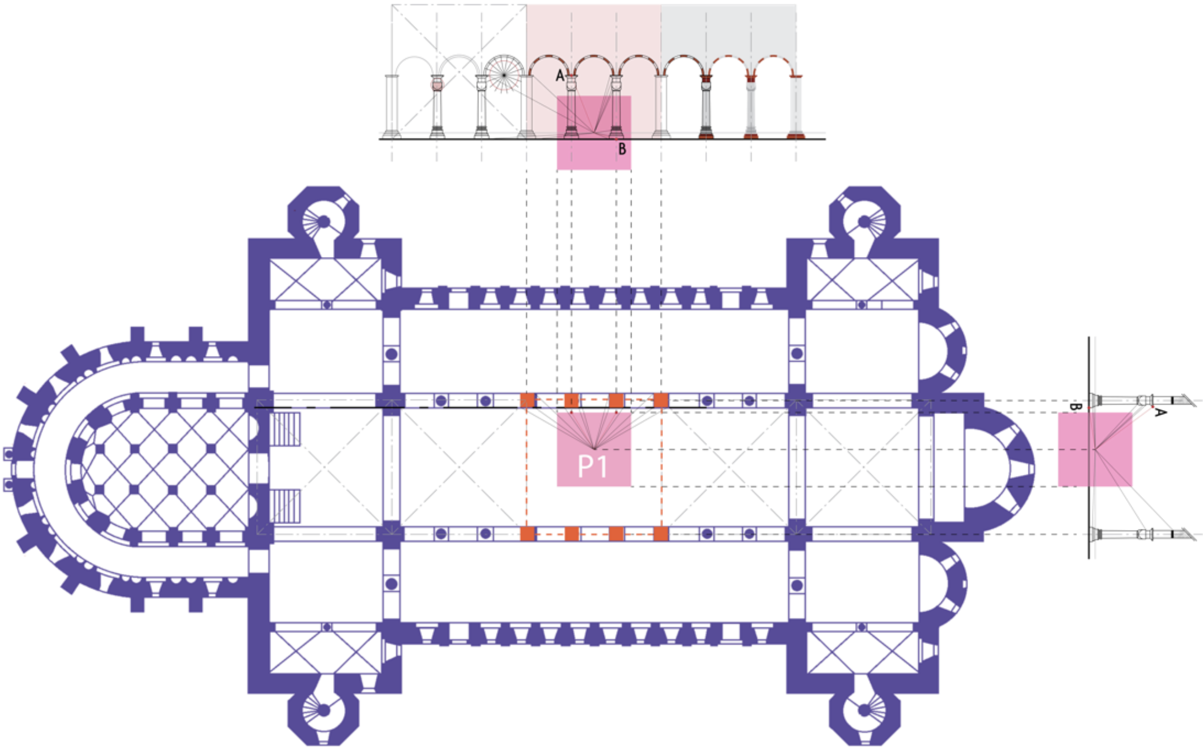


Figure 112 . Floorplan of St. Michael in Hildesheim (bottom). Image distributed under Creative Commons License [221]. Section and graphic elaboration by the author

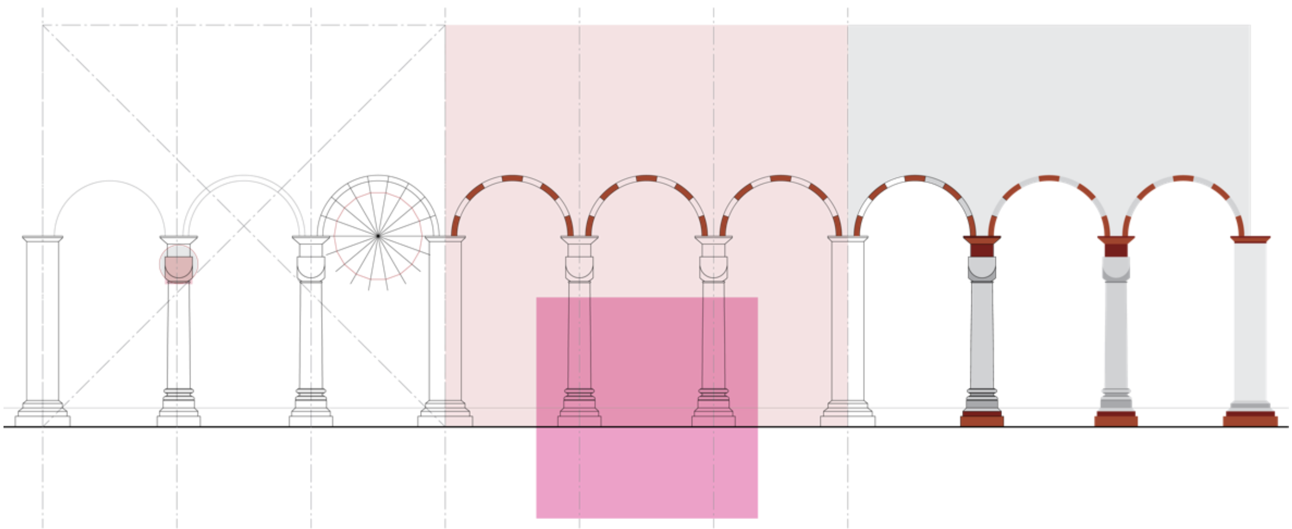


Figure 113 . Schema of the squared unit of measure

V.4.3 Deductions

The final immersive drawing preserves the linearity of classical perspective but with the elegance of all lines having exactly two vanishing points (Figure 115).

Note that constructing the same scene using classical perspective will put the vanishing points of diagonals and lines outside the drawing (sometimes by quite a lot). This worsens as the angle with F vanishes. Instead, using geodesics, I can draw the scene in a compact way by using whichever of the two vanishing points is more convenient, as is guaranteed that all vanishing points and their construction diagrams are inside the bounds of the drawing. This makes cubical perspective an efficient way of drawing not only immersive views but even some wide views obtainable but awkward in classical perspective.

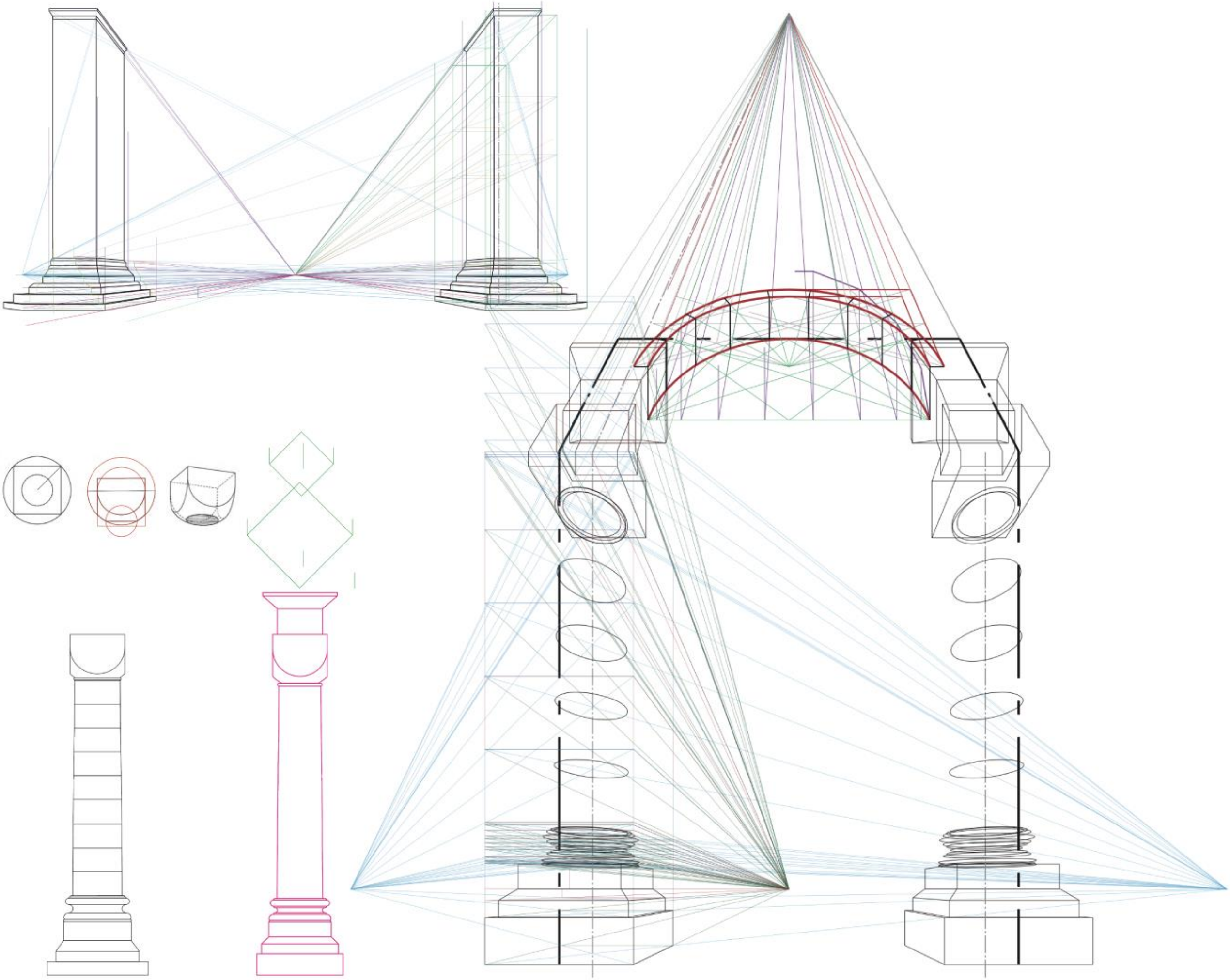


Figure 114 . Columns' geometric relationship and composition using vanishing points

The case study illustrates the application of cubical perspective to architectural survey of heritage sites and introduces practical aspects of the perspective process. I also highlight the facility for drawing given by following the second method and using shortcuts. Regardless the columns' fragmentation, the VR visualisation verifies the correctness of the anamorphosis' construction showing the right shape within the linked vision (Figure 116).

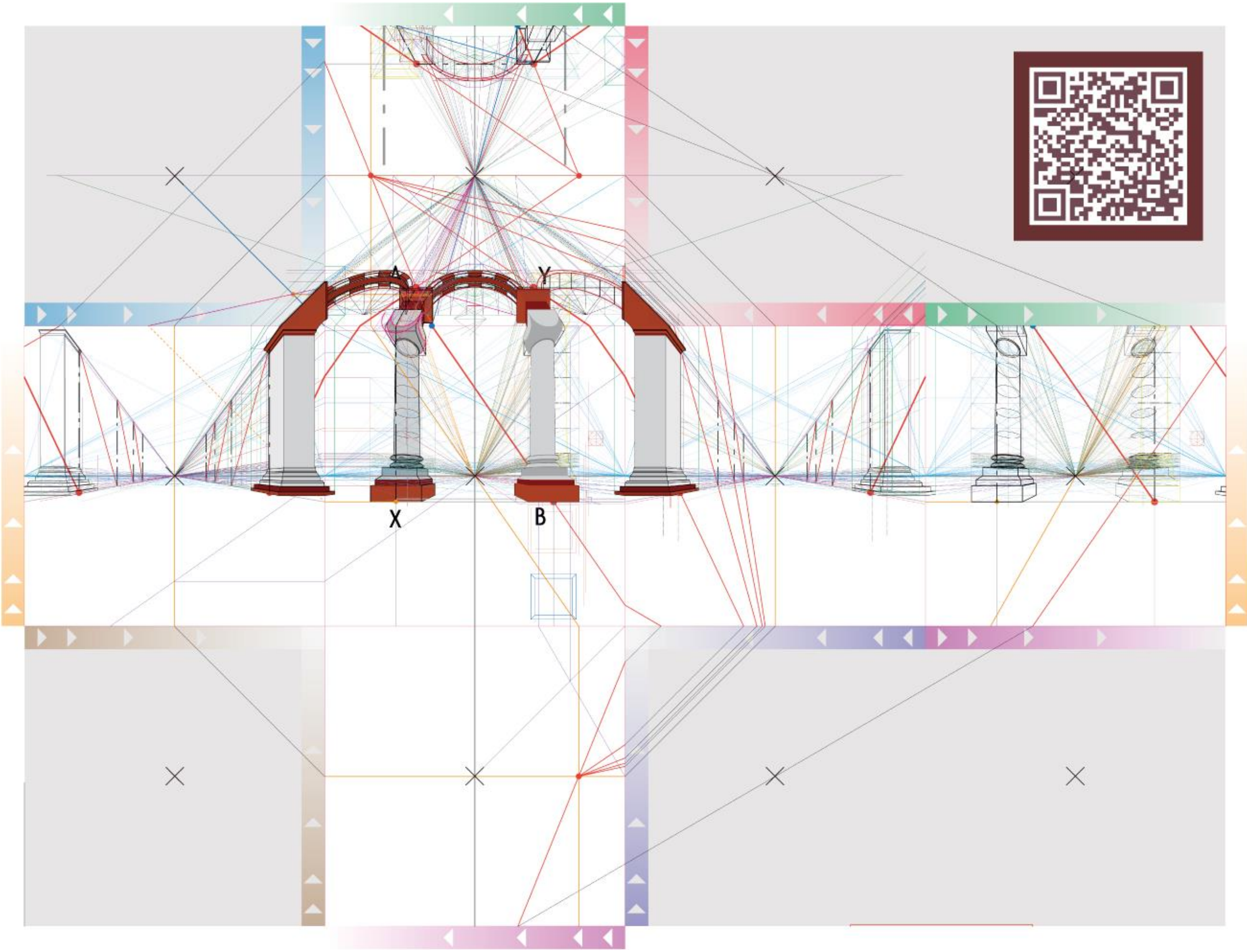


Figure 115 . Final reconstruction

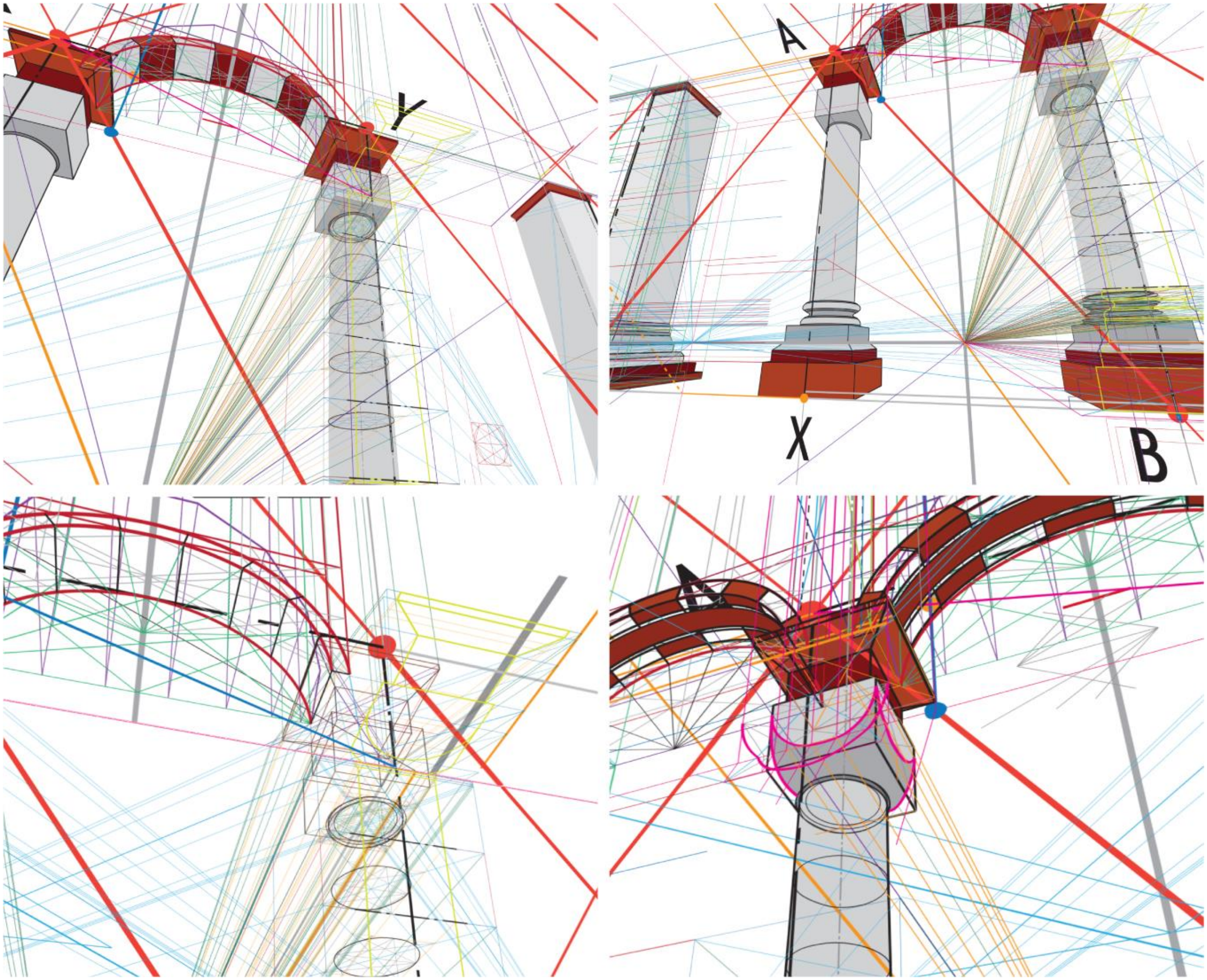
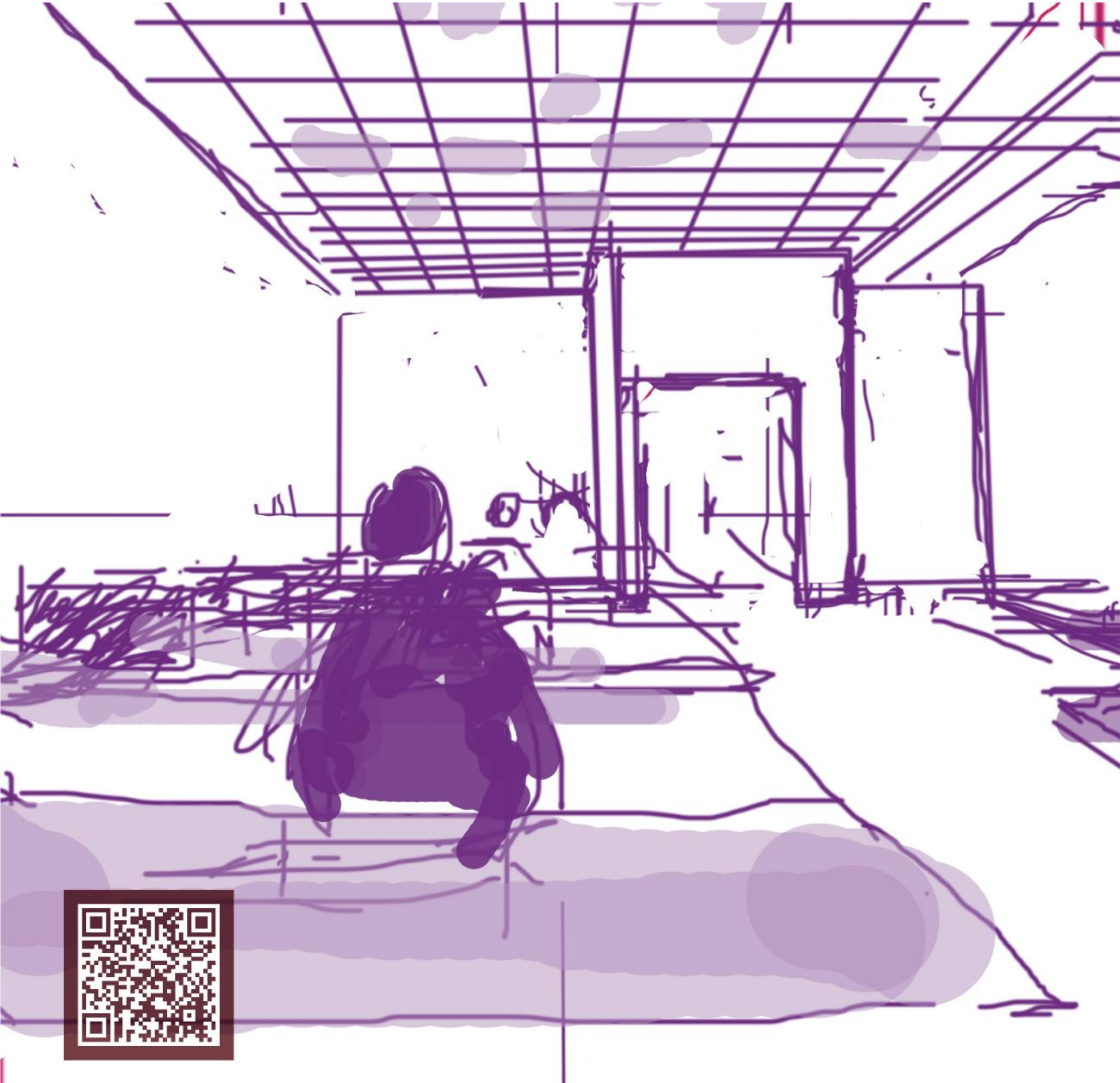


Figure 116 . VR navigation. Details of the geometrical construction and modules



CHAPTER VI OTHER APPLICATIONS

VI.1 PRODUCT DESIGN

After its creation, a product is normally inserted in a real environment, interacting with physical context and users. Such relation between object and environment can be tested during the early conceptual design using immersive representation. If the product designer understands the relationship object-environment, then it will be less likely for the product to unfit or result useless to some context.

Designers can project both object and environment when they are yet an idea using HIMs. This way, product and context are thought contextually, the one in relation to the other. To such a purpose it also contributes the fact that these drawings are normally made from the final user's point of view, which stimulates the perception in a daily-life logic.

In the same line of thought, immersive representation can be useful for architects to understand the building in its context with other buildings. Also, landscape architects and urbanists can use it to analyse the urban project within the surrounding landscape.

VI.1.1 Exercises

The program of exercises previews the creation of environments using HIM with cubical perspective and the construction of a final virtual tour with them.

First, the operator is introduced with essential definitions, such as HIM definition, the basic principle of cubical perspective, the definition of anamorphosis, etcetera. Then, since the content targets designers who are not always prepared and familiarised with classical perspective, the operator follows the simplest case: a series of exercises that focus on the representation of horizontal and vertical lines. In such specific case, the basic principle can be reduced to a simple rule: when the representation of a line is segmented, if one of those segments does not vanish in one face, it will do it in the next one (and vice versa). When a segment vanishes, the vanishing point will match the geometrical centre of the face where the segment belongs to. The case study consists into build five environments: a free construction in cubical perspective, a full spherical picture, a tracing of the previous shot, another tracing on a panoramic picture (which in this case is given), and finally an environment created to show a specific product.

VI.1.1.1 Room 1

The neophyte creates a free room following the basic principle (Figure 117). For a better comprehension of the principle, the operator may use the artefact presented in Second Part IV.2.2, the two first practices from the introduction of V.2, and exercises E I to E IV. During the case study application, one of the operators found an interesting exercise: how to compose the anamorphosis of a column that results fragmented in four faces? The followed reasoning (as previously made) consisted into taking minimal references from floorplan and section, bringing them to the cubical map and then using 45° vanishing points (Figure 118).

VI.1.1.2 Room 2

This exercise introduces the neophyte to panoramic pictures made with spherical projections. In particular, the operator may use free source apps, such as Google Street View and shot with its own phone. These apps allow the acquisition of full panoramas by shooting several individual pictures within the full visual sphere around the observer. The single pictures are then stitched automatically in equirectangular format. In a second step, the equirectangular format is converted to cubical map using, for example, *Hugin*.

VI.1.1.3 Room 3 and 4

These rooms are tracings of two panoramic pictures: the one got for Room 2 and a panorama taken with advanced instrumental and controlled conditions of shooting, i.e., using a reflex camera, setting manually the different parameters of acquiring and operating an automatised head such as Gigapan previously calibrated for a controlled parallax effect.

Operatively, the operator uses a vector-based software and traces on top of both pictures, making its own selection, that is, as in E X from the previous case study. The aim of this exercise is to train the neophyte in the critical selection of information: while the photography captures all the visual information, the traced drawing contains just the wireframe structure of objects and buildings and schemes of proportions and relations among elements. The exercise was also helpful to have a first contact with the representation of curved and more complex objects.

VI.1.1.4 Room 5

The operator must apply the practiced knowledge for creating an exhibition room oriented to the promotion of a product. In the case study, the neophytes used artworks of M. C. Escher, taking inspiration from a previous visited exhibition.

Figure 119 shows an output example where the operator applies Escher's influence in the formal language and textures of the room. Particularly, this example also shows optical games within the floor and mixes the exercise of the columns divided in four faces (Figure 119).

VI.1.1.5 Virtual tour

Finally, it is requested the design of a virtual tour, intended as graphical/informative product. To such an end, the five environments are converted from cubical perspective to equirectangular and inserted in the virtual visit. Then, according to the orientation and definition of the virtual tour, the operator adds interactive elements, such as a menu, welcome screen, navigation bar, hotspots, videos, 3D models, sounds, etcetera. All these media are accessible through hotspots sometimes personalised as well (Figure 120).

The final output is an HTML file and an associated folder containing scripts, XML and media files. All the media information (graphics, videos, sounds) can be navigated interactively while visiting the virtual tour. During the navigation, the technology of tiles allows a full quality navigation of the panoramas using few resources of the computer thanks to the use of the JavaScript three.js.

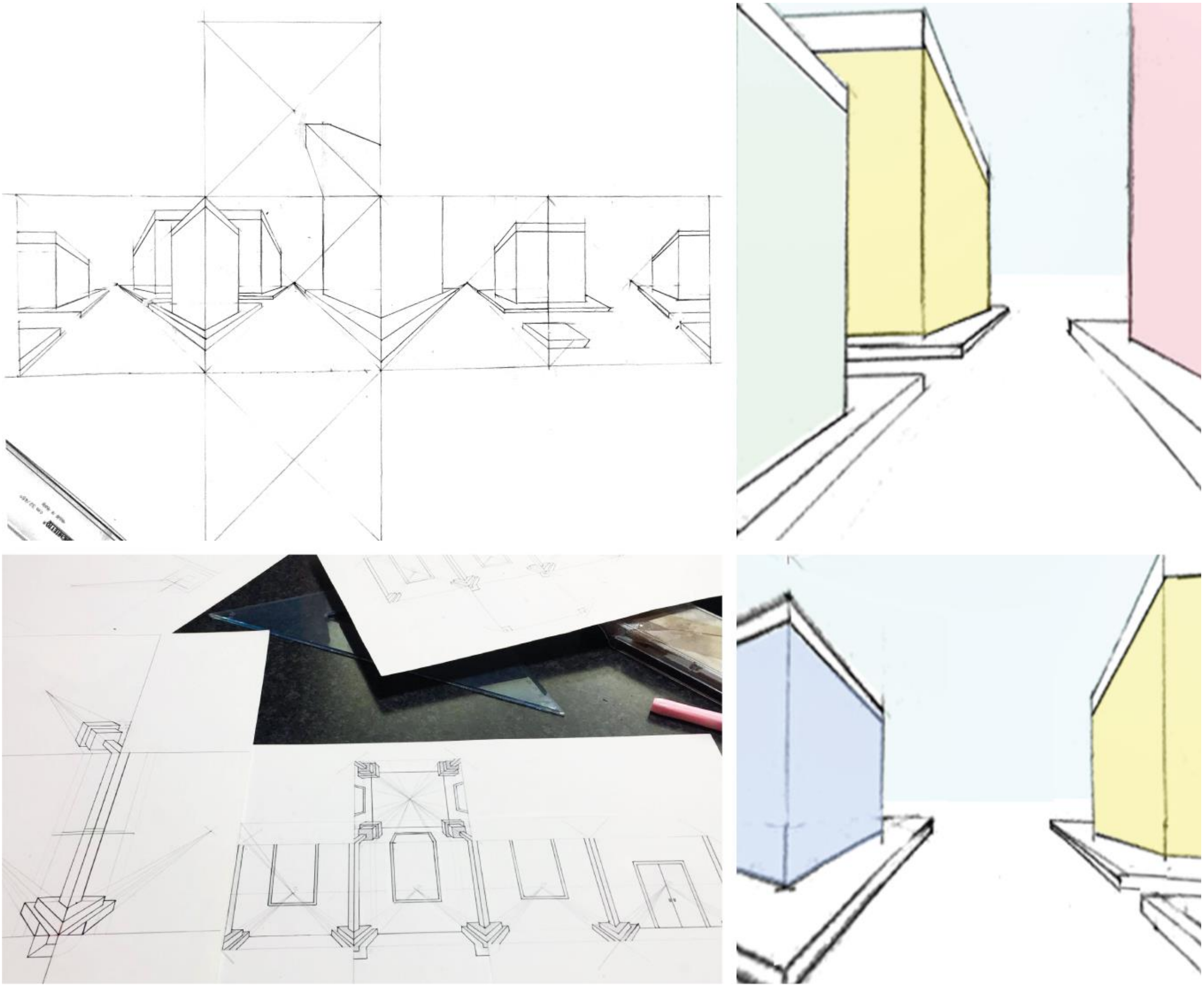


Figure 117 . Handmade analogical drawing made with ruler (left). Digital colouring and VR navigation (right) (2019), by Serena Saggese and Giuseppina Rao. T.A.R. Unicampania a.a. 2018-19 (prof. Adriana Rossi)

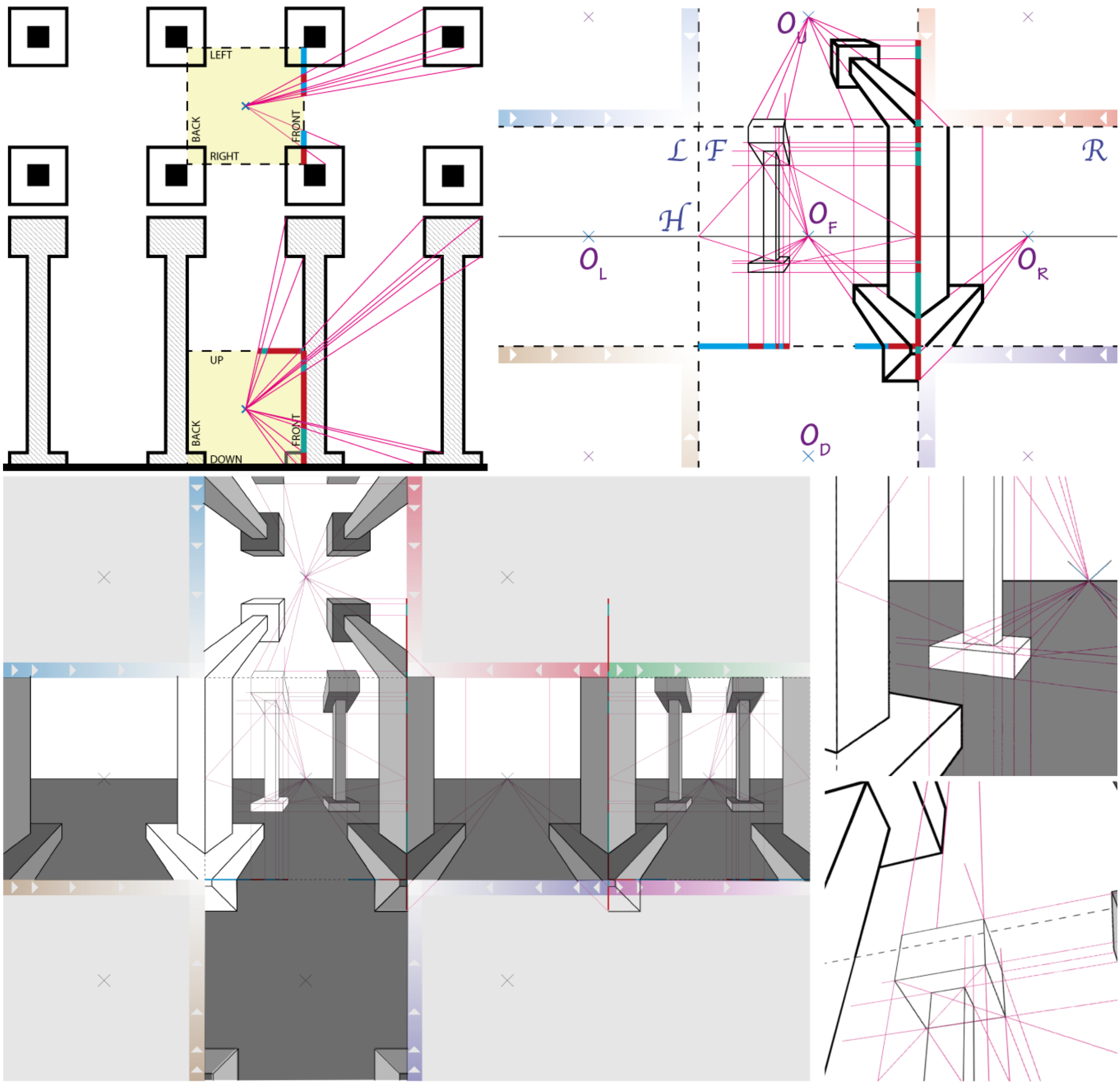


Figure 118 . Exercise of representing a column which image results fragmented in four faces

Figure 119 . Room 5 inspired in M. C. Escher's artworks (left). VR navigation (right) (2019), by Giuseppina Rao. Note the result of the anamorphosis in the floor and the columns, which are divided in four faces. T.A.R. Unicampania a.a. 2018-19 (prof. Adriana Rossi)

VI.1.2 Deductions

HIM results useful for representing the extended relationship between object, building and landscape. It proposes an immersive navigation made with simple tools and from the final user's point of view. Furthermore, the final informative model supports the sharing and interaction with other kinds of documentations, such as a 3D models, videos, texts, etcetera. According to the case study operator's specialisation, the representation is more "object-oriented", adding one more functionality to the space-oriented utility.

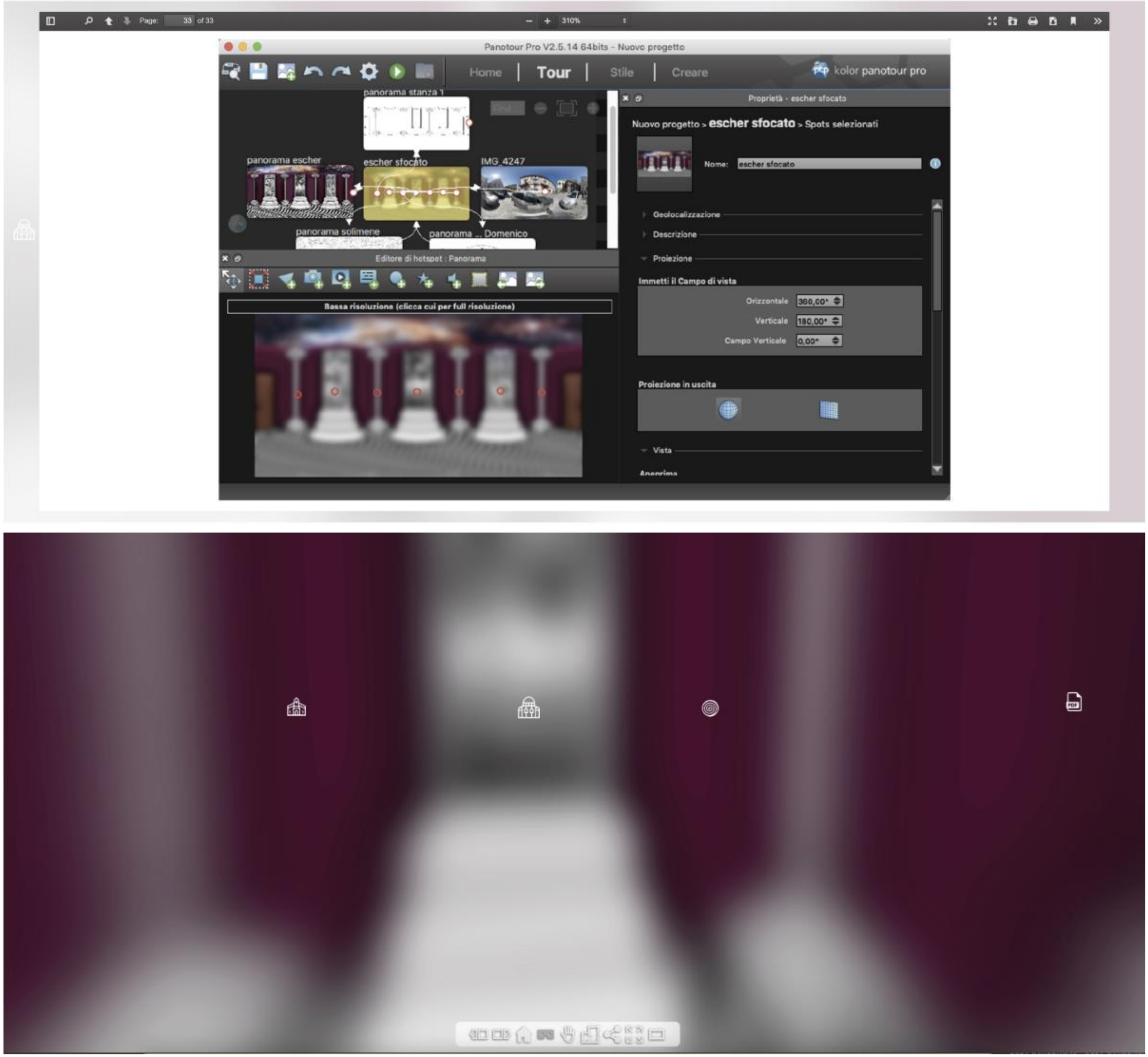


Figure 120 . Virtual tour setup and media content addition (up).
VR navigation (bottom) (2019), by Giuseppina Rao. T.A.R. Unicampania a.a. 2018-19 (prof. Adriana Rossi)

VI.2 COMPUTER SCIENCES

In this chapter I try to narrow down the knowledge in the field of computer sciences, developing essential dynamic and interactive practical applications. I introduce algorithms for the practice of cubical perspective, made with the theoretical definitions of the Second Part. The intention is not to give full solutions, I could not since this research has not such a goal, but I can open the door to IT specialists that may find stimulating to code a complete program. On the other hand, these small solutions help neophytes to learn small concepts one at a time.

VI.2.1 A first algorithm

This first algorithm results a corollary of the first method's application (see Second Part IV.4). It organises the general workflow for defining a cubical representation obtaining the input data from floorplan and section (Figure 121).

It uses the vanishing points of 45° diagonals to find the right height and position of every element. Thus, it performs a double verification using correspondences given by rays OP_x (

Figure 122).

I applied the "program" in an ideal case study composed by two buildings with the same height located in the front and back of the observer (Figure 123). The general setup of the example includes also fragmented representations, that is, contained in more than one face of the cube (

Figure 122 and Figure 124 left). The immersive visualisation confirms the correctness of the whole anamorphosis (Figure 124 right).

Note that this algorithm solves lines with vanishing points at the centres of the faces, that is, lines parallels to the faces of the cube. Nevertheless, the same logic could eventually be applied for non-parallel lines.

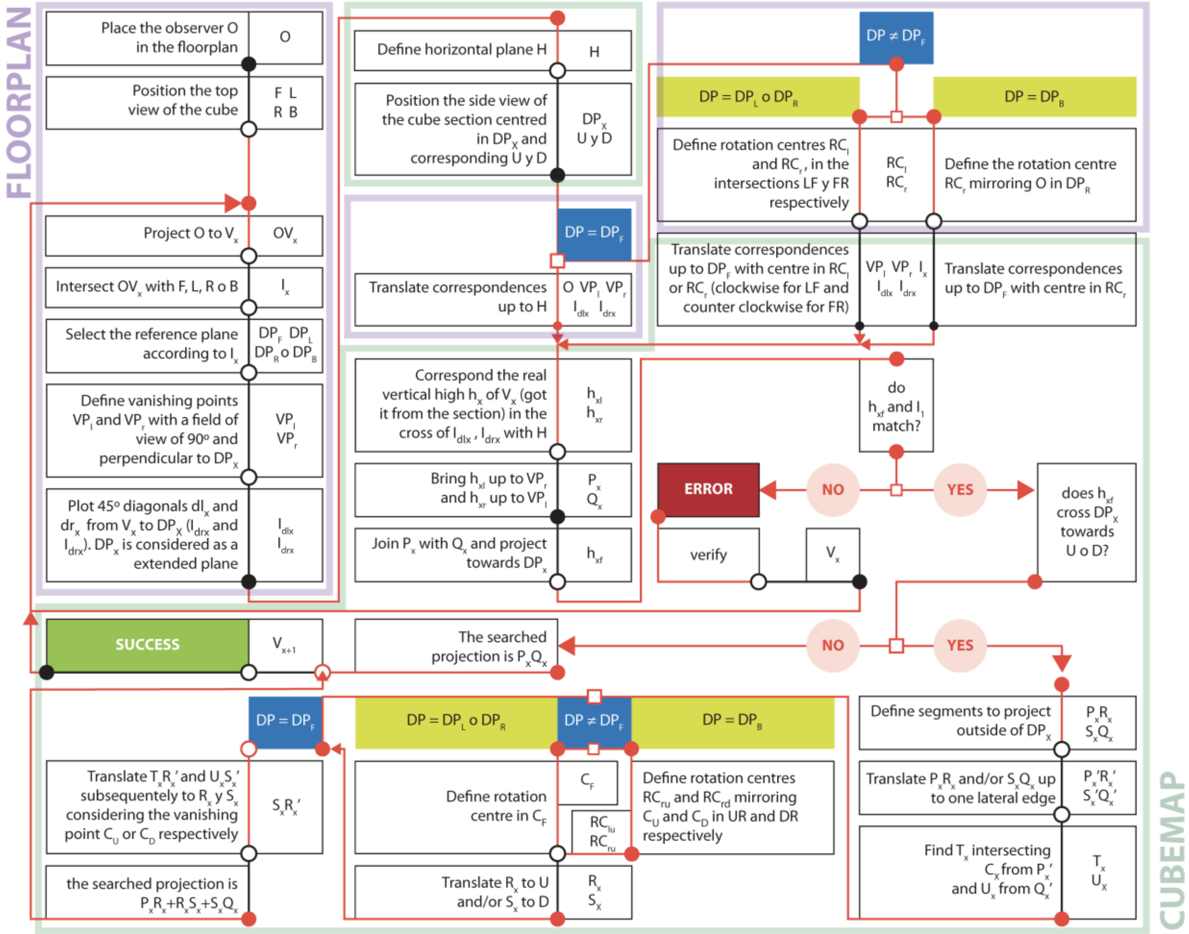


Figure 121 . General layout of an algorithm for horizontal and vertical lines parallel to the faces of the cube

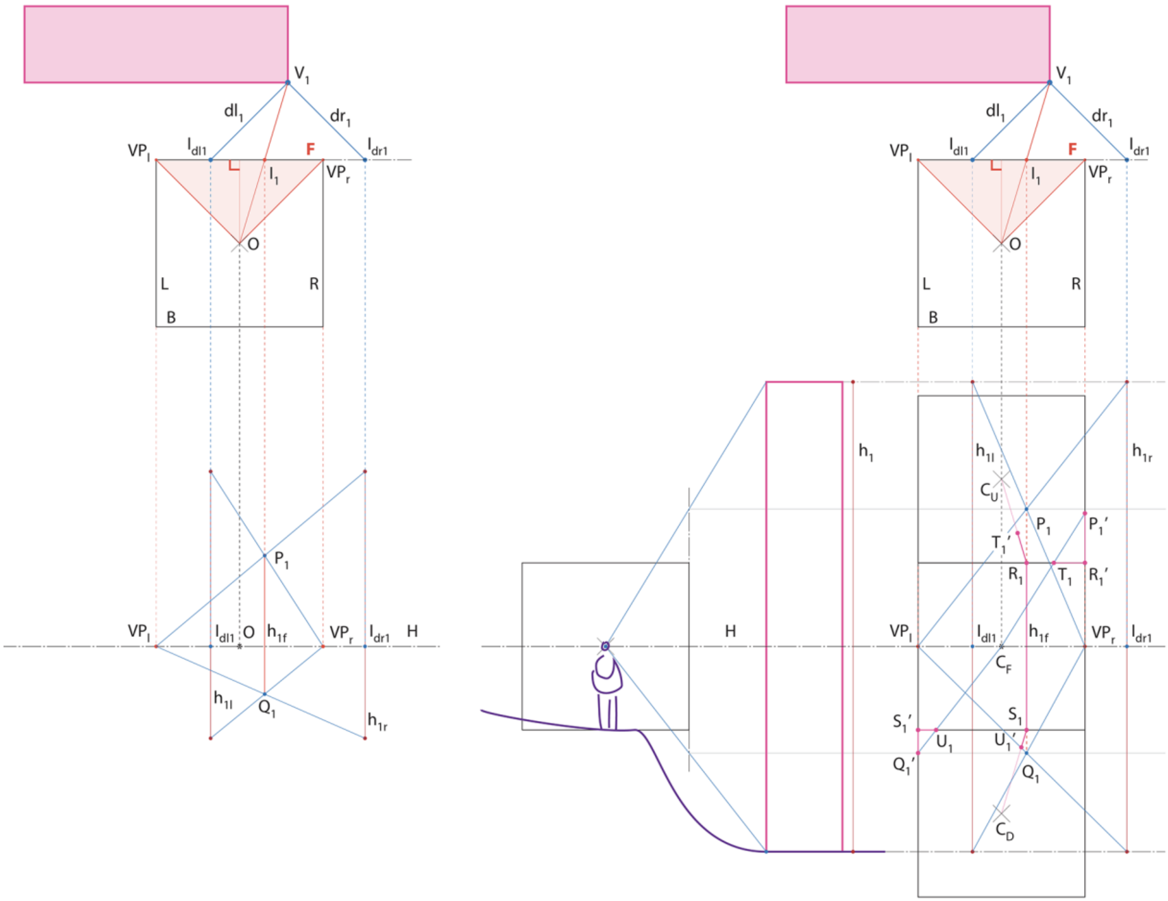


Figure 122 . Application of the algorithm to a case study. Solution of a line contained in one face (left) and of a line divided in three segments (right)

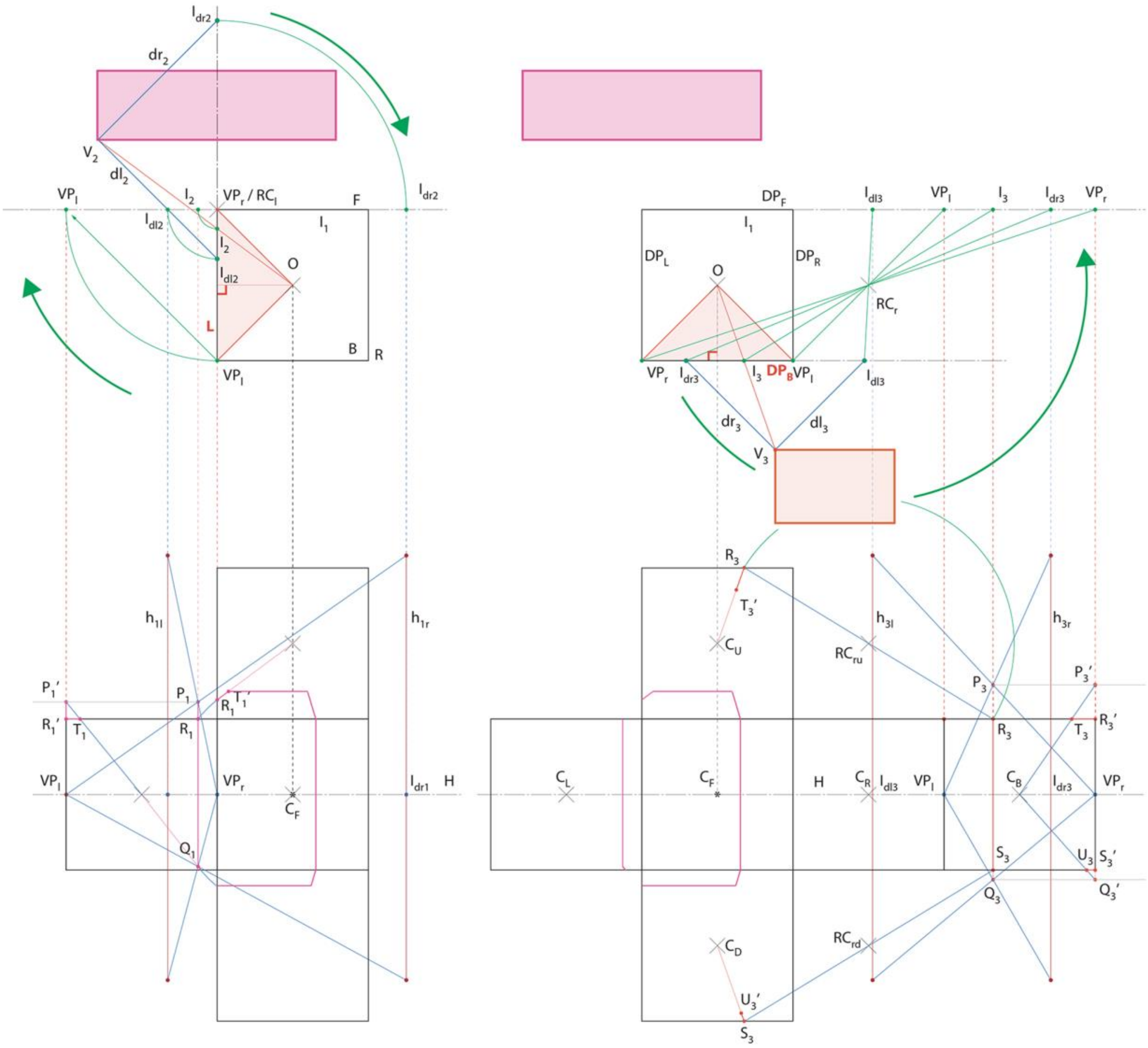


Figure 123 . Applying the algorithm to vertices positioned in planes different than the frontal one: projection in L and R (left), projection in B (right)

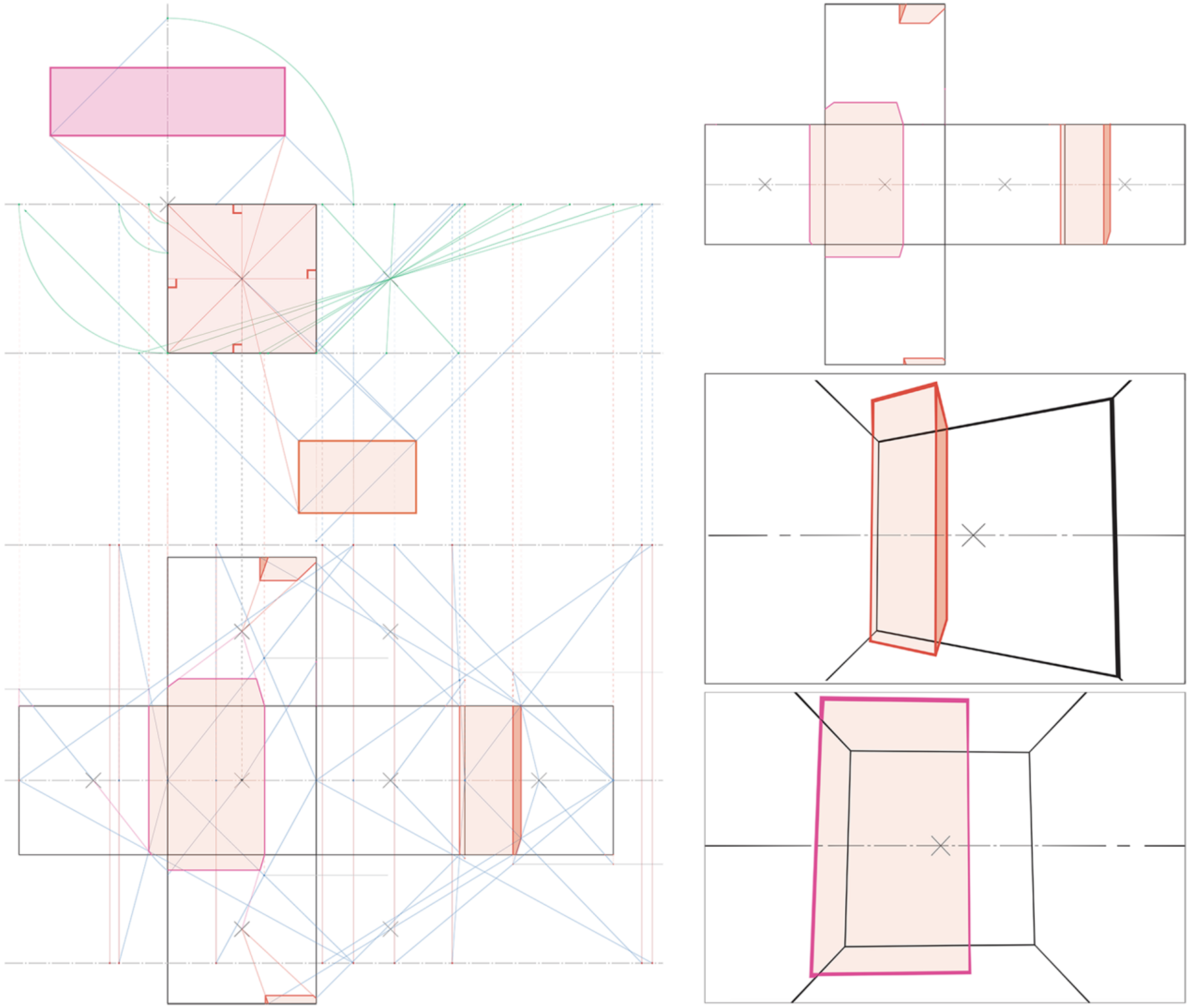


Figure 124 . Resolution of a case study applying the algorithm (left). Final drawing and VR navigation (right)

VI.2.2 Geogebra algorithms

These algorithms show constructions made with graphic-mathematic interfaces. In this case, I used *Geogebra Classic 6*, a free source dynamic mathematics software [222]. One interesting advantage of using Geogebra is the possibility of integrating JavaScript codes.

VI.2.2.1 Solving automatically a geodesic

This script orients to solve the most difficult case for finding a geodesic, that is, when we have two points in two adjacent faces. It gets automatically the auxiliary point and draws the geodesic between them. To do it, I created a personalised tool that requires as input two points and two faces. Then the algorithm constructs the shortcut defined at Second Part IV.6.2 and calculates of the position auxiliary point in the shared edge. Finally, it joins the auxiliary point with the two input points (Figure 125).

VI.2.2.2 Spatial and flat equivalences

A further programming introduces a tool for inserting a point with already pre-formatted code. In such a way, it is possible to define a priori some properties of the point. When the user inserts a new point, a code built in JavaScript assigns correspondences between three different viewports: an ordinary plane, within the cubical map and in space with the cube in 3D (Figure 126). If the user moves one of these points, the other two correspondences also move following the movement. It is also possible the personalisation of some parameters, such as the definition of the field of view (depth, wide, height) and the orientation of the visual cone.

VI.2.2.3 Others

I defined some more algorithms for exploring cubical drawing properties and understand the spatial relation between the sphere, the cube, the great circle and its polygonal development in the cube (Figure 127 bottom).

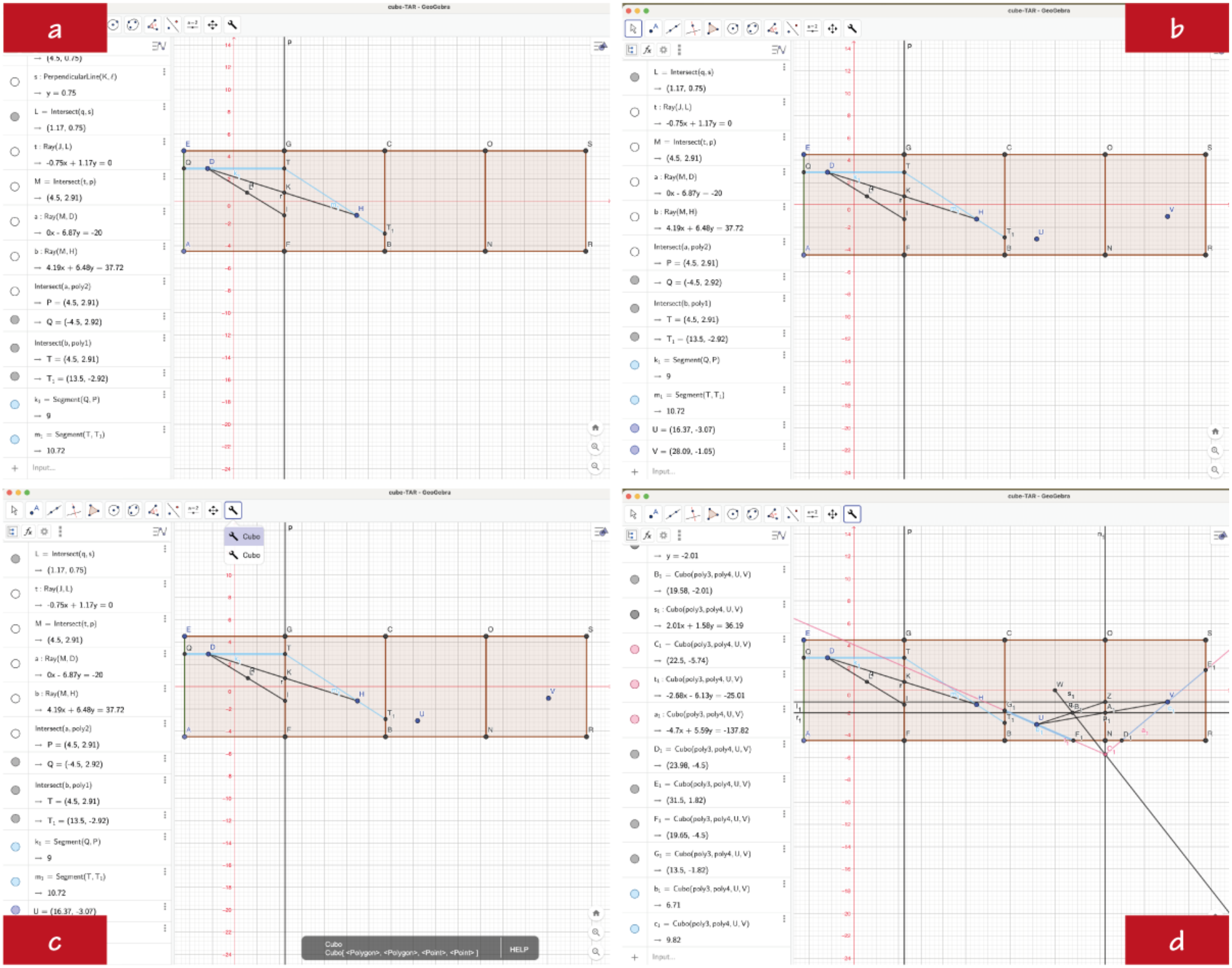


Figure 125 . Geogebra script for constructing the auxiliary point. a) Construction used for calculating the point (got from Second Part IV.6.2). b) Introducing two aleatory points in Right and Back faces. c) Selecting "Cube" tool and necessary inputs. d) Automatic construction of the right geodesic

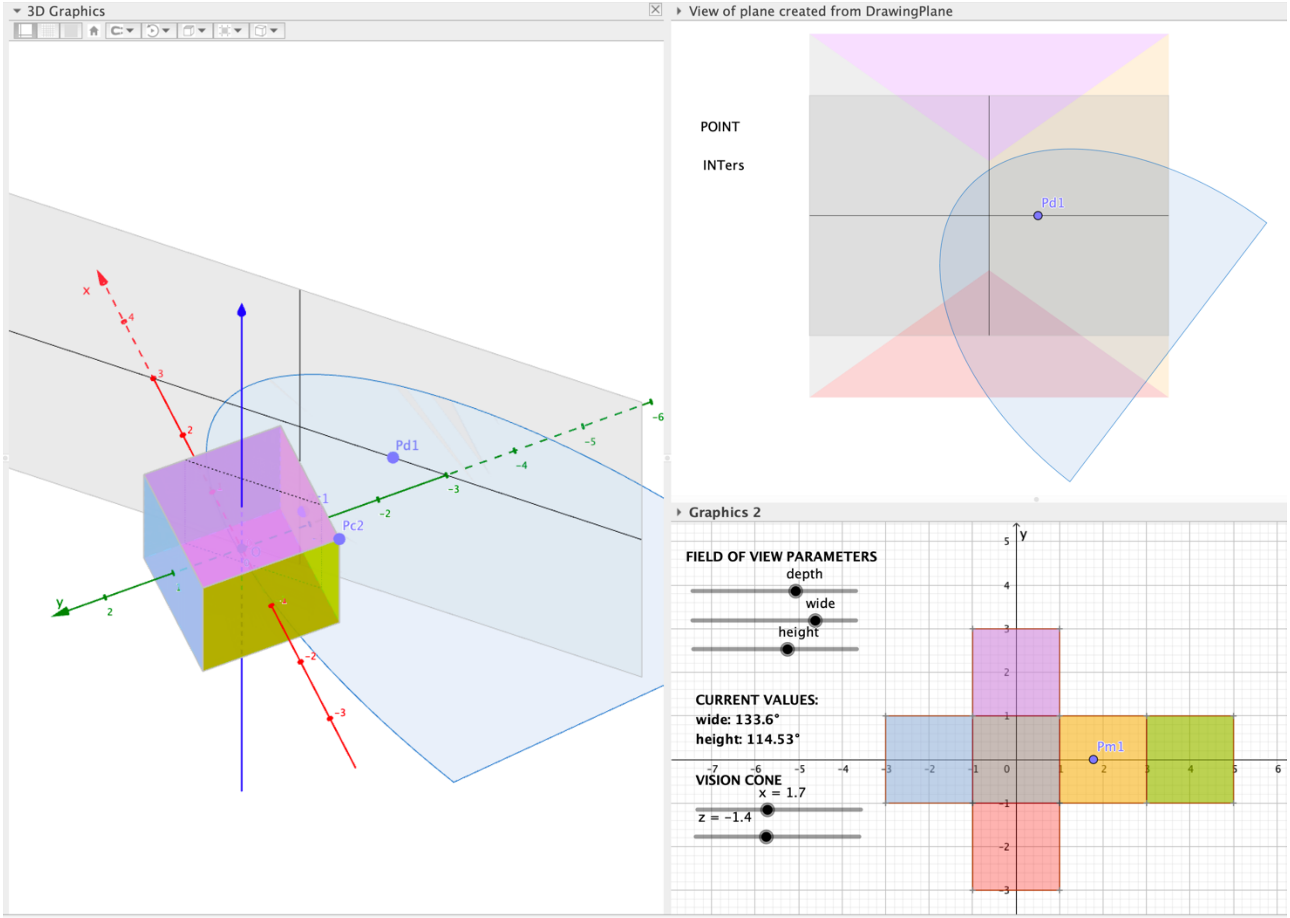


Figure 126 . Geogebra script for inserting a point and seeing its correspondence within the cubical map, an ordinary projecting plane and in the spatial setup

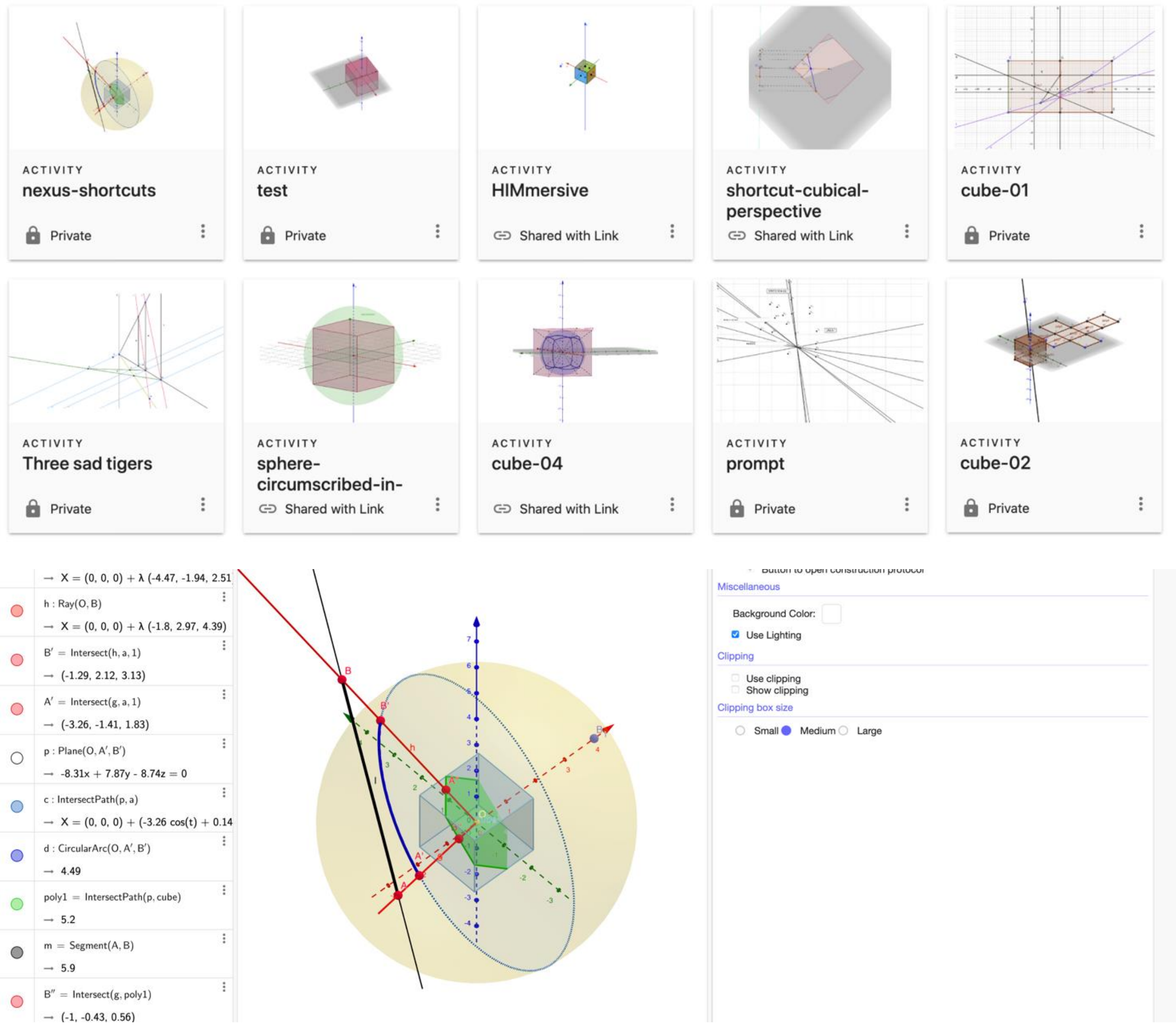


Figure 127 . Other algorithms made with Geogebra

VI.2.3 Deductions

The definition of algorithms for composing perspectives and homologies facilitate possibilities for exploring dynamically geometric transformations, and therefore spatial setups [193, p. 1]. These first steps into coding help to an interactive understanding of the basic concepts of cubical perspective.

It remains a further and more comprehensive development, intentioned to reach a parallelism to what *Eq A Sketch 360* promotes for equirectangular drawing. With such an end, I expect a further collaboration with IT specialists since my knowledge in the area is still very limited.

FIRST CONCLUSIONS

REACHED RESULTS

Current times guide us to see things dynamically and not statically anymore. Walking in such direction the hybrid immersive model consents a renewed sight above already-consolidated practices such as classical perspective, anamorphosis, panoramas and virtual reality. The first steps towards the definition of cubical perspective brought us to discover the theoretical foundations at the base of this new hybrid way with promising results:

A HIM can help to design the extended relation object-building / building-building / building-landscape. This can apply for both external and internal spaces. In the former case, the image does not focus just on the single building ignoring the context, but in the relation and the definition of the building with others and with the landscape around it. In the latter, the architecture is represented concerning the furnishing and the objects to be used in such space. Thanks to the promotion of traditional drawing, the experimentation with anamorphosis and digital technology, the knowledge was successfully transfer and spread, highlighting the impact of this specialistic tool.

Cubical perspective connects linear classical perspective with spherical perspectives with both practical and theoretical advantages. In effect, we can (now) create a scientifically defined immersive visualization, extending known methods and instruments already used to define both architecture and objects. Thanks to the principle of linearity, the representation is accurate, quantitative and measurable.

In the didactic field, is our wish that cubical perspective, as a special case of spherical perspectives, could find a place among the exams of design courses as the complex system that they are more than something exceptional: *"if spherical perspective is mentioned, then it is usually presented as an "outside the box" model (…) rather than as what it should be viewed as: A meta-class model of perspective representation"* [223, p. 345].

Applied to architecture, an HIM extends the vision of the architect from the concept of framing something to a spatial positioning. In the case of the designer, as the representation is centred in the observer, the draughtsman is being “forced” to see the design from the user’s points of view. Both spatial positioning and user’s points of view put the designer into a more quotidian experience, making the design result closer to the user’s daily perception.

The compressed **exercises** and the attractiveness of the VR visualization, supported the effort in propagating cubical perspective’s theory, suppressing a possible banalisation behind its current use and heading it instead to an informative and productive reasoning.

In the discipline of drawing, a hybrid immersive model connects traditional drawing with digital interaction. Thanks to the former, this means a sensitive and personal selection of the main representative elements behind an idea (both copying a reality or drawing what it does not exist yet); and thanks to the latter, it has the potentiality of being a shareable model and standardized product.

A hybrid immersive model encourages new procedures for transferring and acting “where and how” thoughts get shaped. Hence, proposing the use of an “intellectual picklock”, it allows the designer to lead the final output to cognitive and conformational aims through procedures directly connected to the architecture’s definition mechanisms. It stands to reason that such developments must be done in mindfulness, remembering what history already verified as valuable for drawing but not leaving behind the innovation offered by the digital technologies. Immersion might be more than just a fun game with technology: might also be a way to think and perceive space differently through the amalgamation of traditional and innovative techniques. Indeed, perception and representation are closely linked in one same expressive mean, and the search of an representation system matches the exploration of the means to do it [224, p. 13], [225].

EXPECTED RESULTS

Some recent studies about spherical perspective led the definition of grids and templates with isocurves [34], bringing the well-known grids and templates for classical perspective to curved systems. Artefacts as these, speed up perspective elaboration in the following step of users already in contact with the theory. The cubical perspective has no such a dynamic grid for the moment and therefore it could be a further development.

In contrast with the large number of methods existing in the CGI field, other current uses of cubical mapping are a minor and dependant adaptation of the full representation system, a narrow interpretation that can bring to an approximative use. The spread of the presented knowledge aims to open the disciplinary sector for further deepening studies and method developments, enriching the state of the art.

I expect to contribute with this research to the exercise of handmade drawing, leading to a trained and holistic overlook of the reality since “intuitive” operations are actually inspired by the subjective ways of perceiving the known heritage. During architectural survey, such a critical sight gives resources to be recalled during a further project stage. The definition of a criterium, allows the discussion of the reading (or hypothetical intervention) in concrete terms that can be refused or accepted.

This research belongs to the field of experimental drawing and it aims to conceive and use representation as a tool for exploration, verification and communication of architecture and design. Yet, architectural drawing suffers a diversification of the expression within the computer science’s influence and dynamic. Therefore, the experimental drawing here proposed open such aspect *a priori*, keeping the right distance with the fashion behind ICT and truncate knowledge diffusion. Thus, I walk aware of avoiding forced efficiency, often superficial and lacking didactic and cultural content.

Immersive and interactive representation is becoming more and more common. Technology is playing an important role in this process, leading a forced update from classical to interactive immersive representation. The sciences of representation may have an important role in the current situation, being the nexus and connection during the adaptation of all the disciplines behind immersive representation, namely mathematics, architecture, engineering, design, computer sciences, arts. Such an intervention could provide a scientific support for the new creative processes in current developed.

This research seeks to promote a healthy and balanced approach, defining a representation system entailing an exhaustive spatial conceptual resolution (theory) and a wide application casuistry (practice). If this were not the case, the system would risk remaining as an ideal model not verified in practice, or as a mere accumulation of empirical experiences without theoretical development of geometric definition.

Finally, I also emphasize the important role played by the publication and debate of theories, and practices to spread knowledge and creativity. The lack of communication on the importance of the use of immersive perspectives begins perhaps in the academy itself, where its teaching is practically non-existent: *"However, the spherical perspective (as the curvilinear perspective par excellance) has not been widely introduced into design education; literature on applied perspective construction is often confined to straight linear perspective"* [223, p. 3].

ADVANCEMENT STATE

The presence of traditional drawing techniques focusses on the capacity to abstract the material aspect of an element in its geometrical shape. Thanks to this structural research through drawing, we can deduce the formative principles of an idea. Furthermore, the results are defined following a specific criterion, which allow their discussion.

Hence, cubical perspectives can be confirmed as a key to read, interpret or shape the space. The digital side of a HIM is used as a "*grimaldello*" (a picklock) to force the limits between the material and the immaterial. Digital and traditional techniques are not alternative but complementary means. The cognitive analysis of drawing results in a meditative exercise in which the hybrid model allows to touch the limit among what can and what cannot be done [217].

This way, the user becomes both a critical observer and visitor of its product. From this dual position, the designer can verify the correctness of the project *before* a meticulous developing. Besides, nothing stops the visit of the scene by some hypothetical client, so to give a first immersive impression since a very early conceptual stage of the project.

Therefore, a hybrid immersive model helps the economy of resources for project elaboration, saving a considerable amount of time required to build a detailed 3D model just for a first preview. At the same time, the full immersive view enables the perception of the project in context, allowing the illusion of having the construction inserted in its surroundings.

The hybrid model can be converted into a platform to organise informative systems, or also be a container for digital media interaction. Sensorial receptivity is enhanced in the virtual tour, thanks to the inside first-person navigation of the immersive perspectives. Thanks to the self-perception, the operator/creator can also become visitor, interacting with a collaboration space that goes beyond the geographic barriers and the cultural and linguistic diversities.

The construction of hybrid panoramas allows the creation of digital immersive media, which, combined with the flexibility of classic drawing in scientific illustration gets as a result artefact of both scientific and artistic value.

Finally, in the widely open world of our days, hybrid immersive models induced a multi-disciplinary work, stimulating the complex thought but taking care of the results' unicity as well. Indeed, it is required the thorough study of the theory for learning and constructing a HIM (mathematics, architecture, computer sciences), but the final product is a unique representation, result of the personal selection of information. With all, HIM applications made with cubical perspective, demonstrate an equitable and accessible way for creating immersive representations, not anymore as the solo achievement of the virtuoso draughtsman of a time.

BIBLIOGRAPHY

CURRENT SITUATION OF CENTRAL PROJECTIONS: FROM LINEAR PERSPECTIVE TO SPHERICAL PANORAMAS + Introduction[39]

[1] P. Della Francesca, *De prospectiva pingendi*, vol. I. Venice, Italy: Università Ca' Foscari Venezia, Italia, 2016.
[2] A. Barre, A. Flocon, and G. Bouligand, *La Perspective curviligne: de l'espace visuel à l'image construite*. Paris: Flammarion, 1967.
[3] C. Boddien *et al.*, 'SketchTab3D: A Hybrid Sketch Library Using Tablets and Immersive 3D Environments', in *Proceedings of the 2017 ACM Symposium on Document Engineering*, New York, NY, USA, 2017, pp. 101–104, doi: 10.1145/3103010.3121029.
[4] D. A. Schön, *The Reflective Practitioner: How Professionals Think in Action*. Routledge, 2017.
[5] D. Tran Luciani and J. Lundberg, 'Enabling Designers to Sketch Immersive Fulldome Presentations', in *Proceedings of the 2016 CHI Conference Extended Abstracts on Human Factors in Computing Systems*, New York, NY, USA, 2016, pp. 1490–1496, doi: http://doi.acm.org/10.1145/2851581.2892343.
[6] J. H. Israel, E. Wiese, M. Magdalena, S. Rainer Georg, and Z. Christian, 'Investigating three-dimensional sketching for early conceptual design—Results from expert discussions and user studies', *Comput. Graph.*, vol. 33, no. 4, pp. 462–473, Aug. 2009, doi: 10.1016/j.cag.2009.05.005.
[7] A. B. Araújo, 'Ruler, compass, and nail: constructing a total spherical perspective', *J. Math. Arts*, vol. 12, no. 2–3, pp. 144–169, 2018, doi: https://www.doi.org/10.1080/17513472.2018.1469378.
[8] K. Andersen, *The Geometry of an Art: The History of the Mathematical Theory of Perspective from Alberti to Monge*. New York: Springer-Verlag, 2007.
[9] H. Damisch, *L'Origine de la perspective*, Cons. L'origine della prospettiva 1992, Flammarion. Napoli: Guida, 1987.
[10] D. Gioseffi, *Perspectiva artificialis per la storia della prospettiva spigolature e appunti*. Trieste: Università Degli Studi di Trieste, Facoltà di Lettere e Filosofia, 1957.
[11] M. Kemp, *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat*, Reprint edition. New Haven London: Yale University Press, 1992.
[12] M. Kemp, *The Oxford history of western art*. Oxford University Press, USA, 2000.
[13] E. Panofsky, *Perspective as Symbolic Form*, First Edition. New York, NY, USA: Zone Books, 1991.
[14] N. J. Wade, 'Early history of illusions.', in *The Oxford compendium of visual illusions.*, New York, NY, US: Oxford University Press, 2017, pp. 3–37.
[15] C. Candito, *Il disegno e la luce. Fondamenti e metodi, storia e nuove applicazioni delle ombre e dei riflessi nella rappresentazione.*, Alinea Editrice. Florence, Italy: Grafiche PDB - Tavarnelle Val di Pesa, 2010.
[16] L. Vagnetti, *De naturali et artificiali perspectiva: bibliografia ragionata delle fonti teoriche e delle ricerche di storia della prospettiva : contributo alla formazione della conoscenza di un'idea razionale, nei suoi sviluppi da Euclide a Gaspard Monge*. Edizione della Cattedra di composizione architettonica IA di Firenze e della L.E.F., 1979.

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[39] **References** are formatted in **IEEE** (The Institute for Electrical and Electronics Engineers) and divided by chapter accordingly to their first citation. The section "Other" includes **complementary bibliography formatted in APA 7th Edition** and follows an alphabetical order. All the bibliography was generated automatically using Zotero

[17] S. Barba, A. Rossi, and L. F. Olivero, 'CubeME, a variation for an immaterial rebuilding', in *Rappresentazione / Materiale / Immateriale. Drawing as (in)tangible representation*, Rome, Italy, 2018, pp. 31–36, [Online]. Available: http://hdl.handle.net/11591/392282.
[18] M. Maschietto, 'I prospettografi: dalla storia alla scuola', in *Atti del Convegno "Matematica, Arte e Industria Culturale"*, Cetraro, Cosenza, Italy, 2005.
[19] M. Masetti, *La prospettiva e la costruzione dello spazio figurativo*, Youcanprint. 2014.
[20] H. Damisch and J. C. Leal, 'Entrevista com Hubert Damisch', vol. 3, pp. 7–18, 2007.
[21] K. H. Veltman, 'Piero della Francesca and the two methods of Renaissance perspective.', in *Piero della Francesca tra arte e scienza*, Urbino, Arezzo, Sansepolcro, 1996, pp. 407–419, [Online]. Available: https://www.worldcat.org/title/piero-della-francesca-and-the-two-methods-of-renaissance-perspective/oclc/886822999.
[22] J. Romor, 'Righe di legno, righe di carta e fili di seta: per una "costruzione" della prospettiva secondo Pierodella Francesca', in *Le teorie, le tecniche, i repertori fi gurativi nella prospettiva d'architettura tra il '400 e il '700*, M. T. Bartoli and M. Lusoli, Eds. Florence, Italy: Firenze University Press, 2015, pp. 25–44.
[23] J. R. Katinsky, 'Florentine perspective and the development of modern science', *Llull Rev. Soc. Esp. Hist. Las Cienc. Las Téc.*, vol. 23, no. 48, pp. 599–642, 2000.
[24] C. Leopold, 'Points of view and their interrelations with space and image', in *Geometrias & graphica 2015 proceedings*, Porto, Portugal, 2016, vol. 1, pp. 365–373.
[25] J. Romor, 'La prospettiva nel Novecento', Accessed: Nov. 29, 2018. [Online]. Available: https://www.academia.edu/22049441/La_prospettiva_nel_Novecento.
[26] J. Romor, 'Per una storia della Prospettiva del XX Secolo - Repertorio dei testi sulla rappresentazione prospettica in Italia, dalla fine dell'Ottocento ai giorni nostri', Accessed: Nov. 29, 2018. [Online]. Available: https://www.academia.edu/4752035/Per_una_storia_della_Prospettiva_del_XX_Secolo_-_Repertorio_dei_testi_sulla_rappresentazione_prospettica_in_Italia_dalla_fine_dell_Ottocento_ai_giorni_nostri.
[27] A. Pagliano, 'Anamorphic perspectives for archeological heritage', presented at the III Congreso Internacional sobre Documentación, Conservación, y Reutilización del Patrimonio Arquitectónico, Valencia, Spain, Oct. 2015.
[28] A. Rossi and P. M. Cabezos Bernal, 'Tecniche di musealizzazione virtuale. Galleria 3D per la fruizione della cultura romanica | Virtual musealization techniques. A 3D gallery for the contemporary fruition of Romanesque culture', in *UID PER IL DISEGNO*, 2017, pp. 597–604, Accessed: May 18, 2018. [Online]. Available: https://books.google.it/books?id=2U03DwAAQBAJ&pg=PR8&lpg=PR8&dq=rossi+%22Virtual+musealization+techniques%22&source=bl&ots=4IEONn13SH&sig=o-T0c7Fu9ShHHHte-V08uzumxzo&hl=en&sa=X&ved=0ahUKEwiJ6cmPpo_bAhWGkywKHbwFBVoQ6AEIQDAD#v=onepage&q=rossi%20%22Virtual%20musealization%20techniques%22&f=false.
[29] A. Rossi, *Immersive high resoluttion photographs for cultural heritage*, Adriana Rossi., vol. 2. Padova, Italy: libreriauniversitaria edizioni.it, 2017.
[30] M. Fasolo and R. Migliari, 'Decio Gioseffi and Perspective as "symbolic form"', *Disegnare Idee Immagini*, no. 57–2, pp. 46–57, 2018, doi: 10.36165/2679.
[31] R. Migliari, 'La prospettiva e Panofsky', *Disegnare Idee Immagini*, vol. 31, pp. 28–41, 2005.
[32] C. Bernal and P. Manuel, 'Imágenes estereoscópicas aplicadas a la representación arquitectónica', *Riunet*, Feb. 2015, doi: https://doi.org/10.4995/Thesis/10251/46640.
[33] P. M. Cabezos Bernal and J. J. Cisneros Vivó, 'Panoramas esféricos estereoscópicos', *EGA Expr. Gráfica Arquit.*, vol. 21, no. 28, pp. 70–81, Sep. 2016, doi: 10.4995/ega.2016.6264.
[34] A. B. Araújo, 'Drawing Equirectangular VR Panoramas with Ruler, Compass, and Protractor', *J. Sci. Technol. Arts*, vol. 10, no. 1, pp. 15–27, Apr. 2018, doi: https://www.doi.org/10.7559/citarj.v10i1.471.
[35] J. van Eyck, *The Virgin of Chancellor Rolin*. 1434.
[36] J. Elkins, 'On the Arnolfini Portrait and the Lucca Madonna: Did Jan van Eyck Have a Perspectival System?', *Art Bull.*, vol. 73, no. 1, pp. 53–62, 1991, doi: 10.2307/3045778.

[37] G. Ferrero, C. Cotti, M. Rossi, and C. Tedeschi, ‘Geometries of Imaginary Space: Architectural Developments of the Ideas of M. C. Escher and Buckminster Fuller’, *Nexus Netw. J.*, vol. 11, no. 2, pp. 305–316, Jul. 2009, doi: 10.1007/s00004-008-0090-1.

[38] A. Flocon, *A la frontière de l’art graphique et des mathématiques: Maurits-Cornelis Escher*. Jardin des Arts, 1965.

[39] M. Piller, ‘Hand with Reflecting Sphere’, *Escher in Het Paleis*, 2020. https://www.escherinhetpaleis.nl/story-of-escher/hand-with-reflecting-sphere/?lang=en/ (accessed Sep. 25, 2020).

[40] ‘Knots with Albert Flocon’, *Escher in Het Paleis*, Aug. 11, 2018. https://www.escherinhetpaleis.nl/escher-today/knots-with-albert-flocon/?lang=en/ (accessed Sep. 25, 2020).

[41] B. Ernst, ‘Selection is Distortion’, in *M.C. Escher’s Legacy: A Centennial Celebration*, D. Schattschneider and M. Emmer, Eds. Berlin, Heidelberg: Springer, 2003, pp. 5–16.

[42] M. C. Escher and M. A. Esparaz (photographer), *Convex and Concave*. 1955.

[43] M. C. Escher and J. W. Vermeulen, *Escher on Escher: exploring the infinite*. Harry N. Abrams, 1989.

[44] K. Rehkämper, *Bilder, Ähnlichkeit und Perspektive: Auf dem Weg zu einer neuen Theorie der bildhaften Repräsentation*. Deutscher Universitätsverlag, 2002.

[45] M. Docci and R. Migliari, *Scienza della rappresentazione. Fondamenti e applicazioni della geometria descrittiva*. Roma: Carocci, 1992.

[46] A. Comolli, *Bibliografia storico-critica dell’ architettura civile ed arti subalterne*. Stamperia Vaticana, 1791.

[47] N. Germinal Poudra, *Histoire de la perspective ancienne et moderne: contenant l’analyse d’un très grand nombre d’ouvrages sur la perspective et la description des procédés divers qu’on y trouve : faisant suite au Cours de perspective professé à l’École d’état-major*. France: Corréard, 1864.

[48] Gino. Loria, *Storia della geometria descrittiva dalle origini sino ai giorni nostri*. Milano: Ulrico Hoepli, 1921.

[49] A. Chiappelli, ‘Della vita di Filippo Brunelleschi attribuita ad Antonio Manetti con un nuovo frammento di essa. Tratto da un codice Pistoiese del Sec. XVI’, *Arch. Stor. Ital.*, vol. 17, no. 202, pp. 241–278, 1896.

[50] A. Rossi, *Disegno Design. Natura morta e vita metafisica*, vol. I. Rome, Italy: Officina edizioni, 2005.

[51] C. Brisbin, ‘Spatial transfiguration: anamorphic mixed-reality in the Virtual Reality panorama’, presented at the Panorama to Paradise: Scopic Regimes in Architectural and Urban History and Theory, Adelaide, South Australia, 2007, Accessed: Mar. 08, 2020. [Online].

[52] O. Grau, *Virtual Art: From Illusion to Immersion*. MIT Press, 2003.

[53] P. M. Cabezos Bernal, J. Cisneros Vivó, and F. Soler Sanz, ‘Anamorfosis, su historia y evolución’, *EGA Expr. Gráfica Arquit.*, vol. Núm. 23 (2014): Conversando con... JUAN NAVARRO BALDEWEG, 2014, doi: https://www.doi.org/10.4995/ega.2014.2184.

[54] L. Dictionaries, ‘Definition of Anamorphosis’, *Lexico Dictionaries | English*, 2019. https://www.lexico.com/en/definition/anamorphosis (accessed Nov. 09, 2019).

[55] L. Oxford, ‘Anamorfosis’, *Lexico Dictionaries | Spanish*. 2019, Accessed: Nov. 09, 2019. [Online]. Available: https://www.lexico.com/es/definicion/anamorfosis.

[56] D. de la lengua española RAE, ‘Anamorfosis’, *Diccionario de la lengua española - Edición del Tricentenario*. 2019, Accessed: Nov. 09, 2019. [Online]. Available: https://dle.rae.es/anamorfosis.

[57] A. B. Araújo, ‘Anamorphosis Reformed: from Optical Illusions to Immersive Perspectives’, in *Handbook of the Mathematics of the Arts and Sciences*, B. Sriraman, Ed. Cham: Springer International Publishing, 2020, pp. 1–67.

[58] A. B. Araújo, ‘Anamorphosis: optical games with perspective’s playful parent’, in *Proceedings of Recreational Mathematics Colloquium V - G4G (Europe)*, Jorge Nuno Silva., Lisboa: Associação Ludus, 2017, pp. 71–86.

[59] A. Bortot, ‘Il dinamismo percettivo nel refettorio di Andrea Pozzo’, in *Le teorie, le tecniche, i repertori figurativi nella prospettiva d’architettura tra il ’400 e il ’700: dall’acquisizione alla lettura del dato*, M. T. Bartoli and M. Lusoli, Eds. Florence, Italy: Firenze University Press, 2015, pp. 119–126.

[60] ‘Quadraturismo’, *Enciclopedia Treccani*. 2020, Accessed: May 21, 2020. [Online]. Available: http://www.treccani.it//enciclopedia/quadraturismo.

[61] ‘Quadraturisti’, *Enciclopedia Treccani*. 2020, Accessed: May 21, 2020. [Online]. Available: http://www.treccani.it//enciclopedia/quadraturismo.
[62] F. Mazzoccoli, Ed., ‘L’arte della prospettiva e i mirabili disinganni di Andrea Pozzo’, *Fondazione Geom. Ital.*, vol. 9, no. II. 05-06/2010, p. 86, May 2010.
[63] D. Barbaro, *La pratica della perspettiva*. Venice, Italy: Appresso Camillo & Rutilio Borgominieri fratelli, 1569.
[64] J. F. Nicéron, *La perspective curieuse, ou Magie artificielle des effets merveilleux de l’optique, par la vision directe, la catoptrique, par la réflexion des miroirs plats, cylindriques et coniques, la dioptrique, par la réfraction des crystaux*, BnF Gallica. Paris, France: P. Billaine, 1638.
[65] A. Pozzo, *Perspectiva pictorum et architectorum*. Romae: Typis Joannis Jacobi Komarek ..., 1693.
[66] I. B. da Vignola, *Le dve regole della prospettiva pratica*, Getty Research Institute. Rome, Italy: In Roma, Per Francesco Zannetti, 1583.
[67] A. Callandriello, ‘Andrea Pozzo a Roma: nuove ipotesi fruitive del Refettorio di Trinità dei Monti’, in *Le teorie, le tecniche, i repertori figurativi nella prospettiva d’architettura tra il ’400 e il ’700: dall’acquisizione alla lettura del dato*, M. T. Bartoli and M. Lusoli, Eds. Florence, Italy: Firenze University Press, 2015, pp. 127–134.
[68] E. Huhtamo, *Illusions in Motion: Media Archaeology of the Moving Panorama and Related Spectacles*. MIT Press, 2013.
[69] E. Markman, ‘“Spectacles within doors”: Panoramas of London in the 1790s’, *Romanticism*, vol. 14, no. 2, pp. 133–148, Jul. 2008, doi: 10.3366/E1354991X0800024X.
[70] F. Gay and I. Cazzaro, ‘Venetian perspective boxes: when the images become environments. Low-tech high-knowledge media to teach the historical heritage of the (interior/exterior) environments’, *Proc. 1st Int. Interdiscip. Conf. Digit. Environ. Educ. Arts Herit. Brixen 5-6 July 2018 Springer Int. Publ.*, 2018, doi: https://doi.org/10.1007/978-3-030-12240-9_11.
[71] J. Baltrusaitis, *Anamorphic art*. Chadwyck-Healey Translated by WJ Strachan. Cambridge, England, 1977.
[72] O. Fasolo, *Fondamenti geometrici della rappresentazione progettuale e tecnica dell’architettura 1*, Kappa., vol. I. Rome, Italy, 1980.
[73] A. Pozzo and F. C. Panini Editore SpA, *Gloria di Sant’Ignazio*. 1691.
[74] C. Bigg, ‘The Panorama, or La Nature A Coup d’Oeil’, in *Observing Nature - Representing Experience. The Osmotic Dynamics of Romanticism 1800-1850*, Erna Fiorentini., Germany: Dietrich Reimer Verlag GmbH, 2007, pp. 73–96.
[75] R. Barker, ‘Repertory of arts and manufactures - Specification of Mr Barkers Patent for Displaying Views of Nature at Large’, 258.d.13–27, 1796.
[76] M. I. Pascariello, *Oltre il punto di vista*. Italy: Alinea, 2005.
[77] R. Mitchell, *Plans, and views in perspective, with descriptions, of buildings erected in England and Scotland: and also an essay, to elucidate the Grecian, Roman and Gothic architecture, accompanied with designs*. London: Printed by Wilson & Co. for the author, 1801.
[78] A. Taylor, *Plate 14: Section of the Rotunda, Leicester Square, in which is exhibited the Panorama.* 2019.
[79] N. Penny, ‘Review of The Panorama: History of a Mass Medium, ; The Panorama’, *AA Files*, no. 39, pp. 80–82, 1999.
[80] S. Bordini, *Storia del panorama: la visione totale nella pittura del XIX secolo*. Roma: Officina, 1984.
[81] J. van Eyck, ‘The Arnolfini Portrait’, *Wikipedia*. Dec. 20, 2020, Accessed: Jan. 18, 2021. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Arnolfini_Portrait&oldid=995353758.
[82] G. F. M. Mazzola (Parmigianino), ‘Self-portrait in a Convex Mirror’, *Wikipedia*. Dec. 09, 2020, Accessed: Jan. 18, 2021. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Self-portrait_in_a_Convex_Mirror&oldid=993142155.
[83] J. L. P. Macnair, ‘Spherical Perspective’, *J. R. Soc. Arts*, vol. 105, no. 5010, pp. 762–780, 1957.
[84] R. Hansen, ‘This Curving World: Hyperbolic Linear Perspective’, *J. Aesthet. Art Crit.*, vol. 32, no. 2, pp. 147–161, 1973, doi: 10.2307/429032.
[85] United States. War Dept. Division of Military Aeronautics, *Panoramic Drawing: One-point and Cylindrical Perspective. Department of ...* Govt. print. off., 1918.

[86] T. Luhmann, ‘A Historical Review on Panorama Photogrammetry - Semantic Scholar’, in *Panoramic Photogrammetry Workshop*, Dresden, Germany, Feb. 2004, vol. XXXIV PART 5/W16, Accessed: May 22, 2018. [Online]. Available: /paper/A-Historical-Review-on-Panorama-Photogrammetry-Luhmann/6e4224c8ef297ced99e0c896a7f3fe50185e1b8f.

[87] J. van Eyck and Sailko, *The Arnolfini Portrait (Portrait of Giovanni Arnolfini and his Wife)*. 1434.

[88] F. R. Casas, ‘Flat-Sphere Perspective’, *Leonardo*, vol. 16, no. 1, pp. 1–9, 1983, doi: 10.2307/1575034.

[89] G. M. Catalano, *Prospettiva sferica: punto di vista al centro del quadro sferico*, Università di Palermo. Palermo, italy: Co.Gra.S, 1986.

[90] D. Chelsea, *Extreme Perspective! For Artists*, Pap/Cdr edition. New York: Watson-Guptill, 2011.

[91] G. Michel, ‘L’œil, au Centre de la Sphere Visuelle’, *Bol. Aproged*, vol. 30, pp. 3–14, 2013.

[92] D. A. Termes, ‘Six-Point Perspective on the Sphere: The Termesphere’, *Leonardo*, vol. 24, no. 3, pp. 289–292, 1991, doi: 10.2307/1575568.

[93] A. B. Araújo, ‘A Construction of the Total Spherical Perspective in Ruler, Compass and Nail’, *ArXiv151102969 Math*, Nov. 2015, Accessed: May 21, 2018. [Online]. Available: http://arxiv.org/abs/1511.02969.

[94] M. Moose, ‘Guidelines for Constructing a Fisheye Perspective’, *Leonardo*, vol. 19, no. 1, pp. 61–64, 1986, doi: 10.2307/1578303.

[95] A. B. Araújo, ‘Anamorphosis Reformed: from Optical Illusions to Immersive Perspectives’, in *Handbook of the Mathematics of the Arts and Sciences*, B. Sriraman, Ed. Cham: Springer International Publishing, 2020, pp. 1–67.

[96] A. B. Araújo, ‘Spherical Perspective’, in *Handbook of the Mathematics of the Arts and Sciences*, B. Sriraman, Ed. Cham: Springer International Publishing, 2020, pp. 1–61.

[97] ‘Hugin - Panorama photo stitcher’. http://hugin.sourceforge.net/ (accessed Jan. 18, 2021).

[98] N. Greene, ‘Environment Mapping and Other Applications of World Projections’, *IEEE Comput. Graph. Appl.*, vol. 6, no. 11, pp. 21–29, Nov. 1986, doi: https://doi.org/10.1109/MCG.1986.276658.

[99] P. Bourke, ‘Converting to/from cubemaps’, *Paul Bourke’s website*, 2016. http://paulbourke.net/miscellaneous/cubemaps/ (accessed May 21, 2018).

[100] M. Lambers, ‘Survey of cube mapping methods in interactive computer graphics’, *Vis. Comput.*, Jun. 2019, doi: 10.1007/s00371-019-01708-4.

[101] A. M. Dimitrijević, M. Lambers, and D. Rančić, ‘Comparison of shperical cube map projections used in planet-sized terrain rendering’, *Facta Univ. Ser. Math. Inform.*, vol. 31, no. 2, Art. no. 2, Jun. 2016.

[102] ‘FSPViewer’. http://www.fsoft.it/FSPViewer/ (accessed Jan. 18, 2021).

[103] I. V. P. A. IVRPA, ‘RoundMe. Virtual Tours Made Simple’. http://roundme.com (accessed May 04, 2018).

[104] ‘Equirectangular projection’, *Wikipedia*. Jan. 03, 2021, Accessed: Jan. 20, 2021. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Equirectangular_projection&oldid=998076451.

[105] A. B. Araújo, ‘Eq A Sketch 360, a Serious Toy for Drawing Equirectangular Spherical Perspectives’, in *Proceedings of the 9th International Conference on Digital and Interactive Arts*, New York, NY, USA, Oct. 2019, pp. 1–8, doi: 10.1145/3359852.3359893.

[106] D. Swart, ‘Drawing a Spherical Panorama’, *D. M. Swart*, Jun. 28, 2016. https://dmswart.com/2016/06/28/drawing-a-panorama/ (accessed Oct. 12, 2020).

[107] B. Sucurado, ‘360-degrees drawings’, *Roundme*, 2019. https://roundme.com/@bruno_0310 (accessed Jul. 13, 2019).

[108] A. Hartmann, ‘Grafik · Architektur · Webdesign’, 2019. http://www.arnohartmann.de/sonstiges-zeichnungen.html (accessed Jul. 13, 2019).

[109] D. Termes, *New perspective systems. Seeing the total picture: one through six point perspective*, 1st edition. Author’s edition, 1998.

[110] M. Scherotter, *Sketch 360*. United States: Microsoft Store, 2018.

[111] C. Masiero Sgrinzatto, ‘Illustrated 360º panoramas’, 2021. https://www.chiaramasierosgrinzatto.com/ (accessed Jan. 18, 2021).

[112] V. Kurbatov, 'Draw Sketches for Virtual Reality Like a Pro', *Inborn Experience (UX in AR/VR)*, Jan. 18, 2017. https://medium.com/inborn-experience/vr-sketches-56599f99b357 (accessed May 21, 2018).
[113] F. E. Sandnes, 'PanoramaGrid – A graph paper tracing framework for sketching 360-degree immersed experiences', *2016*, pp. 342–343, 2016.
[114] F. E. Sandnes, 'Communicating Panoramic 360 Degree Immersed Experiences: A Simple Technique for Sketching in 3D', in *Universal Access in Human-Computer Interaction. Interaction Techniques and Environments*, Jul. 2016, pp. 338–346, doi: 10.1007/978-3-319-40244-4_33.
[115] F. E. Sandnes, Y. Lianguzov, O. V. Rodrigues, H. Lieng, F. O. Medola, and N. Pavel, 'Supporting Collaborative Ideation Through Freehand Sketching of 3D-Shapes in 2D Using Colour', in *Cooperative Design, Visualization, and Engineering*, 2017, pp. 123–134, doi: https://doi.org/10.1007/978-3-319-66805-5_16.
[116] L. Theng Seng, '4 Steps to create a 360 VR illustration / painting in Photoshop (with pictures) – Studio Behind 90 – Indie Motion Graphic Design Studio', *Behind 90 Studio*, Dec. 25, 2016. https://studiobehind90.com/2016/12/25/how-to-create-360-panorama-painting-in-photoshop/ (accessed Feb. 17, 2018).
[117] K. Hohler, '360° images (photographies and illustrations)', *Kuula*. https://kuula.co/profile/KevinHohler (accessed Jan. 21, 2021).
[118] K. Hohler, '360 Drawing - Dessinez à 360° de A à Z'. https://360-drawing.com (accessed Jan. 21, 2021).
[119] L. F. Olivero and B. Sucurado, 'Analogical immersion: discovering spherical sketches between subjectivity and objectivity', *ESTOA Rev. Fac. Arquit. Urban. Univ. Cuenca*, vol. 8, no. 16, pp. 47–59, Jul. 2019, doi: https://www.doi.org/10.18537/est.v008.n016.a04.
[120] L. F. Olivero, A. B. Araújo, and A. Rossi, 'Applications of Cubical Perspective in Architecture, Engineering and Product Design', in *4.º Seminário Internacional de Arquitectura e Matemática*, Universidade Lusíada., vol. 1, M. Fátima Silva, M. A. Carreira, E. Negas, and R. Seco, Eds. Lisbon, Portugal: Universidade Lusíada, 2020, pp. 63–86.
[121] Chiara Masiero Sgrinzatto, *Tribuna Grimani VR | behind the scenes*. 2020.
[122] AutrePointdeVue, 'Visite virtuelle du Cinéma Sauvenière -Gérard Michel'. http://autrepointdevue.com/blog/wp-content/vv/vv-gm-sauveniere/vv-gm-sauveniere.html (accessed Oct. 12, 2020).
[123] G. Michel, 'Bureau visite virtuelle', *Virtual tour*, 2018. http://autrepointdevue.com/blog/wp-content/vv/vv-gm-bureau/vv-gm-bureau.html (accessed Oct. 12, 2020).
[124] M. Marrazzo, *How to draw curvilinear perspective*. 2018.
[125] A. B. Araújo, 'Guidelines for Drawing Immersive Panoramas in Equirectangular Perspective', in *Proceedings of the 8th International Conference on Digital Arts*, New York, NY, USA, 2017, pp. 93–99, doi: https://www.doi.org/10.1145/3106548.3106606.
[126] R. Migliari, 'La Geometria descrittiva: evoluzione di una teoria', in *Elogio della teoria. Identità delle discipline de disegno e del rilievo*, Rome, Italy, 2012, pp. 23–29, Accessed: Feb. 14, 2019. [Online].
[127] Adobe, *Photoshop: Photo, image & design editing software*. Adobe.
[128] L. Theng Seng, 'How to creating 360 Virtual Reality (VR) Illustration with Adobe Photoshop CC (2017 & 2018) – Studio Behind 90 – Indie Motion Graphic Design Studio', *Behind 90 Studio*, Nov. 01, 2018. https://studiobehind90.com/2018/11/01/how-to-360-virtual-reality-vr-illustration-photoshop-cc17-cc18/ (accessed Feb. 17, 2019).
[129] 'Pacific Domes Home Page', *Pacific Domes, Inc.*, Apr. 25, 2016. https://pacificdomes.com/ (accessed Oct. 11, 2020).
[130] 'Diana Reichenbach - M.O.D.E.' https://dianareichenbach.com/mode (accessed Oct. 11, 2020).
[131] P. Vistisen, D. Luciani, and P. Ekströmer, 'Sketching Immersive Information Spaces: Lessons learned from experiments in "sketching for and through virtual reality"', in *Proceedings of the 7th eCAADe Regional International Symposium: Virtually Real*, 2019, pp. 147–157, Accessed: Aug. 26, 2019. [Online]. Available: https://vbn.elsevierpure.com/en/publications/sketching-immersive-information-spaces-lessons-learned-from-exper.
[132] D. Tran Luciani, 'Designing for sketching to support concept exploration', Doctoral thesis, comprehensive summary, Linköping University Electronic Press, Linköping, 2019.

INTRODUCING HYBRID IMMERSIVE MODELS

[133] M. Gaiani, ‘La rappresentazione dell’architettura: sapere e saper fare’, *MeTis - Mondi Educ. Temi Indagini Suggest.*, vol. 8, no. 2, Art. no. 2, Dec. 2018.
[134] P. Polato, *Il modello nel design, la bottega di Giovanni Sacchi*. Milano: Hopeli, 1991.
[135] T. Maldonado, ‘Questioni di similarità’, *Rassegna*, vol. 32, pp. 57–61, 1987.
[136] J. S. Ackerman and J. Slosburg-Ackerman, *Origins, imitation, conventions: representation in the visual arts*. mit Press, 2002.
[137] ‘Scripting - GeoGebra Manual’. https://wiki.geogebra.org/en/Scripting (accessed Jan. 15, 2021).
[138] ‘Tutorial:Introduction to GeoGebraScript - GeoGebra Manual’. https://wiki.geogebra.org/en/Tutorial:Introduction_to_GeoGebraScript (accessed Jul. 06, 2019).
[139] A. Casale *et al.*, *Disegno come Modello. Riflessioni sul disegno nell’era informatica*. Rome, Italy: Edizioni Kappa, 2004.
[140] R. Migliari, Ed., *Prospettiva dinamica interattiva. La tecnologia dei videogiochi per l’esplorazione dei modelli 3D di architettura*. Roma: Kappa, 2008.
[141] J. Jerald, *The VR Book: Human-Centered Design for Virtual Reality*. Association for Computing Machinery and Morgan & Claypool, 2015.
[142] O. Grau, *On the visual power of digital arts: for a new archive and museum infrastructure in the 21st century*. Ediciones de la Universidad de Castilla-La Mancha, 2016.
[143] V. Treccani, ‘Conformazione’, *Treccani*. 2020, Accessed: Oct. 11, 2020. [Online]. Available: https://www.treccani.it/vocabolario/conformazione.
[144] A. Rossi, L. F. Olivero, and A. B. Araújo, ‘For Representation, a New Reality: Hybrid Immersive Models’, in *Faces of Geometry 2020*, Milan, Italy, 2020.
[145] R. Arnheim, *Art and Visual Perception. A psychology of the creative eye.*, Second Edition. California, USA: University of California Press, 1954.
[146] C. Cruz-Neira, D. J. Sandin, T. A. DeFanti, R. V. Kenyon, and J. C. Hart, ‘The CAVE: Audio Visual Experience Automatic Virtual Environment’, *Commun ACM*, vol. 35, no. 6, pp. 64–72, Jun. 1992, doi: https://www.doi.org/10.1145/129888.129892.
[147] R. Oxford, ‘Suspension of Disbelief’, *Oxford Reference*. doi: 10.1093/oi/authority.20110803100544310.
[148] D. Chandler and R. Munday, *A Dictionary of Media and Communication*. Oxford University Press, 2011.
[149] S. N. Osorio García, ‘Ciencias de la complejidad, pensamiento complejo y conocimiento transdisciplinar: Re-pensando la Humana Coditio en un mundo tecnocientífico’, in *La Crisis axiológica raíz de todas las crisis que sufre nuestro mundo: Cómo manejarnos con ella*, Bubok Publishing S.L., Barcelona, Spain: CETR, 2012, pp. 223–259.
[150] J. Wagensberg, *Ideas sobre la complejidad del mundo*. Tusquets Editores, 2003.
[151] S. Muñoz Viñas, *Teoría Contemporánea de la Restauración*. Madrid, Spain: Sintesis S.A., 2004.
[152] G. Fangi, C. Pisa, and F. Zeppa, ‘Spherical Photogrammetry for Cultural Heritage—San Galgano Abbey and the Roman Theater, Sabratha’, *J Comput Cult Herit*, vol. 4, no. 3, p. 9:1-9:15, Dec. 2011, doi: 10.1145/2069276.2069278.
[153] K. Kwiatek, ‘Generation of a virtual tour in the 3D space applying panoramas, exercised on the sites of Dresden and Cracow’, Unpublished Diploma Thesis, AGH University of Science and Technology, Cracow, Poland, 2005.
[154] K. Kwiatek, ‘360° film brings bombed church to life’, in *ISPRS - International Society for Photogrammetry and Remote Sensing Information Sciences*, Trento, Italy, Sep. 2012, vol. XXXVIII-5-W16, pp. 69–76, doi: https://doi.org/10.5194/isprsarchives-XXXVIII-5-W16-69-2011.
[155] C. García-García, J. Galán Serrano, and J. M. Arce Martínez, ‘The hybrid drawing as a way of architectural representation’, in *Dibujar, Construir, Soñar. Drawing, Building, Dreaming*, I., Castellón, Spain: Tirant lo Blanch, 2016, pp. 1037–1049.
[156] P. Virilio, ‘Il futuro nello spazio “stereoreale”’, Mar. 22, 1999.
[157] G. Matta Clark, *Conical Intersect*. 1975.

[158] P. Delivery, 'Why Gordon Matta-Clark cut holes in buildings - Conical Intersect', Jan. 05, 2020. https://publicdelivery.org/matta-clark-conical-intersect/ (accessed Jan. 28, 2021).
[159] L. Prestinenza Puglisi, 'Corpo e mente: scenari tradizionali e digitali nella ricerca architettonica', *prestinenza.it*, Mar. 30, 2014. https://www.prestinenza.it/2014/03/corpo-e-mente-scenari-tradizionali-e-digitali-nella-ricerca-architettonica/ (accessed Jun. 04, 2020).
[160] G. C. Argan, *Il Rinascimento. Storia dell'arte italiana*, Paola Argan, Cristina Boer, Lucia Lazotti., vol. 3, 1968. Florence, Italy: Sansoni, 2001.
[161] G. Matta Clark and B. Maggio (photographer), *Conical Intersect*. 1975.
[162] S. Koslow, 'De wonderlijke Perspectyfkas: An Aspect of Seventeenth Century Dutch Painting', *Oud Holl. – J. Art Low Ctries.*, vol. 82, no. 1, pp. 35–56, Jan. 1967, doi: 10.1163/187501767X00206.

STATE OF THE ART OF CUBICAL PROJECTIONS + CUBICAL PERSPECTIVE

[163] J. Spencer, 'Illusion as ingenuity. Dutch perspective boxes in the Royal Danish Kunstkammer's "Perspective Chamber"', *J. Hist. Collect.*, vol. 30, no. 2, pp. 187–201, Jul. 2018, doi: https://www.doi.org/10.1093/jhc/fhx024.
[164] S. van Hoogstraten, *Inleyding tot de hooge schoole der schilderkonst: anders de zichtbaere werelt; verdeelt in negen leerwinkels, yder bestiert door eene der zanggodinnen; ten hoogsten noodzakelijk, tot onderwijs, voor alle die deeze edele, vrye, en hooge konst oeffenen, of met yver zoeken te leeren, of anders eenigzins beminnen*. Davaco, 1969.
[165] M. Fukuoka, 'Contextualising the peep-box in Tokugawa Japan', *Early Pop. Vis. Cult.*, vol. 3, no. 1, pp. 17–42, May 2005, doi: https://www.doi.org/10.1080/17460650500056998.
[166] A. Verweij, 'Perspective in a box', *Nexus Netw. J.*, vol. 12, no. 1, pp. 47–62, Apr. 2010, doi: https://www.doi.org/10.1007/s00004-010-0023-7.
[167] The National Gallery, London (photograph) and S. van Hoogstraten, *A Peepshow with Views of the Interior of a Dutch House*. 1655.
[168] J. P. Nakamura, 'Seeing Outside the Box: Reexamining the Top of Samuel van Hoogstraten's London Perspective Box', *Journal of Historians of Netherlandish Art*, 2020. https://jhna.org/articles/seeing-outside-the-box-reexamining-the-top-of-samuel-van-hoogstratens-london-perspective-box/ (accessed Jan. 30, 2021).
[169] The National Gallery, London, 'Samuel van Hoogstraten (1627 - 1678)', *National Gallery, London*. https://www.nationalgallery.org.uk/artists/samuel-van-hoogstraten (accessed Jan. 30, 2021).
[170] 'Skybox (video games)', *Wikipedia*. Feb. 13, 2018, Accessed: Jan. 29, 2019. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Skybox_(video_games)&oldid=825433763.
[171] P. Donnelly, 'Video game play using panoramically-composited depth-mapped cube mapping', US7256779B2, Aug. 14, 2007.
[172] V.A., 'Cube mapping', *Wikipedia*. Apr. 19, 2018, Accessed: Oct. 17, 2018. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Cube_mapping&oldid=837243861.
[173] T. Wong, L. Wan, and C. Leung, 'Isocube: Exploiting the Cubemap Hardware', *IEEE Trans. Vis. Comput. Graph.*, vol. 13, no. 4, pp. 720–731, 2007, doi: 10.1109/TVCG.2007.1020.
[174] C. M. Grimm and B. Niebruegge, 'Continuous Cube Mapping', *J. Graph. Tools*, vol. 12, no. 4, pp. 25–34, Jan. 2007, doi: 10.1080/2151237X.2007.10129250.
[175] M. Lambers and A. Kolb, 'Ellipsoidal Cube Maps for Accurate Rendering of Planetary-Scale Terrain Data', 2012, doi: 10.2312/PE/PG/PG2012short/005-010.
[176] L. Baglioni, M. F. Mancini, J. Romor, and M. Salvatore, 'Proposta di uno standard di acquisizione per il rilievo delle quadrature su superfici piane', in *Prospettive architettoniche. Conservazione digitale, divulgazione e studio*, Università La Sapienza., vol. I, 2 vols, G. M. Valenti, Ed. Rome, Italy: Sapienza Università Editrice, 2014, pp. 687–715.
[177] A. Bortot, 'Rappresentare misurando, misurare rappresentando: rilievo ed elaborazione dei dati del Refettorio del Convento di SS. Trinità del Monti a Roma', in *Prospettive architettoniche. Conservazione digitale, divulgazione e studio*,

Università La Sapienza., vol. II, 2 vols, G. M. Valenti, Ed. Rome, Italy: Università degli Studi La Sapienza, 2016, pp. 351–360.
[178] M. Horvath, *Example scene demonstrating cube mapped panoramic images.* 2009.
[179] Arieee, *An example of a skybox and how the faces can be aligned*. 2011.
[180] M. Zucker and Y. Higashi, 'Cube-to-sphere projections for procedural texturing and beyond', *J. Comput. Graph. Tech. JCGT*, vol. 7, no. 2, pp. 1–22, Jun. 2018.
[181] S. Amalfitano, S. Barba, E. Civale, E. Feo, L. Fernández, and C. Ferreyra, *Villa Rufolo. Una scoperta che dura da mille anni*. 2019.
[182] A. Rossi, E. Carbone, and F. Fiorillo, 'Remote fruition of material and non material goods', *ScieConf*, no. 1, Jun. 2016, doi: 10.18638/scieconf.2016.4.1.388.
[183] M. L. Cimatti, 'La realidad virtual en el proceso de interpretación patrimonial. Diagnóstico de implementación de una CAVE en el Teatro Argentino de La Plata', Tesis, Universidad Nacional de La Plata, 2018.
[184] C. Battini and R. Vecchiattini, 'Immagini HDR per la documentazione tridimensionale dei manufatti storici', presented at the 40° International Conference of Representation Disciplines Teachers. XV Congress of Unione Italiana per il Disegno. Milan 13 to 15 of September 2018, Rome, Italy, Sep. 2018.
[185] Z. Song and J. Li, 'A Dynamic Tiles Loading and Scheduling Strategy for Massive Oblique Photogrammetry Models', in *2018 IEEE 3rd International Conference on Image, Vision and Computing (ICIVC)*, Jun. 2018, pp. 648–652, doi: 10.1109/ICIVC.2018.8492731.
[186] M. Hancher, 'Introduction to Map Projections with Google Earth Engine: Part 1', Dec. 10, 2016. https://medium.com/google-earth/introduction-to-map-projections-with-google-earth-engine-part-1-7840e4ca6264 (accessed Jun. 19, 2018).
[187] C. Davis, 'Osmose', 1995. http://www.immersence.com/osmose/ (accessed Oct. 14, 2020).
[188] 'Cave automatic virtual environment', *Wikipedia*. Apr. 20, 2018, Accessed: Jul. 17, 2018. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Cave_automatic_virtual_environment&oldid=837436463.
[189] Gcsnow, *A virtual reality cave installation at EVL, University of Illinois in Chicago*. 2016.
[190] D. F. Keefe, D. A. Feliz, T. Moscovich, D. H. Laidlaw, and J. L. Viola, 'CavePainting: A fully immersive 3D artistic medium and interactive experience', in *Proceedings of the Symposium on Interactive 3D Graphics*, Jan. 2001, pp. 85–93, Accessed: Jan. 23, 2021. [Online]. Available: https://experts.umn.edu/en/publications/cavepainting-a-fully-immersive-3d-artistic-medium-and-interactive.
[191] J. H. Israel, C. Zöllner, and A. Müller, 'Sketching in space', *The constitute*, 2010. http://theconstitute.org/sketching_in_space/ (accessed Apr. 25, 2018).
[192] I. N. Laboratory, *Engineering teams can use the CAVE at INL's Center for Advanced Energy Studies to tour a virtual nuclear reactor, train staff, orient subcontractors and consider new designs.* 2010.
[193] M. Fernández Rodríguez, 'Homología entre figuras de tercera categoría', presented at the XIV Congreso Internacional de Ingeniería Gráfica de INGEGRAF, Santander, Spain, 7/06 2002, [Online]. Available: http://ingegraf.es/wp-content/uploads/2017/03/XIVCONGRESO.ISO.
[194] D. Hernández Falagán, F. Signes Orovay, and A. Berdié Soriano, 'Sistema cónico-diédrico'. SdR - Sistemes de representació - Escola Massana, Mar. 03, 2015, Accessed: Oct. 03, 2019. [Online]. Available: https://sistemesderepresentacio.files.wordpress.com/2015/03/03.pdf.
[195] Oniride Art 360, *Oniride Plugin for Photoshoph*. Italy: Oniride S.R.L., 2016.
[196] M. Marrazzo, 'Cubic panorama', *Biodomotica*, 2020. http://www.biodomotica.com/cubic_panorama.htm (accessed May 20, 2020).
[197] J. Wehby, *Smustard(TM) - the Companion to Sketchup(TM) - CubicPanoOut*. Smustard.
[198] Outerspace, *Bixorama | 360° photo software*. Outerspace Software.
[199] L. F. Olivero, A. Rossi, and S. Barba, 'A codification of cubical projection for the generation of immersive models', *diségno*, no. 4, pp. 53–63, Jun. 2019, doi: https://www.doi.org/10.26375/disegno.4.2019.07.

[200] A. B. Araújo, 'Topologia, anamorfose, e o bestiário das perspectivas curvilíneas', *Convoc. Rev. Ciênc. Arte Arte E Geom.*, vol. 2, pp. 51–69, Sep. 2016.
[201] A. B. Araújo, 'Anamorphosis Reformed: from Optical Illusions to Immersive Perspectives', in *Handbook of the Mathematics of the Arts and Sciences*, B. Sriraman, Ed. Cham: Springer International Publishing, 2020, pp. 1–67.
[202] Adobe, *Adobe Bridge | Digital asset management software*. Adobe, 2021.
[203] O. Alshaker, '360Toolkit - Convert and View Panorama Pictures Instantly', *360Toolkit*, Jan. 2018. https://www.360toolkit.co (accessed May 04, 2018).
[204] 'Photo stitching software 360 degree Panorama image software - PTGui Stitching Software'. https://www.ptgui.com/ (accessed Feb. 01, 2021).
[205] 'Exif Fixer Online'. https://exiffixer.com/ (accessed Feb. 01, 2021).
[206] 'Embedding 360° Photos and Virtual Reality (VR) Content', *Support — WordPress.com*, Dec. 09, 2016. https://wordpress.com/support/embedding-360-photos-and-virtual-reality-vr-content/ (accessed Feb. 01, 2021).

APPLICATIONS + Acknowledgments

[207] A. Rossi, L. P. Iglesias, and U. Palmieri, 'De la digitalización laser hacia el H-BIM: un caso de estudio', *EGA Expr. Gráfica Arquit.*, vol. 25, no. 38, Art. no. 38, Mar. 2020, doi: 10.4995/ega.2020.12835.
[208] A. Rossi and U. Palmieri, 'Experimentation of an Information Model', *Vitr. - Int. J. Archit. Technol. Sustain.*, vol. 5, no. 1, Art. no. 1, Jun. 2020, doi: 10.4995/vitruvio-ijats.2020.13639.
[209] A. Rossi and U. Palmieri, 'LOD for Architectural Heritage: BIM Modeling for the Solimene Factory', *diségno*, no. 4, pp. 213–224, Jun. 2019, doi: 10.26375/disegno.4.2019.20.
[210] A. Rossi and U. Palmieri, 'Modelling Based on a Certified Level of Accuracy: The Case of the Solimene Façade', *Nexus Netw. J.*, vol. 22, no. 3, pp. 615–630, Sep. 2020, doi: 10.1007/s00004-019-00474-z.
[211] A. Rossi and U. Palmieri, 'Processi conoscitivi a supporto di metodologie BIM / KNOWLEDGE PROCESS FOR THE BIM METHODOLOGIES', *Paesaggio Urbano*, vol. 3, pp. 68–78, 2019.
[212] A. Rossi, L. Palmero Iglesias, and S. De Gregorio, 'El análisis de la forma en el diseño arquitectónico: desde el proyecto a la ejecución', in *EGA. Revista de Expresión Gráfica Arquitectónica*, Mar. 2018, vol. 23, no. 32, pp. 186–197, doi: 10.4995/ega.2018.8942.
[213] A. Rossi, 'Mathematical Models for typical terracotta infill walls | Aticipi Tompagni: modelli matematici e numerici', *Sustain. Mediterr. Constr.*, vol. Eight, pp. 15–21, 2018.
[214] A. Rossi, 'The ' physical' dimension of light, in Fire', *Sustain. Mediterr. Constr.*, vol. Five, pp. 17–23, 2018.
[215] A. Rossi, 'The Façade of Paolo Soleri's Solimene Factory', *Nexus Netw. J.*, vol. 19, no. 2, pp. 503–520, Jul. 2017, doi: https://www.doi.org/10.1007/s00004-017-0336-x.
[216] F. Venezia and G. Petrusch, 'In forma elicoidale', *GRAN BAZAAR Riv. Bimest. Ambienti Immagini Luoghi Archit. Pers. Des. Cose E Parole Nn*, no. 5–6, pp. 166–167, 1983.
[217] F. Purini, *Una lezione sul disegno*, First Edition edition. Roma: Arte, Arredamento, Disegno, 1996.
[218] Q. De Quincy and A. Chrysostôme, *Dizionario storico di architettura. Le voci teoriche Quatremere de Quincy Antoine Chrysostôme*. Venice, Italy: Polis, Marsilio editori S.p.A, 1985.
[219] K. Krüger, 'St Michael's at Hildesheim: Scripture Networks and the Perception of Sacred Space', in *Sacred Scripture / Sacred SpaceThe Interlacing of Real Places and Conceptual Spaces in Medieval Art and Architecture*, vol. 23, Berlin, Boston: De Gruyter, 2019.
[220] C. Ulricht, 'Sankt Bernwards Würfel / Saint Bernward`s Cube', *Daidalos*, vol. Der gequälte Würfel / The Tortured Cube, no. 35, pp. 88–91, Mar. 1990.
[221] Jchancerel, *Eglise Saint-Michel d'Hildesheim, plan restitué de l'église ottonienne*. 2017.
[222] 'About GeoGebra', *GeoGebra*. https://www.geogebra.org/about (accessed Feb. 03, 2021).

[223] M. Kulcke, 'Spherical perspective in design education', *FME Trans.*, vol. 47, pp. 343–348, Jan. 2019, doi: https://www.doi.org/10.5937/fmet1902343K.
[224] A. De Rosa and A. Giordano, 'La geometria, lo spazio, la configurazione: un incontro con Anna Sgrosso', *diségno*, no. 2, pp. 009–015, Jun. 2018, doi: 10.26375/disegno.2.2018.2.
[225] A. Sgrosso, *Il problema della rappresentazione e dello spazio attraverso i tempi*, Stabilimento poligrafico IEM. Casoria, Naples, Italy, 1969.
[226] L. I. Kahn, *Louis Kahn: Essential Texts*, First. New York: W. W. Norton & Company, 2003.
[227] L. F. Olivero, M. Alí, A. B. Zucotti, and V. Ferraris, 'Los bits del lápiz y el grafito del pixel', in *III Congreso Internacional y VII Congreso Nacional de Expresión Gráfica en Ingeniería, Arquitectura y Carreras Afines*, Córdoba, Argentina, 2010, pp. 224–228.
[228] L. F. Olivero, G. Utrera, J. Aldapi, and S. Lafourcade, 'Propuesta 08', in *Publicación Workshop El discurso de la Sombra. FAPyD. UNR.*, UNR Editora., M. I. Rucq, Ed. Rosario, Argentina: Editorial de la Universidad Nacional de Rosario. Facultad de Arquitectura, Planeamiento y Diseño, 2010, pp. 89–93.
[229] L. F. Olivero, 'Dibujando, aprendiendo, enseñando', in *IV Congreso Internacional y IX Congreso Nacional de Expresión Gráfica en Ingeniería, Arquitectura y Carreras Afines*, La Plata, Argentina, 2012, pp. 424–430.
[230] R. Ferraris, S. Barba, and L. F. Olivero, 'La bitácora de viaje: aprehensión y transferencia en los procesos de aprendizaje', in *El dibujo de viaje de los arquitectos*, Gran Canaria, Spain, May 2014.
[231] 'Laboratorio di rilievo a cielo aperto… l'appendice estiva in Villa Rufolo'. https://villarufolo.com/laboratorio-di-rilievo-a-cielo-aperto-lappendice-estiva-in-villa-rufolo/ (accessed Aug. 14, 2020).
[232] A. B. Araújo, L. F. Olivero, and A. Rossi, 'A Descriptive Geometry Construction of VR panoramas in Cubical Spherical Perspective', *diségno*, no. 6, pp. 35–46, 2020, doi: https://doi.org/10.26375/disegno.6.2020.06.

OTHER

Adams, R. (2011, April 17). Spherical Perspective [Blog]. *Rob Adams a Painter's Blog*.
Agisoft PhotoScan Professional 1.4.1. (2018, March 18).
Alonso Sarría, F. (2013). *Sistemas de Información Geográfica*. Universidad de Murcia.
Araújo, António Bandeira, Olivero, L. F., & Antinozzi, S. (2019). HIMmaterial: Exploring new hybrid media for immersive drawing and collage. In P. Arantes, V. J. Sá, P. A. Da Veiga, & F. M. Adérito (Eds.), *Proceedings of ACM ARTECH conference (ARTECH2019)* (pp. 247–251). ACM Press. https://www.doi.org/10.1145/3359852.3359950
Araújo, António Bandeira, Olivero, L. F., & Rossi, A. (2019). Boxing the Visual Sphere: Towards a systematic solution of the cubical perspective. *REFLECTIONS the Art of Drawing | the Drawing of Art*, 33–40. https://www.doi.org/10.36165/1004
Araújo, António Bandeira. (2018c). Let's Sketch in 360º: Spherical Perspectives for Virtual Reality Panoramas. In Eve Torrence Bruce Torrence, Carlo Séquin & K. Fenyvesi (Eds.), *Proceedings of Bridges 2018: Mathematics, Art, Music, Architecture, Education, Culture* (pp. 637–644). Tessellations Publishing.
Automatic Panoramic Image Stitching using Invariant Features | SpringerLink. (n.d.). Retrieved 25 November 2018, from https://link.springer.com/article/10.1007/s11263-006-0002-3
Baker Eddy, M. (n.d.). *Mapparium*. Mary Baker Eddy Library. Retrieved 8 July 2019, from https://www.marybakereddylibrary.org/project/mapparium/
Barazzetti, L., Previtali, M., & Roncoroni, F. (2017). 3D modelling with the Samsung Gear 360. *Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci.*, *XLII-2/W3*, 85–90. https://doi.org/10.5194/isprs-archives-XLII-2-W3-85-2017
Barra, S., & Merlo, C. (Eds.). (2016). *Presente, pasado y futuro de la expresión gráfica* (Facultad de Arquitectura, Urbanismo y Diseño). Universidad Nacional de Córdoba.
Beckers, B., & Beckers, P. (2014). Discovering the Central Perspective. In *Reconciliation of Geometry and Perception in Radiation Physics* (pp. 1–30). John Wiley & Sons, Ltd. https://doi.org/10.1002/9781118984482.ch1

Bellos, A. (2014, October 30). Pumpkin geometry: Stunning shadow sculptures that illuminate an ancient mathematical technique | Alex Bellos. *The Guardian*.
Bianchi, N., Rucq, M., & Brebbia, V. (2015). Ocupándonos del vacío. *Entre Formas: Teoría, Investigación, Concreciones y Experiencias*. Congreso de ARS Buenos Aires, Mar del Plata, Argentina.
Bistagnino, E. (2018). *Il disegno nella Scuola di Ulm*. FrancoAngeli.
Borges, J. L. (2017). *Borges esencial. Edicion Conmemorativa / Essential Borges: Commemorative Edition* (Commemorative ed. edition). Real Academia Española / Alfaguara.
Bortot, A. (2016b). From the light's point of view: The concept of projection in anamorphosis and gnomonics. *Geometrias & Graphica 2015 Proceedings*, *1*, 355–363.
Boscolo, P. (2006). *Psicologia dell'apprendimento scolastico. Aspetti cognitivi e motivazionali*. Utet Università.
Bringing pixels front and center in VR video. (2017, March 14). Google.
Brown, M., & Lowe, D. (2003). Recognising Panoramas. *Proceedings of the Ninth IEEE International Conference on Computer Vision - Volume 2*, 1218-. http://dl.acm.org/citation.cfm?id=946247.946772
Brown, M., & Lowe, D. (2007). Automatic Panoramic Image Stitching Using Invariant Features. *Int. J. Comput. Vision*, *74*(1), 59–73. https://doi.org/10.1007/s11263-006-0002-3
Brown, M., Hartley, R., & Nister, D. (2007). Minimal Solutions for Panoramic Stitching. *2007 IEEE Conference on Computer Vision and Pattern Recognition*, 1–8. https://doi.org/10.1109/CVPR.2007.383082
Bruno Sucurado. (2020). Https://Roundme.Com. Retrieved 14 August 2020, from https://roundme.com/@bruno_0310/about
Cabezos Bernal, P. M., & Cisneros Vivó, J. (2013). La restituzione fotogrammetrica 2D/3D di elementi architettonici e l'integrazione dei modelli virtuali sulle fotografie dell'intorno reale, mediante programmi CAD, software liberi e fotocamere convenzionali. *DISEGNARECON*, *6*(12), 1–11. https://doi.org/10.6092/issn.1828-5961/3849
Càndito, C. (2003). *Le proiezioni assonometriche. Dalla prospettiva isometrica all'individuazione dei fondamenti del disegno assonometrico* (Alinea editrice s.r.l.). Lito Terrazzi - Cascine del Riccio.
Cardone, V. (2012). La Geometria descrittiva oggi. *Elogio della teoria. Identità delle discipline de disegno e del rilievo*, 15–22. http://hdl.handle.net/11386/3888193
Cardone, V. (2014). *Viaggiatori d'architettura in Italia: Da Brunelleschi a Charles Garnier* (1. ed.). Università degli Studi di Salerno.
Cardone, V. (2015). *Modelli grafici dell'architettura e del territorio: Vol. III edizione* (S. Barba, Ed.). Maggioli Editore.
Correia, J., Costa, M., Guerreiro, A., & Romao, L. (2015). Eyesight cartographies-unfolding the visual sphere. *Journal for Geometry and Graphics*, *19*(1), 117–130.
Cozzi, P. (n.d.). *Introducing 3D Tiles | cesium.com*. Cesium. Retrieved 15 September 2020, from https://cesium.com/blog/2015/08/10/introducing-3d-tiles/
Da Vinci, L., Richter, I. A., Wells, T., & Kemp, M. (Eds.). (2008). *Notebooks* (New Edition). Oxford University Press.
Damisch, H. (1994). *The Origin of Perspective* (J. Goodman, Trans.). Cambridge, Mass.: MIT Press.
Davis, C. (1995). *Osmose*. https://www.youtube.com/watch?v=54O4VP3tCoY&t=616s
De Simone, M. (1982). Presentazione. In *Palermo: La memoria costruita* (Vol. 1, pp. 9–29). S.F. Flaccovio.
Dipartimento di Storia dell'Arte. (2018). *Fra le Nuvole. Giornata di studi in ricordo di Hubert Damisch*. Facoltà di Lettere e Filosofia, Sapienza Università di Roma.
Docci, M., Maestri, D., & Migliari, R. (2017). *Scienza del disegno* (UTET Università). UTET Università.
Dos Santos, J. (2010). *GeoGebra, Internet, y intercomunicación con JavaScript*. https://doi.org/10.13140/RG.2.1.2647.0567
Eroglu, S., Gebhardt, S., Schmitz, P., Hausch, D., & Kuhlen, T. W. (2018). Fluid Sketching—Immersive Sketching Based on Fluid Flow. *2018 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*, 475–482.
Facebook 360, C. (n.d.). *Editing 360 Photos & Injecting Metadata*. Retrieved 4 May 2018, from https://facebook360.fb.com/editing-360-photos-injecting-metadata/
Falanga, M., & Battaia, L. (2000, 2020). *Geodetiche*. Batmath. Sito per Amanti Della Matematica e Della Fisica. http://www.batmath.it/matematica/a_ageo/capn/geodetiche.htm

Fangi, G., & Nardinocchi, C. (2013). Photogrammetric Processing of Spherical Panoramas. *The Photogrammetric Record*, *28*(143), 293–311. https://doi.org/10.1111/phor.12031

Felip Miralles, F., Gual Ortí, J., Cabeza González, M., & García-García, C. (Eds.). (2016). *Dibujar, Construir, Soñar. Drawing, Building, Dreaming* (I). Tirant lo Blanch.

Fernández Rodríguez, M. (2005). Secciones en cuádricas: Homología entre sección y contornos aparentes. In ABEG (Ed.), *Graphica 2005*. ABEG.

Figueiredo, M., Böhm, K., & Teixeira, J. (1993). Advanced interaction techniques in virtual environments. *Computers & Graphics*, *17*(6), 655–661. https://doi.org/10.1016/0097-8493(93)90114-O

Fong, C. (2019). *Homeomorphisms Between the Circular Disc and the Square*. https://doi.org/10.1007/978-3-319-70658-0_27-1

French, R. (1987). The Geometry of Visual Space. *Noûs*, *21*(2), 115–133. JSTOR. https://doi.org/10.2307/2214910

Friedberg, A. (2009). *The virtual window: From Alberti to Microsoft*.

Fundación, G.-S. D. (n.d.). *Dalí. Estereoscopías. La pintura en tres dimensiones.* Dali exhibitions. Retrieved 29 January 2019, from https://exhibitions.salvador-dali.org/es/daliestereoscopies/

Giordano, A., & Maggio, F. (2017). La figurazione dello spazio architettonico di Gaspare De Fiore. *diségno*, *1*, 159–164. https://doi.org/10.26375/disegno.1.2017.17

Gómes Rivero, Á., Guerrero Santos, P. L., & Jiménez Rueda, A. (n.d.). *Dibujo Técnico para Ingeniería Industrial y Civil—Homología, Homotecia* [University website]. Dibujo Técnico Industrial y Civil. Retrieved 8 July 2019, from /geomplana/transformacionesplanas/

Gómez, R. C., & Calabuig, D. D. (2013). El valor y el propósito de un dibujo de Louis I. Kahn. *EGA Expresión Gráfica Arquitectónica*, *18*(22), 242–251. https://doi.org/10.4995/ega.2013.1253

Goodman, N. (1968). *Languages of Art. An approach to a theory of symbols* (Second printing). The Boobs-Merrill Company, Inc.

Hand with Reflecting Sphere. (2020). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Hand_with_Reflecting_Sphere&oldid=963427173

Harrison, E., Mahdavi-Amiri, A., & Samavati, F. (2011). Optimization of Inverse Snyder Polyhedral Projection. *2011 International Conference on Cyberworlds*, 136–143. https://doi.org/10.1109/CW.2011.36

Houbraken, A. (1753). *De groote schouburgh der Nederlantsche konstschilders en schilderessen* (Heidelberg University Library). J. Swart, C. Boucquet, M. Gaillard. https://doi.org/10.11588/diglit.44224

Immersion (virtual reality). (2018). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Immersion_(virtual_reality)&oldid=844519485

Irles Más, F. (2015). *Transformaciones homográficas de modelos tridimensionales. Análisis proyectivo para el desarrollo de una aplicación infográfica.* http://dspace.umh.es//handle/11000/1751

Ivanova, O. V. (n.d.). *VR Oxygen Case Study*. Retrieved 16 October 2018, from https://ol-iva.com/VRO2-CaseStudy.html

Javascript Graphics Library jsDraw2D draw line, circle, rectangle, polygon, curve, ellipse. (n.d.). Retrieved 6 July 2019, from http://jsdraw2d.jsfiction.com/

Jiang, D., & Kim, J.-W. (2018). *Artwork Recognition for Panorama Images Based on Optimized ASIFT and Cubic Projection* (Vol. 8). https://doi.org/10.18178/ijmlc.2018.8.1.663

Kahn, Louis Isadore. (1931). *The Value and Aim in Sketching*. T-Square Club Journal of Philadelphia.

Kamperman, K. (2019). *Kasperkamperman/MobileCameraTemplate* [JavaScript]. https://github.com/kasperkamperman/MobileCameraTemplate (Original work published 2018)

Kauhanen, H., Rönnholm, P., & Lehtola, V. (2016). Motorized panoramic camera mount & calibration and image capture. *ISPRS Annals of Photogrammetry, Remote Sensing and Spatial Information Sciences*, *III–5*, 89–96. https://doi.org/10.5194/isprs-annals-III-5-89-2016

Korsmeyer, C. (1989). Review of Curvilinear Perspective: From Visual Space to the Constructed Image [Review of *Review of Curvilinear Perspective: From Visual Space to the Constructed Image*, by A. Flocon, A. Barre, & R. Hansen]. *The Journal of Aesthetics and Art Criticism*, *47*(2), 190–191. JSTOR. https://doi.org/10.2307/431839

Krishnamurti, R. (2011). *Basic Concepts of Descriptive Geometry*. RK School of Architecture, Carnegie Mellon University.

Kwang, T. S. (n.d.). *Embedding GeoGebra Apps into SLS – Physics Lens*. Retrieved 3 July 2019, from https://physicslens.com/2018/10/19/geogebra-in-sls/

Lange-Küttner, C., & Ebersbach, M. (2013). Girls in detail, boys in shape: Gender differences when drawing cubes in depth. *British Journal of Psychology*, *104*(3), 413–437. https://doi.org/10.1111/bjop.12010

Lazzarini, P. (1997, 2017). *Geometria sulla sfera. Un approccio elementare alla geometria non euclidea*. Home Page di Paolo Lazzarini. http://www.paololazzarini.it/geometria_sulla_sfera/geo.htm

Liu, L. (2018). *Sketch-to-VR: Personal fun project - turn your mocks into VR* [JavaScript]. https://github.com/auxdesigner/Sketch-to-VR (Original work published 2016)

Lolas, C. (2014). Inmersividad, orígenes y nuevos medios. *Escáner Cultural. Revista Virtual de Arte Contemporáneo y Nuevas Tendencias.*, *XVI-number 173*. http://revista.escaner.cl/node/7473

Lu, M., Wang, X., Liu, X., Chen, M., Bi, S., Zhang, Y., & Lao, T. (n.d.). Web-based real-time visualization of large-scale weather radar data using 3D tiles. *Transactions in GIS*, *n/a*(n/a). https://doi.org/10.1111/tgis.12638

Maldonado, T. (1984). Ulm rivisitato. In *RASSEGNA 19. Il Contributo Della Scuola Di ulm/ the Legacy of the School of Ulm* (C.I.P.I.A. s.r.l.). Rassegna.

Marr, D. (2010). Vision: A Computational Investigation into the Human Representation and Processing of Visual Information. In *Vision*. The MIT Press.

Messina, B. (2020). Women and Descriptive Geometry in Italian University. In P. Magnaghi-Delfino, G. Mele, & T. Norando (Eds.), *Faces of Geometry. From Agnesi to Mirzakhani* (pp. 141–154). Springer International Publishing. https://doi.org/10.1007/978-3-030-29796-1_13

Migliari, R. (2000). *Fondamenti della rappresentazione geometrica e informatica dell'architettura*. Kappa.

Migliari, R. (2003). *Geometria dei Modelli. Rappresentazione grafica e informatica per l'architettura e per il design*. Kappa.

Migliari, R. (2018). Lo specchio alle origini della prospettiva—The mirror at the origin of perspective. *Disegnare Idee Immagini / Drawing Ideas Images*, *Year XXIX, Number 56*, 42–51.

Migliari, R. (Ed.). (2004). *Disegno come Modello. Riflessioni sul disegno nell'era informatica*. Edizioni Kappa.

Mobile First Camera App Template (HTML, CSS, JS and WebRTC) ⍰ Kasper Kamperman. (n.d.). Retrieved 5 September 2019, from https://www.kasperkamperman.com/blog/camera-template/

Montes-Serrano, C. (2005). *Louis Kahn en la costa de Amalfi (1929)*. http://dadun.unav.edu/handle/10171/18020

Morton Heilig's Sensorama (Interview).mov. (2011, January 5). https://www.youtube.com/watch?v=vSINEBZNCks

Nakamura, J. P. (2020b). Seeing Outside the Box: Reexamining the Top of Samuel van Hoogstraten's London Perspective Box. *Journal of Historians of Netherlandish Art*, *12*(2), 1–25. https://doi.org/10.5092/jhna.12.2.3

Next-generation video encoding techniques for 360 video and VR. (2016, January 21). *Facebook Code*. https://code.fb.com/virtual-reality/next-generation-video-encoding-techniques-for-360-video-and-vr/

Pacific Domes Virtual Reality VR Sphere, 360 Immersion, Dubai 360 – VIDEO. (n.d.). Retrieved 11 October 2020, from https://www.youtube.com/watch?v=kxIn1Nj6C_8

PAN, P. delle A. D. N. (2019). *Exhibition in Naples, Italy extended till 22nd of April 2019*. M.C. Escher – The Official Website. https://mcescher.com/exhibition-in-naples/

Pérez Ramos, A., & Robleda Prieto, G. (2016). Only image based for the 3D metric survey of gothic structures by using frame cameras and panoramic cameras. *ISPRS - International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, *XLI-B5*, 363–370. https://doi.org/10.5194/isprs-archives-XLI-B5-363-2016

Pérez, J. H. (2012). Matemáticas en el Arte: La Geometría del Espacio en Las Meninas. *Pensamiento Matemático*, *2*(2 (Octubre)), 157–165.

Pestinenza Puglisi, L. (2004). Corpo e mente: Scenari tradizionali e digitali nella ricerca architettonica. In R. Migliari (Ed.), *Disegno come modello. Riflessioni sul disegno nell'era informatica* (pp. 20–25). Kappa.

Poly. (n.d.). Poly. Retrieved 14 January 2019, from https://poly.google.com

Puglisi, L. P. (2001). *Silenziose Avanguardie: Una Storia Dell'architettura 1976-2001*. Testo & Immagine.

Purini, F. (1992). Nota introduttiva a 'Arti elettroniche' di Lorenzo Taiuti. *XY Dimensioni del disegno*, *14–15*, 45.

Pyramid (geometry). (2019). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Pyramid_(geometry)&oldid=905288046

Reflection mapping—Wikipedia. (n.d.). Retrieved 14 July 2019, from https://en.wikipedia.org/wiki/Reflection_mapping
Richardt, C. (2017). Stereoscopic 3D Videos and Panoramas. *SIGGRAPH 2017*, 51.
Richardt, C., Tompkin, J., Halsey, J., Hertzmann, A., Starck, J., & Wang, O. (2017). Video for virtual reality. *ACM SIGGRAPH 2017 Courses on - SIGGRAPH '17*, 1–159. https://doi.org/10.1145/3084873.3084894
Rieuf, V., Bouchard, C., Meyrueis, V., & Omhover, J.-F. (2017). Emotional activity in early immersive design: Sketches and moodboards in virtual reality. *Design Studies*, *48*, 43–75. https://doi.org/10.1016/j.destud.2016.11.001
Roberto de Rubertis. (2018). Towards which Representation? *Diségno*, *0*(2). https://doi.org/10.26375/disegno.2.2018.5
Rory, R. (2012, January 15). Cubemap Texel Solid Angle – CodeItNow. *CodeItNow*. http://www.rorydriscoll.com/2012/01/15/cubemap-texel-solid-angle/
Rossi, A. (1991). *Il costruito 'ad quadratum' tra memoria e ragione. PhD Thesis in 'Rilievo e rappresentazione del costruito'. Advisor: Rosa Penta. Coadvisor: Emma Mandelli.* [Università degli Studi di Palermo]
Rossi, A. (1999). Study the works of Eisenman? Why ?! *Nexus Network Journal*, *I*(n°1), 65–74
Rossi, A. (2014). *Introduzione al disegno informatico per l'architettura e l'ingegneria edile* (Edizioni Scientifiche e Artistiche, pp. 0–232). https://iris.unicampania.it/handle/11591/161615?mode=full.14#.Xu30n_JS924
Rossi, A. (2015). Il Vero si prolunga nel Verosimile. In M. T. Bartoli & M. Lusoli (Eds.), *Le teorie, le tecniche, i repertori figurativi nella prospettiva d'architettura tra il '400 e il '700: Dall'acquisizione alla lettura del dato* (pp. 335–346). Firenze University Press. https://iris.unicampania.it/handle/11591/337237?mode=full.330#.Xu3zk_JS924
Rossi, A. (2017a). Abitare la prospettiva: Aspirazione e traguardo della ricerca artistica. In G. M. Valenti (Ed.), *Prospettive architettoniche. Conservazione digitale, divulgazione e studio: Vol. II* (Università La Sapienza, pp. 251–276). Università La Sapienza. https://doi.org/10.13133
Rossi, A. (2017b). *Territori e Frontiere della Rappresentazione. Territories and Frontiers of Representation.* (Gangemi, Vol. 6).
Rossi, A., & Olivero, L. F. (2018). Immersive models from analogical sketches applied to Solimene's Factory. *VII Congreso Internacional y XV Congreso Nacional de Expresión Gráfica En Ingeniería, Arquitectura y Carreras Afines*, 55. http://hdl.handle.net/11591/396733
Rossi, D. (2018). Brand new: Panorama. L'immagine continua per il disegno della realta virtuale / Brand new: Panorama. Continuous image to shape virtual reality. *Rappresentazione / Materiale / Immateriale. Drawing as (in)tangible representation*, 1389–1396.
Rossi, D. (2019). First Person Shot: La prospettiva dinamica interattiva negli ambienti virtuali immersivi. *REFLECTIONS the Art of Drawing / the Drawing of Art*, 977–984.
Rossi, M., Mele, G., & Buratti, G. (2018). La prospettiva come architettura immateriale. Il finto coro di Santa Maria presso San Satiro/The Perspective as immaterial Building. The fake Choir of Santa Maria by San Satiro. *Rappresentazione / Materiale / Immateriale. Drawing as (in)Tangible Representation*, 215–224.
Schneider, D., & Maas, H.-G. (2004). Photogrammetric processing of digital 360° panoramic camera image data. *GIM International*, *7*(04), 68–71.
Scolari, M. (1982). Considerazioni e aforismi sul disegno. *Rassegna*, *9*, 79–83.
Screech, T. (2018). *The Lens Within the Heart: The Western Scientific Gaze and Popular Imagery in Later Edo Japan*. Routledge. https://doi.org/10.4324/9781315029313
Scully, V. (1991). *The Paintings and Sketches of Louis I. Kahn* (Jan Hochstim). Rizzoli Intl Pubns.
Self-portrait in a Convex Mirror. (2019). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Self-portrait_in_a_Convex_Mirror&oldid=905719499
Shannon, S. (2016). *Panoramas: 19th-Century Virtual Reality* [Critic history]. Shannon Selin. Imagining the Bounds of History. https://shannonselin.com/2016/11/panoramas-19th-century/
Silva, E. E., & Ávila Fuenmayor, F. (2002). Random and Art Concepts in Jorge Wagensberg. *2002*, *19*, 93–111.
Snapshot. (n.d.-a). Retrieved 20 January 2021, from https://en.wikipedia.org/w/index.php?title=Equirectangular_projection&oldid=998076451
Snapshot. (n.d.-b). Retrieved 16 May 2018, from http://artes.ucp.pt/citarj/article/view/471

Sriraman, B., Michelsen, C., Beckmann, A., & Freiman, V. (Eds.). (2007). *Interdisciplinary Educational Research In Mathematics and Its Connections to The Arts and Sciences* (The Montana Mathematics Enthusiast). IAP Information Age Publishing, Inc.
Stavness, I., Lam, B., & Fels, S. (2010). pCubee: A Perspective-corrected Handheld Cubic Display. *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, 1381–1390. https://doi.org/10.1145/1753326.1753535
Stevens, K. A. (2012). The Vision of David Marr. *Perception*, *41*(9), 1061–1072. https://doi.org/10.1068/p7297
Tate. (n.d.). *Medium – Art Term TO MYSELF*. Tate. Retrieved 1 June 2019, from https://www.tate.org.uk/art/art-terms/m/medium
TIME. (n.d.). This Is the Future of Art. TIME introduces Google's Tilt Brush. *Time*. Retrieved 14 January 2019, from http://time.com/vr-is-for-artists/
Tribuna Grimani VR – Chiara Masiero Sgrinzatto. (n.d.). Retrieved 19 January 2021, from https://www.chiaramasierosgrinzatto.com/2020/05/07/tribuna-grimani-vr/
Triggianese, A. (2013). *Epifanie della visione. Geometrie e interpretazioni delle trascrizioni anamorfiche.* [Tesi di dottorato, Università degli Studi di Napoli Federico II]. https://doi.org/10.6092/UNINA/FEDOA/9185
Uki-e. (2017). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Uki-e&oldid=759715948
Ukiyo-e. (2019). In *Wikipedia*. https://en.wikipedia.org/w/index.php?title=Ukiyo-e&oldid=902595829
Venice, Bird's eye view – Chiara Masiero Sgrinzatto. (n.d.). Retrieved 20 January 2021, from https://www.chiaramasierosgrinzatto.com/2021/01/01/venice-birds-eye-view/
Viana, V., Gonzalez, F., Braviano, G., Couceiro Costa, M., & Murtinho, V. (Eds.). (2016). *Geometrias & Graphica 2015 Proceedings* (Universidade Lusíada de Lisboa, Vol. 1). APROGED - Associação dos Professores de Desenho e Geometria Descritiva. http://www.aproged.pt/geg2015/proceedings1.pdf
Vizible | Virtual Reality Collaboration Software. (n.d.). Retrieved 16 September 2020, from https://www.worldviz.com/virtual-reality-collaboration-software
Wahbeh, W., & Nardinocchi, C. (2015). Toward the Interactive 3D Modelling Applied to Ponte Rotto in Rome. *Nexus Network Journal*, *17*(1), 55–71. https://doi.org/10.1007/s00004-015-0238-8
Weisstein, E. W. (n.d.). *Stereographic Projection* [Text]. Retrieved 21 January 2019, from http://mathworld.wolfram.com/StereographicProjection.html
Wisniewski, M. (2018). *Globus Map Projector*. https://www.winski.net/globus-projector/simplified/
Zonno, M., Maiellaro, N., & Capotorto, S. (2012). *Fotografia immersiva applicata ai beni culturali*. Aracne.

ACKNOWLEDGMENTS

This book closes either a long research about drawing and a big cycle of my life. The investigation started many years ago as an “intuition” and it arrives to the full development of a new perspective system. The book is about the theory and the practice for drawing in cubical perspective and has the intention to give a full theoretical background, guidelines, good practices, algorithms, and the first case studies of its application. My hope is that this knowledge would contribute to open the debate and to generate new methods for this representation system that is just coming to the light in its full potential.

I wanted to write a preface at “Vito Cardone’s way”, that is, summarising facts, persons and dates to humbly try to make justice to the history and people that brought me until here. Yet, I finally decided to include this chapter in the end, so to do not deviate the focus of the research. So, for those curious – such as grandmas or non-experts of the drawing field - I will recapitulate in the next paragraphs about my personal experience and how I arrived here.

This research arrived at my life getting shaped step by step since my early experiences and studies. I had my first drawing lessons during my childhood in Salta, Argentina[40] with that kind of that have more heart than technical skills[41]. With these humble art lovers, I exercised basic notions about composition. I feed my skills with their teachings and painted “*Naturaleza Muerta”* (Still Life), vessels, angels and Christian scenes made in ceramic, artworks[42]. I easily

[40] My hometown city is actually Córdoba, but during the period 1989-1994 I was living in Salta, in the North of Argentina. During those five years I travelled to Córdoba mainly for holidays. In one of these travels, when I was maybe six or seven, I asked to my parents to stay alone for some weeks at my grandparents’ place. They agreed and I dedicated great part of that moments to drawing and painting

[41] I had lessons in Salta in which I painted animals, fruits and Christ’s face. My grandmother, Santina Giordano de Chirio, taught me to paint with pencils filling children’s books during my holidays in Córdoba

[42] From 1994 to 2001 (approximately) I spent every holiday at my godmother’s place taking painting lessons with her. Edith Mafalda Olivero de García, who I’m deeply thankful to, is still being a devoted Christian believer, so that is the reason behind this kind of ceramics

coloured existing figures or copied from reality. Still, I remember to feel frustrated when drawing something new: I could *see* machines, mechanisms and environments in my mind, but I could barely materialise any of them on the paper. With the lessons I got some skills with acrylics and watercolours, but it was not enough, I was yet very far from the structure and order of any representation system. It may sound sappy, but my never-stop research started with these questions: **How to illustrate accurately what I *see* in my mind? How to draw what I *feel*?**

Such a frustration, I will get to know later, was the feeling that every line was already a restriction, a mistake. My scratches were *reducing* and not *representing* the "perfection" of the thing. Many years after, reading Louis Kahn's dialog "Form and Design", I found this: "*The first line on paper is already a measure of what cannot be expressed fully. The first line on paper is less*" [226, p. 63]. Kahn's teachings would had been particularly helpful during my youth to avoid ripping and trashing so many pages from my grandfather's homemade *talonarios*[43]. I performed such a waste of paper with the ferocity and anxiety of someone who feels the risk to lose an idea and still I got almost no one. **As my professors, I had just heart.**

When I was twelve, I got basic notions of perspective by my myself, running a basic software on my mother's computer. During the technical aeronautical high school[44], I acquired the first skills to draw in floorplan, section and axonometric views. I practiced using both traditional and digital techniques[45]. Already in the university, I reinforced this knowledge during my Computer Sciences" first year[46]. After Engineering, I jumped to Architecture[47] in the Faculty of Architecture and Design (FAUD) at the National University of Córdoba (UNC). There,

[43] Notebooks

[44] Period 1998-2003, School IPEM Nº 251 Guarnición Aérea Córdoba, Córdoba, Argentina

[45] As "traditional" technique I used analogical handmade drawings in paper with rulers and compass. By digital, the construction of such pieces in AutoCAD R14 version

[46] Period 2004-2007, Faculty of Engineering (FCEFyN), National University of Córdoba (UNC), Argentina

[47] Period 2008-2015, Faculty of Architecture (FAUD), UNC, Argentina

I strengthened the application of perspective, the double orthogonal projection, axonometry and mixed techniques with photography, digital collage, photomontage and digital drawing.

I saw that the more I practiced and studied, the closer I was to get my thoughts on the paper: on the one hand, every system of representation gave order to my deliberations, making them concrete and measurable "*Physical nature is measurable (…) Feeling and dream has no measure, has no language, and everyone's dream is singular (…) Thought is Feeling and presence of Order (…) To remain in Feeling away from thought means to make nothing*" [226, p. 63]. On the other hand, *every technique gave me the best match between tool, representation system and planning stage*: the naturality of handmade traditional drawing during early conceptual design, the powerfulness of digital modelling tools when precision is a must and the versatility of hybrid techniques for verifying and presenting options. I understood that was very difficult - and reductive for each technique - to use the same tool for capturing the inner dialog when reasoning; for defining and modifying in those moments in which the exactness, accuracy and interrelation among parts matters; and for exploring or presenting alternatives either to the designer itself or to the client.

To this point, the limitation that I found back then as a child turned into a "*defining while drawing*" process, which implied to *already think* in a specific setup of technique and representation system. Playing among techniques and associated perspective / floorplan / sections, I covered more and more angles of what was inside my mind. My main issue was starting to get solved but there was still much to discover so, in parallel to my studies and thirsty of answers, I expanded my research doing academical exchanges, attending congresses and collaborating within teaching activities.

I remember starting to *draw while travelling* in January 2009 during a holiday in Patagonia. After a short time, I visited many more cities in Argentina, exchanged and participated in workshops with Paraguay[48], Uruguay[49], South Africa[50] and Italy[51]. In 2009 also, I began a collaboration as a volunteer with the non-profit institution *EGraFIA* (Graphical Expression in Engineering and Architecture). For the years to come until 2016 and thanks to Roberto Ferraris, I participated and organised many international events with this institution[52]. The contact with new cultures allowed me to see how others were using drawing and the international interaction brought me new opportunities.

In 2011 and 2012, I got selected to travel with a group of students for drawing workshops in Italy. It is known that the situation in Argentina was never easy, and especially my personal situation was in one of the worsens points. Nevertheless, after huge efforts, difficulties, sour disagreements, wonderful surprises and invaluable supports, I finally travelled to Europe. Far away from tourism and holidays, I worked as much as I could, trying to capture as many information as possible while following a sort of *Grand Tour*. I discovered full of joy cities such as Rome, Naples, Amalfi, Positano, Ravello, Salerno, Paestum, Venice, Turin, Milan, Florence, Madrid, Toledo, Barcelona among many other places. The goal for these two trips was the same: to draw the architecture and the city. And to draw is what I did: in 2011 during the first trip[53] I filled almost 10 linear meters of the Japanese Moleskine notebooks, while in 2012[54] I completed 15 meters on similar travelogues, but this time created by my own [bitácoras]. I

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[48] Thanks to a bursary from the exchange program *MARCA* I did five months in Asunción, Paraguay, from August 2012 to January 2013

[49] Architecture planning workshop, Montevideo, Uruguay, September 2010

[50] Academical exchange Argentina/South Africa *"Uniendo dos continentes"* April-May 2011

[51] Academical exchange Argentina/Italy *"Cuaderno de viaje"* March 2012

[52] Organisation committee member of III (Córdoba 2010), IV (La Plata 2012) and V (Rosario 2014) *"Congreso Internacional de Expresión Gráfica en Ingeniería, Arquitectura y Carreras Afines"*. Organisation and coordination of international workshops: *"Cuaderno de viaje. La caravana gráfica con Frank Ching"* (2012) and *"La Caravana Gráfica II Edición: Grafitos + Píxeles"* (2014) with students and teachers from Argentina, Brazil, Peru, Paraguay and Italy

[53] *"Il disegno dei viaggiatori"* organised by FAUD, UNC and Department of Civil Engineering (DICIV), University of Salerno (UNISA). 14.07 to 13.08.2011

[54] *"Dibujando Italia"* and *"Architetture Per Il Cinematografo Tra Ottocento E Novecento: conoscenza e valorizzazione"*, organised by FAUD, UNC and Polytechnic of Turin 11.06 to 01.07.2012

drew using graphite, pencils, crayons, pens, watercolours, markers, acrylics… I tested horizontal and vertical compositions, one, two, seven modules of the sketchbook. I mixed text and drawings, wrote about what I saw, what I listened, what I tasted.

However, I felt that this huge amount of experience drawing was not enough to capture the thousand feelings in front of the Dome of Brunelleschi in Florence, the Cinema Museum in Turin or the Sistine Chapel in the Vatican State. I wanted more, **I wanted to draw everything.** I started to enlarge the field of view of my perspectives and passed from the classical cone with up to two vanishing points to an extended cone with a third one. I did not realise in that moment that I was already tracing the next steps: widening the FOV was the material manifestation that I wanted to literally *enter the drawing*. I lectured in Italy for the first time in my life on July 13th, 2012 at the *Laboratorio Modelli*, UNISA by invitation of Vito Cardone[55], summarising in the presentation graphical answers to my big question.

My choice of drawing over taking pictures glowed exponentially in every new trip as it represented the best and personal way to scrutinise what I was perceiving instantly and for the first time: the specific setup of tool and utility led me to realise that handmade drawings are an excellent instrument to synthetize the huge amount of information arriving when knowing something new. Such a reduction recalls significative elements, filtering just those captured by the intellect. Again, the youthful "limitation" becoming a tool.

Since the beginning in Architecture, I had my first contacts with teaching: I began helping as volunteer in exams *Sistemas Gráficos de Expresión*[56] and *Gráfica Arquitectónica*[57] at FAUD, UNC, Argentina. Already living in Italy permanentely, I collaborated within the *Laboratorio*

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[55] *"Un viaggio di china"*, coordinated by Vito Cardone, lecture in the framework of the academical exchange between UNISA and UNC

[56] Exam of *Arquitectura* career, volunteer from 2009 to 2011

[57] Exam of *Arquitectura* career, volunteer from 2010 to 2011

Modelli[58] from Salerno's University, either helping or as responsible for academical activities in *Rilievo dell'Architettura*[59] and in *Disegno dell'Architettura II*[60]. During these years, I followed as co-advisor Carla Donato's research[61], one of my greatest experiences in the field of teaching and the key for the developments to come.

Until here, representation systems, techniques, travelling, academical exchanges and teaching activities were summarised in my first articles, shaping more concretely some answers to my subject. I published them either as papers in international congresses or as book chapters: in 2010, I wrote about the pros and cons of traditional, hybrid and digital techniques [227]. I also published that year a graphical exploration about the use of the shadows for architecture planning [228]. In 2012, the article pointed to travel drawing and its further influence in the design process, analysing Alvar Aalto and Louis Kahn's examples [229]. In 2014, I wrote about the fluent workflow for thinking and drawing that gives the use of the continuous paper [230].

In November 2013 I left Argentina and moved to Italy to join a double degree agreement between FAUD, UNC and DICIV, UNISA. The program expected students from FAUD, after finishing all the exams except the final work, performing six exams plus the final research in Italy, attaining at once the master's degree in architecture-building engineering (UNISA) and the architecture degree (UNC). In one of these exams, *Rilievo dell'Architettura,* I learned about full-immersive panoramic photography surveying ancient Roman *Villas* near Pompei and

58 Laboratory part of the Civil Engineering Department (DICIV), University of Salerno (UNISA), Italy

59 Exam of *Ingegneria edile-architettura* career, academical years 2015-16, 2016-17, 2017-18 and 2018-19

60 Exam of *Ingegneria edile-architettura* career, academical years 2017-18 and 2018-19

61 Carla Donato was among the three best students of her last academical year at FAUD and attained the double degree program as the first student from Córdoba reaching the maximum qualification in the Italian system (110 and lode) within the mentioned agreement UNC-UNISA. Her research produced a virtual tour that revived Louis Kahn's experience in the Amalfi Coast. During the survey, got consolidated an important – and yet existing – collaboration between the Laboratorio Modelli, DICIV and Villa Rufolo, Ravello, Italy

creating virtual tours[62]. At the final moment of this exam, Paolo Alfonso[63] showed how to switch from the equirectangular projection (commonly used with virtual tours) to a cubical format. One of the disadvantages of the equirectangular projection when editing its content in postproduction, is the management of the curvilinear graphic information: in the poles there is an extreme distortion which is hardly handy and makes almost impossible any modification in that zone. In the cubical format instead, the group of Paolo managed easily to delete the tripod or edit parts of the photography since the distortion was *known*. Indeed, every face of the cube has nothing but a classical perspective!

After the double degree program[64] I worked for a private company[65] and among my responsibilities were the acquisition of giga-panoramas and the creation of virtual tours. I surveyed historical castles in Bisaccia, Monteverde, Morra de Sanctis, Sant'Angelo dei Lombardi and Torella dei Lombardi in Alta Irpinia region, Italy, for the project *Castelli di Storia*; and Vibo Valentia's castle in Calabria region. I also surveyed museums: in Cosenza, the open-air museum Bilotti - MAB and in Naples the *Museo Civico Gaetano Filangieri* and the *Fondazione Circolo Artistico Politecnico*. Many of these experiences were held thanks and in collaboration with the above-mentioned *Laboratorio Modelli* which gave me, beside the opportunity to use updated high-tech instrumental, the technical knowledge about full-immersive photography, the equirectangular format, the software and workflows for virtual tours. In any case, I never forgot my big question, and the utility of all these experiences finished showing up in 2017.

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[62] During the academical year 2013-14 we surveyed Villa San Marco, Villa Ariana, Oplontis o Villa Poppea and the Necropolis of Cuma

[63] One of the members of the group that surveyed Oplontis

[64] On April 27th, 2015 and October 14th, 2015, I attained the master's degree *Ingegniere edile-architetto* (UNISA) and *Arquitecto* (UNC) respectively

[65] NAOS Consulting, period 2015-2017

By 2016, at the beginning of Carla Donato's research, I had a meeting to brainstorming about ideas for thesis. I proposed to the student Lucía Fernández to try *drawing on top of equirectangular pictures*, but Lucía followed instead a documentary research about Villa Rufolo's *Balnea* (with magnificent results[66]). A while later, Carla found the work of Bruno Sucurado who was drawing in equirectangular projection and sharing his drawings on Facebook and RoundMe [107]. His work was analysed and then Bruno got invited to draw for Donato's thesis and to give a workshop in Italy within the lessons of *Rilievo dell'Architettura* 2016-17. During that year I was responsible for the module *Disegno dal Vero* within the same exam, so I found myself talking with Bruno about full-immersive drawings by the end of March 2017.

In some moment of our talk with Bruno and reminding the above-mentioned experiences, a flash crossed my mind: **if I can draw in equirectangular format, then I can also do it in the cubical one!** I ran to my desk, drew anxiously an open cube in a piece of paper and a couple of lines inside of it. I scanned the drawing, switched to the equirectangular format and navigated the results: it worked! Awfully and with graphical errors, but it worked. I explored many other options of opening the cube and practiced with the few knowledge that I had about it. Bruno and me, shared the two formats within classrooms that year. The Engineering students from UNISA and their peers from Architecture UNC developed with dedication and great results their drawings, surveying real spaces of the UNISA's campus.

I was on fire, I had many things to learn yet, but I had two new full-immersive drawing methods to explore and to draw everything: I practiced and practiced, and I quickly got used to

[66] Lucía Fernández found during her research documentary evidence about Villa Rufolo's Balnea that changed the story itself of this iconic place. Her final presentation at UNISA was introduced by the director of Villa Rufolo's Foundation, Dr. Secondo Amalfitano [181], [231]

the equirectangular drawing. It was perfect for my intentions and I started to try it with artistic applications while listening to live music.

Bruno's teachings were based in following a grid with meridian and parallels. Yet, this grid allowed a trustable drawing with a field of view up to +/- 70° approximately, so it was more a cylindrical drawing in equirectangular projection than a proper full-spherical one. One of the pitfalls of this grid is that the guidelines are too far from each other, so the distortion in the poles is managed without suitable accuracy. To overcome this difficulty, I started to build a handmade own grid, since I was "naturally" used to the distribution of the curvilinear guidelines due to my familiarity with equirectangular pictures. I synthetized my experience up here in a journal article shared with Bruno, an innocent article pointing the beginning of a long path to come [119].

For the cubical perspective instead, I just had understood the very basic principles. I did a first bibliographic research, looking to find all the rules and curious to learn this novel representation system. Curiously, nothing appeared except some extremely poor explanations in entry blogs not saying more than I had in mind. My first reaction was ascetical, but I started to suspect: **Is it possible that there is nothing about cubical perspective? Could it be that cubical perspective has not been developed yet?** With these questions in mind, I got an invitation to lecture on 25th May 2017 in the University of Campania *Luigi Vanvitelli* (UNICAMPANIA) for the so-called *Giovedì del Dottorato*[67]. Behind the organization of this seminar, there was Adriana Rossi, who invited Carla Donato, María Josefa Agudo Martínez from Spain and me to talk in the session "*Abitare la prospettiva: scenari nell'era post-digitale*". That day, Argentinian holiday to remember May's revolution, I got to know three very important things for my further developments: first, I heard about the PhD program *Ambiente, Design e*

[67] The seminar "*Il Giovedì del Dottorato*" is a curricular activity part of the PhD program in *Ambiente, Design e Innovazione*, UNICAMPANIA

Innovazione to which I decided to apply (and here we are!). Second, I met personally Adriana after many years of just exchanging e-mails with her[68]. Matter of destiny or not, but after a couple of months she would become my advisor. Last but not least, thanks to Adriana's management of the seminar, I tasted during lunch the best *mozzarella di bufala* of Aversa.

I started this PhD program in November 2017, I wanted to research about the "complexity behind drawing", something that nowadays neither do I understand what it means. I just knew that I wanted to go deeper into drawing practice. The final subject started to shape slowly: with the development of the state of art I realized what I suspected: cubical perspective had no systematic definitions for architecture drawing which, I must confess, was difficult to believe even for myself.

Going ahead with the research, I wrote a first article about this perspective for the conference of the *Unione Italiana per il Disegno* (UID) held at Milan in September 2018 [17]. The article presented a first workflow to produce a simple cubical perspective and to visualize them using free and open-source software. After the congress, the same team was invited to write an extended article to be published in the Journal *Diségno*. In that opportunity, we approach a more defined geometrical method, gave a first algorithm[69] and highlighted the need to extend a collaboration with someone from the mathematical field [199].

Almost one year after the experience in Salerno, Bruno sent me a paper written by a mathematician about equirectangular drawing [34]. I read the paper and found it difficult to understand since my English skills were even more pathetic than now and the knowledge pointed directly to a mathematical interpretation to which I was not used to. After a couple of readings, I shaped the message in my mind, founding it stimulating since it reflected the kind of development

[68] Adriana Rossi attended many international congresses of EGraFIA and the years I collaborated with the institution I was behind the mailbox secretary replying e-mails

[69] Limited to horizontal and vertical lines parallel to the faces of the cube

I aimed for my PhD. I decided to contact the author, António Bandeira Araújo using the "Academical Facebook". In that moment, António was professor at Universidade Aberta of Lisbon in the PhD program Media-Arte Digitale[70] and since the first talks we pointed the idea of an international collaboration. With such a purpose, I spent my first month in Lisbon in July 2018 and I will never forget the first meeting with António in 1800's coffee (dramatized dialog):

(A) - So, what's that you want to do?
(L) – I want to solve the cubical perspective.
(A) – I see, yes sure, we can do it… and in the fifteen days left we can think ok something else to do.

With such a sentence, I thought: *here the answer why there is nothing about cubical perspective: it is banal to solve!* My intentions of a long-term collaboration to "decipher the big problem" collapsed in a moment. According to António's first impression of the problem, cubical perspective should have been solved very quick. But, for surprise of everyone, the solution was not easy at all: when we started to dive into the possible solutions, we found out a work that keeps giving research material until today. The collaboration went ahead and became a Sandwich PhD (I spent 12 months in Lisbon), António finished as international supervisor of this thesis. It took more than one year and a half to publish a first full method [232], working every day and just fuelled by *abatanados* and *paes-de-deus* (Portuguese coffee and pastries) at our favourite "office": the coffeeshop *1800* right next to our less used, official office, at Aberta University.

In parallel with the experience in Lisbon, I continued travelling, attending congresses and extending the network of collaborations. I also had, with the experience and knowledge of Adriana Rossi, the possibility to define this new system in the right terms aligned with the

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[70] António is the current coordinator of the *Digital Media Art* program, period 2020/21

scientific terms of ICAR/17; apply the cubical perspective to important architectural examples (such as the Solimene's Factory); and share the gifts of cubical perspective in *Technique Avanzate per la Rappresentazione*[71] at the Architecture Department, UNICAMPANIA. Regarding the later, the classroom has always been a way to test the few knowledge that I have accumulated during these years, representing a constant "forced but pleasant" update of my drawing skills and was motivation of two more articles [120], [144].

Within this PhD I am reaching my goal and even going further: I know now how to draw accurately, I can draw everything around me, and I'm being part of the creation of a new way to do it. But there is more to say and to do, so I anticipate my further developments in the field of digital art.

Since 2017, this team, Adriana, António, and me, have passed through many difficulties: bureaucracy, disciplinary dissimilarities, linguistic barriers, cultural misunderstandings and personal discrepancies. However, thanks to a constant will for work and research, we managed to succeed all these differences, getting adapted to our potentialities and consolidating a solid and international group of work. The cubical perspective saw the light of many articles and international congresses since 2017, being always well received, with great attention and interest from the graphic community's members, demonstrating to deserve its place in the field of the sciences of representation.

[71] Exam of *Design per l'Innovazione* career, academical years 2017-18, 2018-19 and 2019-20

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Many professors and colleagues marked me strongly during my career. Either finding stimulating their support or challenging their rejection, I got from the interaction with them the courage, will and strength to keep drawing and researching. Thanks to this encouragement I went deeper into the main trigger of this work: the handmade analogical drawing. All the universities that I passed through allowed me to do this, especially in this moment in which architecture representation is turning more and more into the real rendering modality.

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[72] Professor at *Introducción a las problemáticas del diseño y su expresión*, 2008, FAUD, UNC
[73] Professor at *Sistemas Gráficos de Expresión*, 2008, FAUD, UNC
[74] Professor and Full Professor (respectively) of *Arquitectura I*, 2008, FAUD, UNC
[75] Professor at *Morfología I*, 2008, FAUD, UNC

Martinez Paz[76], Liza Arriazu[77], Soledad Guerra[78], Marcos Barboza[79], Marcela Rucq[80], Luz Becaceci[81], Jorge Vidal and Lidia Samar[82], Francisco Solano Benitez and Ricardo Meyer[83], Jonny Gallardo[84], Sebastián Rosa[85], Jorge Ruiz[86], Pablo Oshiro[87], Vito Cardone (and all his team).

I thank to every colleague and friend, to all those persons that were with me during my career and life. There are countless names to give, so I will not even try to do it. Yet, all of you know how much I appreciate your friendship and company.

I will just remind Damaris Balland

and she knows why.

[76] Professor at *Historia II*, 2009, FAUD, UNC
[77] Professor at *Arquitectura II*, 2009, FAUD, UNC
[78] Professor at *Morfologia II*, 2009, FAUD, UNC
[79] Professor at *Morfología III*, 2010, FAUD, UNC
[80] Professor at workshop "*El discurso de la sombra. De la proyección al proyecto*", 2010, FAPyD, UNR
[81] Professor of *Arquitectura III*, 2010, FAUD, UNC
[82] Professor and Full Professor (respectively) of *Historia II*, 2012, FAUD, UNC
[83] Full professor of *Proyecto* and Dean (respectively), 2012, FADA, UNA, Paraguay
[84] Professor of *Equipamiento*, 2013, FAUD, UNC
[85] Professor of *Arquitectura Paisajísta*, 2013, FAUD, UNC
[86] Professor of *Urbanismo II*, 2013, FAUD, UNC
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